With
1 Model Paper &
2 Practice Sets

Chapterwise GATE Solved Papers (2012-2000)

Electrical Engineering

Manish Purbey

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PREFACE

GATE –Graduate Aptitude Test in Engineering – offers a good opportunity to the students who are willing to seek admission to post graduate courses and want to avail government scholarships for a further academic career in Engineering.

This Practice book for GATE has been designed to help students get away with their initial qualms and hiccups about the GATE. This book strictly adheres to the prescribed syllabus for GATE and contains **Chapterwise 13 years' (2012-2000)** Questions with their authentic solutions for Electrical Engineering. which make it a complete practice book. The exercises in the book will help students to find solutions to their problems and get through the exam without any difficulty. After going through the book they can climb the ladder of success and fulfill their dreams.

Manish Purbey

Contents

- **Solved Paper 2012** 1-24

1. Mathematical Engineering 1-18
2. Electric Circuit and Fields 19-62
3. Signals and Systems 63-76
4. Electrical Machines 77-114
5. Power System 115-146
6. Control Systems 147-176
7. Measurements and Instruments 177-194
8. Analog and Digital Electronics 195-245
9. Power Electronic 246-275
10. General Aptitude 276-292

- Crack Paper - 1 295-312
- Crack Paper - 2 313-320
- Crack Paper - 3 321-328

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GATE

About the Test & How to Succeed in It?

GATE - Graduate Aptitude Test in Engineering

GATE - Graduate Aptitude Test in Engineering - is an all India level examination, conducted and controlled by the Indian Institute of Science in cooperation with seven Indian Institutes of Technology on behalf of the National Coordination Board – GATE, Department of Higher Education, Ministry of Human Resource Development (MHRD), and Government of India. The GATE committee which consists of representatives from the governing institutes is the sole authority for conducting the examination and declaring the results.

Zones and Institutes for GATE

The GATE is conducted and controlled through eight zones are as follow :

Zone - 1 : IIS, Bangalore	***Zone - 2*** : IIT, Bombay
Zone - 3 : IIT, Delhi	***Zone - 4*** : IIT, Guwahati
Zone - 5 : IIT, Kanpur	***Zone - 6*** : IIT, Kharagpur
Zone - 7 : IIT, Madras	***Zone - 8*** : IIT, Roorkee

Eligibility Criteria for GATE

The following categories of candidates are eligible to appear in GATE.

1. Candidates with Bachelor Degree in Engineering/Technology/Architecture (4 years after 10+2) and those who are in the final or pre-final year of such programme
2. Candidates with Master Degree in any branch of Science/Mathematics/Statistics/Computer Applications or its equivalent and those who are in the final or pre-final year of such programme
3. Candidates in the second or higher year of the Four-year Integrated Master Degree Programme (Post B.Sc.) in Engineering/Technology or in the third, or higher year of Five-year Integrated Master Degree Programme and Dual Degree Programme in Engineering/ Technology
4. Candidates with qualifications obtained through examinations conducted by professional societies recognized by UPSC/AICTE (e.g. AMIE by IE (I), AMICE (I) by the Institute of Civil Engineers (India) - ICE (I)) as equivalent to B.E. / B. Tech. The students who have completed section A or equivalent of such professional courses are also eligible.

When & How to Apply

Students have to submit their applications complete in all respects usually by the end of October. However, they should keep an eye on the bulletin/news from the concerned authority from time to time.

Students can send their applications through online method or in an offline way.

Pattern of Examination

The GATE consists of a single paper of 3 hours' duration, which contains 65 questions carrying a maximum of 100 marks. The question paper is divided into three sections. Section A (Engineering Mathematics and Technical Subjects) consists of about 25 questions of ONE MARK each. Section B (Engineering Mathematics and Technical Subjects) consists of about 30 questions of TWO MARKS each. Section C (General Ability) consists of about 10 questions of about ONE and TWO MARKS each.

The question paper consists of only multiple choice questions. Each question has four choices for the answer. The candidate needs to mark the correct choice on an Optical Response Sheet (ORS) by darkening the appropriate bubble. Incorrect answers carry negative marks.

GATE Scorecard

GATE scorecard is valid for two years from the date of announcement of the GATE results. The GATE results consist of:

- GATE score
- Marks obtained by the candidate
- All India Rank (AIR)

GATE Score

The GATE score is calculated using a specific formula, where the marks obtained by the candidate, average of marks of all candidates in that paper, standard deviation of marks of all candidates in that paper, global average of marks of all candidates in the last 2 years and the global standard deviation of marks of all candidates in the last 2 years are considered.

The GATE score is set to zero if the value calculated by the formula is less than zero and the GATE score is set to 1000 if the value calculated by the formula is more than 1000.

Formula for GATE Score

GATE Score = $10\,[a_g + s_g\,\{(m-a)/s\}]$

where;

m = marks obtained by the candidate

a = average of marks of all candidates who appeared in the paper, mentioned on the scorecard in GATE for that year

s = standard deviation of marks of all candidates who appeared in the paper, mentioned on the scorecard in GATE for that year

a_g = global average of marks of all candidates who appeared across all papers in the last two years

s_g = global standard deviation of marks of all candidates who appeared across all papers in the last two years

Qualifying Score

Qualifying score is the minimum marks that are essential to become eligible for availing scholarship provided by MHRD. The Qualifying score for general category in each paper is $(a + s)$ or 25, whichever is higher; where a and s are the average and standard deviation of marks of all candidates in a particular paper, with all marks less than zero are converted to zero.

For example,

For qualifying ECE paper for GATE, the score to be obtained is given below.

General	OBC	SC/ST
26.11	23.5	17.41

Benefits of GATE

Candidates who qualify GATE are eligible to get admission to Postgraduate and Doctoral Programmes with MHRD and they can avail scholarships in engineering colleges and institutes also.

Recent Changes in GATE

- There is no separate IT paper now as the IT and CS papers have been merged from 2009.
- The GATE score was valid only for one year till 2009 but from 2010 onwards the GATE score will be valid for 2 years.
- Pharmacology has been removed from the subject list of the GATE from 2010. And it has been replaced with the GPAT. Also, the Biotechnology section of the XL GATE paper has been removed and is replaced with a separate biotechnology paper (paper code BT).

- An additional section of General Aptitude has been introduced in the GATE from 2010 onwards. A total of 10 questions carrying 15 marks make this section of GATE.
- From 2011 and onwards, four of the GATE papers can be attempted using computers through online mode. The four papers are Aerospace Engineering (paper code AE), Geology and Geophysics (paper code GG), Mining Engineering (paper code MN) and Textile Engineering and Fibre Science (paper code TF).
- From 2011 onwards, the GATE will be held in the morning (9 - 12) and afternoon (2 - 5) session. Also the computer based tests will be held on different dates.

Subject Codes for GATE

Subject	Code
Aeronautical/Aerospace Engineering	AE
Agricultural Engineering	AG
Architecture (B. Arch.)	AR
Automobile Engineering	AU
Biochemical Engineering	BI
Biomedical Engineering	BM
Biotechnology	BT
Civil Engineering	CE
Chemical Engineering	CH
Computer Science and Information Tech.	CS
Chemistry	CY
Electronics & Communication Engg.	EC
Electrical Engineering	EE

Subject	Code
Geology and Geo-physics	GG
Instrumentation Engineering	IN
Mathematics	MA
Mechanical Engineering	ME
Mining Engineering	MN
Metallurgical Engineering	MT
Physics	PH
Production and Industrial Engineering	MT
Textile Engineering and Fibre Science	TF
Engineering Science	XE
Life Science	XL
Pharmaceutical Science	PY

How to Crack GATE?

Make a target to get high percentile and all India rank in the GATE

To get a high score in the GATE doesn't mean that you are selected. In this exam candidates are selected on their relative scores. So, getting a score of 95% does not mean a percentile of 95. Here you complete with the best candidates in India. To get through the GATE you should know where you are among the top few candidates.

Follow standard books for GATE

Make an extensive search for standard books in the library and go for the best ones. Try to cover complete syllabus within the time you have (5 or 6 months). If it isn't possible, get expertise in the topics you have studied.

Apply group study method

Group study is one of the best ways for preparation of GATE. Assign a few sections/topics to your friend and you focus on the remaining. Then have a brief session and discussion and exchange what both of you have studied/gained. This not only saves your time and efforts but also enhances understanding on the topics/concepts.

Take actual practice tests

Taking actual practice tests is very important. When you begin to take the test, switch off your cell phone and have a timer which tells you the time of your test. You can take up a mock test series to experience the test in a classroom environment.

Analyze your practice test results

Analyzing the results of your practice tests is very significant. If you do not analyze, it does not add value to your performance. You should check and find out where you have mistaken and could have scored more. Know your accuracy rates in various topics and prepare a topicwise datasheet to make record of your performance in different test papers.

Solve previous years' GATE papers

Solve previous years' GATE papers to understand what the actual paper would be like. It also brushes up your mind and tells you the weaknesses in the subject knowledge. So try to solve as many test papers as you can. This is the best way to prepare and get through the GATE.

GATE Syllabus for Electrical Engineering

Engineering Mathematics

Linear Algebra Matrix Algebra, Systems of linear equations, Eigen values and eigen vectors.

Calculus Mean value theorems, Theorems of integral calculus, Evaluation of definite and improper integrals, Partial Derivatives, Maxima and minima, Multiple integrals, Fourier series. Vector identities, Directional derivatives, Line, Surface and Volume integrals, Stokes, Gauss and Green's theorems.

Differential Equations First order equation (linear and nonlinear), Higher order linear differential equations with constant coefficients, Method of variation of parameters, Cauchy's and Euler's equations, Initial and boundary value problems, Partial Differential Equations and variable separable method.

Complex Variables Analytic functions, Cauchy's integral theorem and integral formula, Taylor's and Laurent' series, Residue theorem, solution integrals.

Probability and Statistics Sampling theorems, Conditional probability, Mean, median, mode and standard deviation, Random variables, Discrete and continuous distributions, Poisson, Normal and Binomial distribution, Correlation and regression analysis.

Numerical Methods Solutions of non-linear algebraic equations, single and multi-step methods for differential equations.

Transform Theory Fourier transform, Laplace transform, Z-transform.

Electrical Engineering

Electric Circuits and Fields Network graph, KCL, KVL, node and mesh analysis, transient response of dc and ac networks; sinusoidal steady-state analysis, resonance, basic filter concepts; ideal current and voltage sources, Thevenin's, Norton's and Superposition and Maximum Power Transfer theorems, two-port networks, three phase circuits; Gauss Theorem, electric field and potential due to point, line, plane and spherical charge distributions; Ampere's and Biot-Savart's laws; inductance; dielectrics; capacitance.

Signals and Systems Representation of continuous and discrete-time signals; shifting and scaling operations; linear, time-invariant and causal systems; Fourier series representation of continuous periodic signals; sampling theorem; Fourier, Laplace and Z transforms.

Electrical Machines Single phase transformer - equivalent circuit, phasor diagram, tests, regulation and efficiency; three phase transformers - connections, parallel operation, [illegible] energy conversion principles; DC machines - [illegible] generator characteristics, armature reaction and [illegible] starting and speed control of motors; three phase induction [illegible] principles, types, performance characteristics, starting and spe[illegible] control; single phase induction motors; synchronous machine[illegible] performance, regulation and parallel operation of generators, m[illegible] starting, characteristics and applications; servo and stepper motors.

Power Systems Basic power generation concepts; transmission line models and performance; cable performance, insulation; corona and radio interference; distribution systems; per-unit quantities; bus impedance and admittance matrices; load flow; voltage control; power factor correction; economic operation; symmetrical components; fault analysis; principles of over-current, differential and distance protection; solid state relays and digital protection; circuit breakers; system stability concepts, swing curves and equal area criterion; HVDC transmission and FACTS concepts.

Control Systems Principles of feedback, transfer function, blok diagrams; steady-state errors; Routh and Niquist techniques; Bde plots; root loci; lag, lead and lead-lag compensation; state spce model; state transition matrix, controllability and observability.

Electrical and Electronic Measurements Bridges and potentiometers; PMMC, moving iron, dynamometer and induction type instruments; measurement of voltage, current, power, energy ad power factor; instrument transformers; digital voltmeters ad multimeters; phase, time and frequency measurement; Q-metrs; oscilloscopes; potentiometric recorders; error analysis.

Analog and Digital Electronics Characteristics of diodes, BJT, FET; amplifiers - biasing, equivalent circuit and frequency respose; oscillators and feedback amplifiers; operational amplifier - characteristics and applications; simple active filters; VCOs ad timers; combinational and sequential logic circuits; multiplexer; Schmitt trigger; multi-vibrators; sample and hold circuits; A/D and DA converters; 8-bit microprocessor basics, architecture, programmig and interfacing.

Power Electronics and Drives Semiconductor power diode, transistors, thyristors, triacs, GTOs, MOSFETs and IGBTs - static characteristics and principles of operation; triggering circuits; phase control rectifiers; bridge converters - fully controlled and half controlled; principles of choppers and inverters; basis concepts of adjustable speed dc and ac drives.

Solved Paper
2012

Electrical Engg G462(22.0 ,2012) Form 1-16 Front

Detailed Analysis
GATE 2012 Electrical Engineering

The Graduate Aptitude Test in Engineering (GATE-2012) was conducted successfully on 12 Feb, 2012 across India. The exam threw no surprises on the candidates and was in accordance with the pattern specified in the exam prospectus. This paper consists of 65 questions with a maximum score of 100 marks. The questionwise breakup of the paper was as follows:

- Engineering Mathematics—16 marks
- General Aptitude (Verbal Ability & Numerical Ability)—15 marks
- Core Engineering Subjects—69 marks

This year, the cut-offs are expected to rise due to an increase in the number of aspirants, as well as the lower level of examination.

The paper was tougher than 2011. There was a stronger emphasis on **Electrical & Electronic Measurements, Power Electronics & Drives and Electric Circuits & Fields.** The questions from **Electrical & Electronic Measurements, Electrical Machine and Analog & Digital Electronics** were relatively easy. The questions from **Power Electronics & Drives and Control System** were of medium difficulty level. The Questions from **Power System and Electric Circuits & Fields** were of higher difficulty level & were a surprise for the test takers.

Subjectwise Analysis

Subjects	No. of 1 Mark Questions	No. of 2 Marks Questions
Engineering Mathematics		
Linear Algebra	0	1
Calculus	0	2
Differential Equations	1	0
Complex Variables	2	0
Probability & Statistics	1	1
Numerical Methods	0	1
Transform Theory	1	0
Total	**5**	**10**
General Aptitude		
Verbal Ability	4	4
Numerical Ability	1	1
Total	**5**	**10**
Electronics & Communication Engineering		
Electric Circuits & Fields	5	6
Signals & Systems	1	1
Electrical Machines	1	3
Power Systems	3	2
Control Systems	1	5
Electrical & Electronic Measurement	3	1
Analog & Digital Electronics	4	4
Power Electronics & Drives		3
Total	**20**	**50**

Marks Distribution as per Difficulty Level

Subjects / Topics	Easy	Moderate	Difficult	Total Marks
Engineering Mathematics				
Linear Algebra	0	2	0	2
Calculus	2	0	2	4
Differential Equations	0	1	0	1
Complex Variables	0	1	1	2
Probability & Statistics	2	1	0	3
Numerical Methods	0	0	2	2
Transform Theory	1	0	0	1
Total	**5**	**5**	**5**	**15**
General Aptitude				
Verbal Ability	1	4	1	6
Numerical Ability	4	3	2	9
Total	**5**	**7**	**3**	**15**
Electrical Engineering				
Electric Circuits & Fields	0	7	10	17
Signals & Systems	0	2	1	3
Electrical Machines	0	1	6	7
Power Systems	0	2	5	7
Control Systems	0	8	3	11
Electrical & Electronic Measurement	1	4	0	5
Analog & Digital Electronics	0	10	2	12
Power Electronics & Drives	0	8	0	8
Total	**1**	**42**	**27**	**70**

GATE

Electrical Engineering

Solved Paper 2012

Time : 3 hrs] **[MM : 100**

Read the following instructions carefully.

1. All questions in this paper are of objective type.
2. There is a total of 65 questions carrying 100 marks.
3. Questions 1 to 25 will carry 1 mark each and questions 26 to 55 will carry 2 marks each.
4. Questions 48 to 51 (2 pairs) are common data questions and questions pairs 52 and 53 and 54 and 55 are linked answer questions. The answer to the second question of the linked answer questions depends on the answer to the first question of the pair. If the first question in the linked pair is wrongly answered or is unattempted, then the answer to the second question in the pair will not be evaluated.
5. Questions 56 to 65 belong to General Aptitude (GA). Questions 56 to 60 will carry 1 mark each and questions 61 to 65 will carry 2 marks each.
6. Unattempted questions will carry zero marks.
7. Wrong answers will carry negatie marks. For questions 1 to 25 and 56 to 60, $\frac{1}{3}$ mark will be deducted for each wrong answer. For questions 26 to 51 and 61 to 65, $\frac{2}{3}$ mark will be deducted for each wrong answer. The question pairs (questions 52 and 53) and (questions 54 and 55) are questions with linked answer. There will be negative marks only for wrong answer to the first question of the linked answer question pair *i.e.,* for questions 52 and 54, $\frac{2}{3}$ mark will be deducted for each wrong answer. There is no negative marking for questions 53 and 55.

Engineering Mathematics

1. Two independent random variables X and Y are uniformly distributed in the interval $[-1, 1]$. The probability that max $[X, Y]$ is less than 1/2, is **[1 Mark]**

(a) 3/4 (b) 9/16 (c) 1/4 (d) 2/3

2. If $x = \sqrt{-1}$, then the value of x^x is **[1 Mark]**

(a) $e^{-\pi/2}$ (b) $e^{\pi/2}$ (c) x (d) 1

3. Given $f(z) = \frac{1}{z+1} - \frac{2}{z+3}$. If C is a counterclockwise path in the Z-plane such that $|z+1| = 1$, the value of $\frac{1}{2\pi j}\oint_C f(z)\,dz$ is **[1 Mark]**

(a) -2 (b) -1 (c) 1 (d) 2

4. With initial condition $x(1) = 0.5$, the solution of the differential equation $t\frac{dx}{dt} + x = t$ is **[1 Mark]**

(a) $x = t - \frac{1}{2}$ (b) $x = t^2 - \frac{1}{2}$

(c) $x = \frac{t^2}{2}$ (d) $x = \frac{t}{2}$

5. The unilateral Laplace transform of $f(t)$ is $\frac{1}{s^2+s+1}$. The unilateral Laplace transform of $t\,f(t)$ is **[1 Mark]**

(a) $-\frac{s}{(s^2+s+1)^2}$ (b) $-\frac{2s+1}{(s^2+s+1)^2}$

(c) $\frac{s}{(s^2+s+1)^2}$ (d) $\frac{2s+1}{(s^2+s+1)^2}$

6. Given that $A = \begin{bmatrix} -5 & -3 \\ 2 & 0 \end{bmatrix}$ and $I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$, the value of A^3 is **[2 Marks]**

(a) $15A + 12I$ (b) $19A + 30I$
(c) $17A + 15I$ (d) $17A + 21I$

7. The maximum value of $f(x) = x^3 - 9x^2 + 24x + 5$ in the interval [1, 6] is **[2 Marks]**

(a) 21 (b) 25 (c) 41 (d) 46

8. A fair coin is tossed till a head appears for the first time. The probability that the number of required tosses is odd, is **[2 Marks]**

(a) $\frac{1}{3}$ (b) $\frac{1}{2}$ (c) $\frac{2}{3}$ (d) $\frac{3}{4}$

9. The direction of vector $\vec{A}$ is radially outward from the origin, with $|\vec{A}| = kr^n$, where $r^2 = x^2 + y^2 + z^2$ and k is a constant. The value of n for which $\nabla \cdot \vec{A} = 0$ is **[2 Marks]**

(a) -2 (b) 2
(c) 1 (d) zero

10. Consider the differential equation:

$$\frac{d^2y(t)}{dt^2} + 2\frac{dy(t)}{dt} + y(t) = \delta(t)$$

with $y(t)\big|_{t=0^-} = -2$

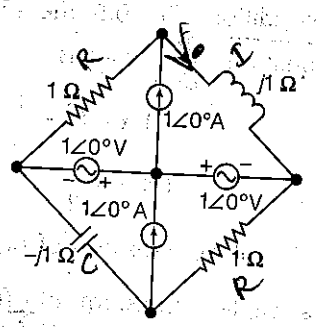
and $\left.\frac{dy}{dt}\right|_{t=0^-} = 0$

The numerical value of $\left.\frac{dy}{dy}\right|_{t=0^+}$ is **[2 Marks]**

(a) -2 (b) -1 (c) zero (d) 1

Network Theory

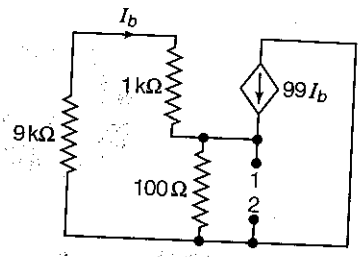
11. In the circuit shown below, the current through the inductor is



[1 Mark]

(a) $\frac{2}{1+j}$ A (b) $\frac{-1}{1+j}$ A
(c) $\frac{1}{1+j}$ A (d) zero

12. The impedance looking into nodes 1 and 2 in the given circuit is



[1 Mark]

(a) 50 Ω (b) 100 Ω (c) 5 kΩ (d) 10.1 Ω

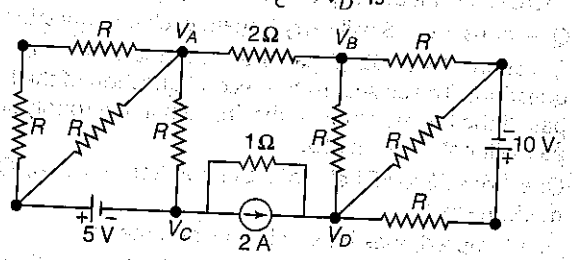
13. A two-phase load draws the following phase currents $I_1(t) = I_m \sin(\omega t - \phi_1)$, $I_2(t) = I_m \cos(\omega t - \phi_2)$. These currents are balanced if ϕ_1 is equal to **[1 Mark]**

(a) $-\phi_2$ (b) ϕ_2
(c) $(\pi/2 - \phi_2)$ (d) $(\pi/2 + \phi_2)$

14. The average power delivered to an impedance $(4 - j3)$ Ω by a current $5\cos(100\pi t + 100)$ A is **[1 Mark]**

(a) 44.2 W (b) 50 W (c) 62.5 W (d) 125 W

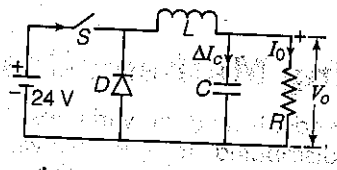
15. If $V_A - V_B = 6$ V, then $V_C - V_D$ is



[2 Marks]

(a) -5 V (b) 2 V (c) 3 V (d) 6 V

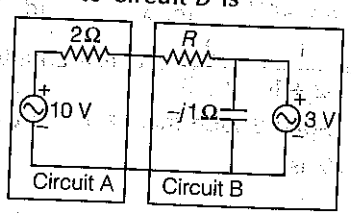
16. In the circuit shown, an ideal switch S is operated at 100 kHz with a duty ratio of 50%. Given that ΔI_c is 1.6 A peak-to-peak and I_0 is 5A DC, the peak current in S is



[2 Marks]

(a) 6.6 A (b) 5.0 A (c) 5.8 A (d) 4.2 A

17. Assuming both the voltage sources are in phase, the value of R for which maximum power is transferred from circuit A to circuit B is



[2 Marks]

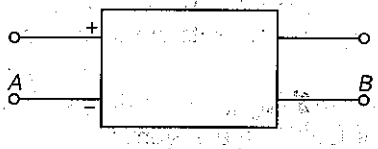
(a) 0.8 Ω (b) 1.4 Ω (c) 2 Ω (d) 2.8 Ω

Common Data for Questions 18 and 19

With 10 V DC connected at port A in the linear non-reciprocal two-port network shown below, the following were observed :

(i) 1 Ω connected at port B draws a current of 3 A

(ii) 2.5 Ω connected at port B draws a current of 2 A



18. For the same network, with 6 V DC connected at port A, 1 Ω connected at port B draws 7 A. If 8 V DC is connected to port A, the open circuit voltage at port B is **[2 Marks]**

(a) 6 V (b) 7 V

(c) 8 V (d) 9 V

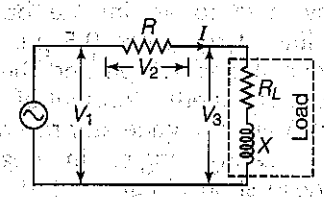
19. With 10 V DC connected at port A, the current drawn by 7 Ω connected at port B is **[2 Marks]**

(a) 3/7 A (b) 5/7 A

(c) 1 A (d) 9/7 A

Statement for Linked Answer Questions 20 and 21

In the circuit shown, the three voltmeter readings are $V_1 = 220\,V$, $V_2 = 122\,V$, $V_3 = 136\,V$.



20. The power factor of the load is **[2 Marks]**

(a) 0.45 (b) 0.50

(c) 0.55 (d) 0.60

21. If $R_L = 5\,\Omega$, the approximate power consumption in the load is **[2 Marks]**

(a) 700 W (b) 750 W (c) 800 W (d) 850 W

Signals and Systems

22. If $x[n] = (1/3)^{|n|} - (1/2)^n u[n]$, then the region of convergence (ROC) of its Z-transform in the Z-plane will be **[1 Mark]**

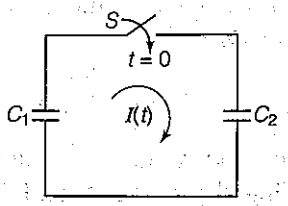
(a) $\frac{1}{3} < |z| < 3$

(b) $\frac{1}{3} < |z| < \frac{1}{2}$

(c) $\frac{1}{2} < |z| < 3$

(d) $\frac{1}{3} < |z|$

23. In the following figure, C_1 and C_2 are ideal capacitors. C_1 has been charged to 12 V before the ideal switch S is closed at $t = 0$. The current $I(t)$ for all t is



[1 Mark]

(a) zero

(b) a step function

(c) an exponentially decaying function

(d) an impulse function

24. Let $y[n]$ denotes the convolution of $h[n]$ and $g[n]$, where $h[n] = (1/2)^n u[n]$ and $g[n]$ is a causal sequence. If $y[0] = 1$ and $y[1] = 1/2$, then $g[1]$ equals **[2 Marks]**

(a) zero (b) $\frac{1}{2}$

(c) 1 (d) 3/2

25. The Fourier transform of a signal $h(t)$ is $H(j\omega) = (2\cos\omega)(\sin 2\omega)/\omega$. The value of $h(0)$ is **[2 Marks]**

(a) 1/4 (b) $\frac{1}{2}$ (c) 1 (d) 2

26. The input $x(t)$ and output $y(t)$ of a system are related as $y(t) = \int_{-\infty}^{t} x(\tau)\cos(3\tau)\,d\tau$. The system is **[2 Marks]**

(a) time invariant and stable

(b) stable and not time invariant

(c) time invariant and not stable

(d) not time invariant and not stable

Electrical Machines

27. The slip of an induction motor normally does not depend on **[1 Mark]**

(a) rotor speed (b) synchronous speed

(c) shaft torque (d) core-loss component

28. A 220 V, 15 kW, 1000 rpm shunt motor with armature resistance of 0.25 Ω, has a rated line current of 68 A and a rated field current of 2.2 A. The change in field flux required to obtain a speed of 1600 rpm while drawing a line current of 52.8 A and a field current of 1.8 A, is **[2 Marks]**

(a) 18.18% increase

(b) 18.18% decrease

(c) 36.36% increase

(d) 36.36% decrease

29. The locked rotor current in a 3-phase, star connected 15 kW, 4-pole, 230 V, 50 Hz induction motor at rated conditions is 50 A. Neglecting losses and magnetizing current, the approximate locked rotor line current drawn when the motor is connected to a 236 V, 57 Hz supply, is **[2 Marks]**

(a) 58.5 A (b) 45.0 A
(c) 42.7 A (d) 55.6 A

30. A single phase 10 kVA, 50 Hz transformer with 1 kV primary winding draws 0.5 A and 55 W, at rated voltage and frequency, on no load. A second transformer has a core with all its linear dimension $\sqrt{2}$ times the corresponding dimensions of the first transformer. The core material and lamination thickness are the same in both transformers. The primary windings of both the transformers have the same number of turns. If a rated voltage of 2 kV at 50 Hz is applied to the primary of the second transformer, then the no load current and power respectively, are **[2 Marks]**

(a) 0.7 A, 77.8 W
(b) 0.7 A, 155.6 W
(c) 1 A, 110 W
(d) 1 A, 220 W

Power System

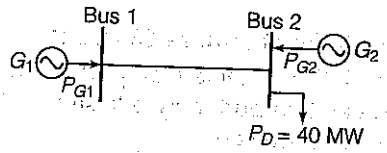
31. The bus admittance matrix of a three-bus three-line system is

$$Y = j\begin{bmatrix} -13 & 10 & 5 \\ 10 & -18 & 10 \\ 5 & 10 & -13 \end{bmatrix}$$

If each transmission line between the two buses is represented by an equivalent π-network, the magnitude of the shunt susceptance of the line connecting buses 1 and 2 is **[1 Mark]**

(a) 4 (b) 2
(c) 1 (d) zero

32. The given figure shows a two-generator system supplying a load of $P_D = 40$ MW, connected at bus 2.



The fuel cost of generators G_1 and G_2 are

$C_1(P_{G1}) = 10000$ ₹/MW-h

and $C_2(P_{G2}) = 12500$ ₹/MW-h

and the loss in the line is $P_{loss\,(pu)} = 0.5\,P_{G1\,(pu)}^2$, where the loss coefficient is specified in pu on a 100 MVA base. The most economic power generation schedule in MW is **[1 Mark]**

(a) $P_{G1} = 20, P_{G2} = 22$ (b) $P_{G1} = 22, P_{G2} = 20$
(c) $P_{G1} = 20, P_{G2} = 20$ (d) $P_{G1} = 0, P_{G2} = 40$

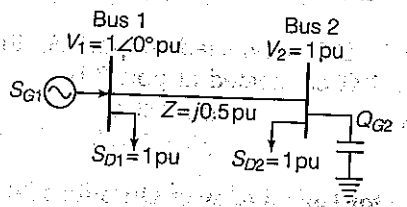
33. The sequence components of the fault current are as follows :

$I_{positive} = j1.5$ pu, $I_{negative} = -j\,0.5$ pu, $I_{zero} = -j1$ pu.

The type of fault in the system is **[1 Mark]**

(a) LG (b) LL (c) LLG (d) LLLG

34. For the system shown below, S_{D1} and S_{D2} are complex power demands at bus 1 and bus 2 respectively. If $|V_2| = 1$ pu, the VAR rating of the capacitor (Q_{C2}) connected at bus 2 is



[2 Marks]

(a) 0.2 pu (b) 0.268 pu (c) 0.312 pu (d) 0.4 pu

35. A cylindrical rotor generator delivers 0.5 pu power in the steady state to an infinite bus through a transmission line of reactance 0.5 pu. The no load voltage of generator is 1.5 pu and the infinite bus voltage is 1 pu. The inertia constant of the generator is 5 MW-s/MVA and the generator reactance is 1 pu. The critical clearing angle, in degrees, for a three-phase dead short circuit fault at the generator terminal is **[2 Marks]**

(a) 53.5 (b) 60.2 (c) 70.8 (d) 79.6

Control System

36. A system with transfer function

$$G(s) = \frac{(s^2 + 9)(s + 2)}{(s + 1)(s + 3)(s + 4)}$$

is excited by $\sin(\omega t)$. The steady-state output of the system is zero at **[1 Mark]**

(a) $\omega = 1$ rad/s (b) $\omega = 2$ rad/s
(c) $\omega = 3$ rad/s (d) $\omega = 4$ rad/s

37. The state variable description of an LTI system is given by

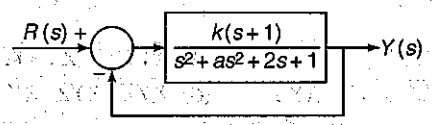
$$\begin{pmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{pmatrix} = \begin{pmatrix} 0 & a_1 & 0 \\ 0 & 0 & a_2 \\ a_3 & 0 & 0 \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} u$$

$$y = (1 \quad 0 \quad 0)\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix}$$

where, y is the output and u is the input. The system is controllable for **[2 Marks]**

(a) $a_1 \neq 0,\ a_2 = 0,\ a_3 \neq 0$
(b) $a_1 = 0,\ a_2 \neq 0,\ a_3 \neq 0$
(c) $a_1 = 0,\ a_2 \neq 0,\ a_3 = 0$
(d) $a_1 \neq 0,\ a_2 \neq 0,\ a_3 = 0$

38. The feedback system shown below oscillates at 2 rad/s when



[2 Marks]

(a) $k = 2$ and $a = 0.75$ (b) $k = 3$ and $a = 0.75$
(c) $k = 4$ and $a = 0.5$ (d) $k = 2$ and $a = 0.5$

Statement for Linked Answer Questions 39 and 40

The transfer function of a compensator is given as

$$G_c(s) = \frac{s+a}{s+b}$$

39. $G_c(s)$ is a lead compensator if **[2 Marks]**

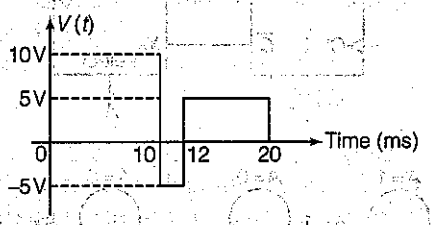
(a) $a = 1, b = 2$ (b) $a = 3, b = 2$
(c) $a = -3, b = -1$ (d) $a = 3, b = 1$

40. The phase of the above lead compensator is maximum at **[2 Marks]**

(a) $\sqrt{2}$ rad/s (b) $\sqrt{3}$ rad/s
(c) $\sqrt{6}$ rad/s (d) $1/\sqrt{3}$ rad/s

Electrical and Electronic Measurement

41. A periodic voltage waveform observed on an oscilloscope across a load is shown. A Permanent Magnet Moving Coil (PMMC) meter connected across the same load reads



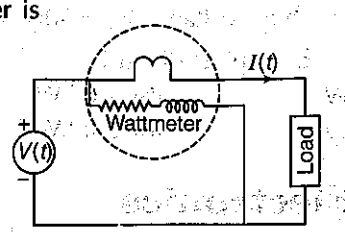
[1 Mark]

(a) 4 V (b) 5 V
(c) 8 V (d) 10 V

42. The bridge method commonly used for finding mutual inductance is **[1 Mark]**

(a) Heaviside Campbell bridge
(b) Schering bridge
(c) De Sauty bridge
(d) Wien bridge

43. For the circuit shown in the figure, the voltage and current expressions are $V(t) = E_1 \sin(\omega t) + E_3 \sin(3\omega t)$ and $I(t) = I_1 \sin(\omega t - \phi_1) + I_3 \sin(3\omega t - \phi_3) + I_5 \sin(5\omega t)$ The average power measured by the Wattmeter is



[1 Mark]

(a) $\frac{1}{2} E_1 I_1 \cos\phi_1$

(b) $\frac{1}{2} [E_1 I_1 \cos\phi_1 + E_1 I_3 \cos\phi_3 + E_1 I_5]$

(c) $\frac{1}{2} [E_1 I_1 \cos\phi_1 + E_3 I_3 \cos\phi_3]$

(d) $\frac{1}{2} [E_1 I_1 \cos\phi_1 + E_3 I_1 \cos\phi_1]$

44. An analog voltmeter uses external multiplier settings. With a multiplier setting of 20 kΩ, it reads 440 V and with a multiplier setting of 80 kΩ, it reads 352 V. For a multiplier setting of 40 kΩ, the voltmeter reads **[2 Marks]**

(a) 371 V (b) 383 V (c) 394 V (d) 406 V

Power Electronics and Drives

45. A half-controlled single-phase bridge rectifier is supplying an RL load. It is operated at a firing angle α and the load current is continuous. The fraction of cycle that the freewheeling diode conducts, is **[1 Mark]**

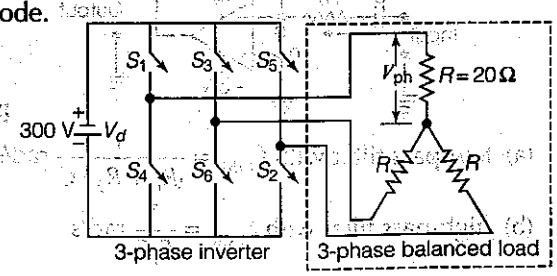
(a) $\frac{1}{2}$ (b) $\left(1 - \frac{\alpha}{\pi}\right)$
(c) $\frac{\alpha}{2\pi}$ (d) $\frac{\alpha}{\pi}$

46. The typical ratio of latching current to holding current in a 20 A thyristor is **[1 Mark]**

(a) 5.0 (b) 2.0 (c) 1.0 (d) 0.5

Common Data for Questions 47 and 48

In the 3-phase inverter circuit shown, the load is balanced and the gating scheme is 180° conduction mode.



47. The rms value of load phase voltage is **[2 Marks]**
(a) 106.1 V (b) 141.1 V
(c) 212.2 V (d) 282.8 V

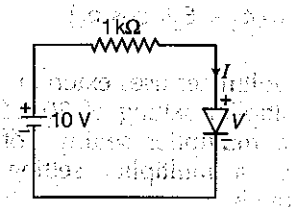
48. If the DC bus voltage $V_d = 300$ V, the power consumed by 3-phase load is **[2 Marks]**
(a) 1.5 kW (b) 2.0 kW
(c) 2.5 kW (d) 3.0 kW

Analog Electronics

49. The I-V characteristics of the diode in the circuit given below are

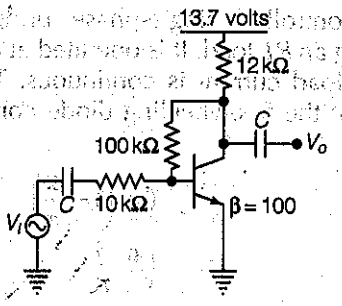
$$I = \begin{cases} \dfrac{V - 0.7}{500}\text{ A}, & V \geq 0.7\text{ V} \\ 0\text{ A}, & V < 0.7\text{ V} \end{cases}$$

The current in the circuit is **[1 Mark]**



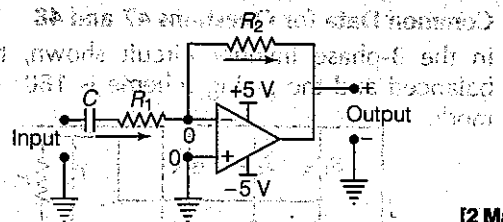
(a) 10 mA (b) 9.3 mA
(c) 6.67 mA (d) 6.2 mA

50. The voltage gain A_V of the circuit shown below is



[2 Marks]
(a) $|A_V| \approx 200$ (b) $|A_V| \approx 100$
(c) $|A_V| \approx 20$ (d) $|A_V| \approx 10$

51. The circuit shown is a



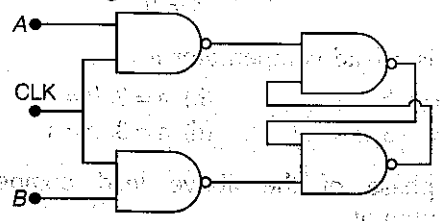
[2 Marks]
(a) low-pass filter with $f_{3\,dB} = \dfrac{1}{(R_1 + R_2)C}$ rad/s
(b) high-pass filter with $f_{3\,dB} = \dfrac{1}{R_1 C}$ rad/s
(c) low-pass filter with $f_{3dB} = \dfrac{1}{R_1 C}$ rad/s
(d) high-pass filter with $f_{3\,dB} = \dfrac{1}{(R_1 + R_2)C}$ rad/s

Digital Electronics

52. In the sum of products function $f(X, Y, Z) = \Sigma(2, 3, 4, 5)$, the prime implicants are **[1 Mark]**
(a) $\bar{X}Y, X\bar{Y}$ (b) $\bar{X}Y, X\bar{Y}\,\bar{Z}, X\bar{Y}Z$
(c) $\bar{X}Y\bar{Z}, \bar{X}\,YZ, X\bar{Y}$ (d) $\bar{X}Y\bar{Z}, \bar{X}YZ, X\bar{Y}\,\bar{Z}, X\bar{Y}Z$

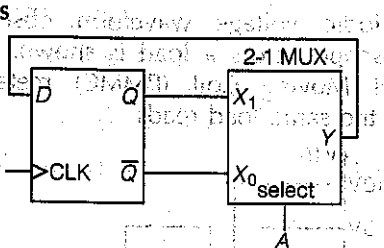
53. The output Y of a 2 bit comparator is logic 1 whenever the 2 bit input A is greater than the 2-bit input B. The number of combinations for which the output is logic 1, is **[1 Mark]**
(a) 4 (b) 6
(c) 8 (d) 10

54. Consider the given circuit :

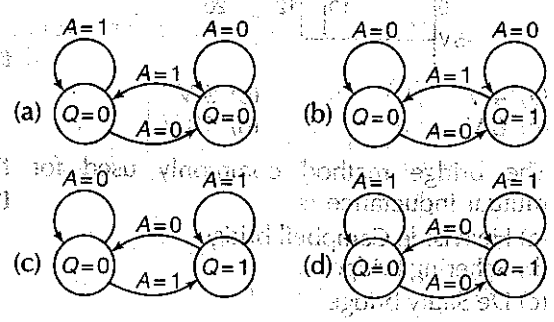


In this circuit, the race around **[1 Mark]**
(a) does not occur
(b) occurs when CLK = 0
(c) occurs when CLK = 1 and $A = B = 1$
(d) occurs when CLK = 1 and $A = B = 0$

55. The state transition diagram for the logic circuit shown is



[2 Marks]



General Aptitude

56. One of the parts (*A, B, C, D*) in the sentence given below contains an error, which one of the following is incorrect?
I requests that he should be given the driving test today instead of tomorrow. **[1 Mark]**
(a) requested that (b) should be given
(c) the driving test (d) instead of tomorrow

57. If $(1.001)^{1259} = 3.52$ and $(1.001)^{2062} = 7.85$, then $(1.001)^{3321}$ equals **[1 Mark]**
(a) 2.23 (b) 4.33
(c) 11.37 (d) 27.64

58. Choose the most appropriate alternative from the options given below to complete the following sentence :
If the tired soldier wanted to lie down, he ..... the mattress out on the balcony. **[1 Mark]**
(a) should take (b) shall take
(c) should have taken (d) will have taken

59. Choose the most appropriate word from the options given below to complete the following sentence :
Given, the seriousness of the situation that he had to face, his ..... was impressive. **[1 Mark]**
(a) beggary (b) nomenclature
(c) jealousy (d) nonchalance

60. Which one of the following options is the closest in meaning to that word given below? **[1 Mark]**
Latitude
(a) Eligibility (b) Freedom
(c) Coercion (d) Meticulousness

61. *A* and *B* are friends. They decide to meet between 1 pm and 2 pm on a given day. There is a condition that whoever arrives first will not wait for the other for more than 15 min. The probability that they will meet on that day is **[2 Marks]**
(a) 1/4 (b) 1/16
(c) 7/16 (d) 9/16

62. One of the legacies of the Roman legions was discipline. In the legions, military law prevailed and discipline was brutal. Discipline on the battlefield kept units obedient, intact and fighting, even when the odds and conditions were against them.
Which one of the following statements best sums up the meaning of the above passage? **[2 Marks]**
(a) Through regimentation was the main reason for the efficiency of the Roman legions even in adverse circumstances
(b) The legions were treated inhumanly as if the men were animals
(c) Discipline was the armies inheritance from their seniors
(d) The harsh discipline to which the legions were subjected to led to the odds and conditions being against them

63. Raju has 14 currency notes in his pocket consisting of only ₹ 20 notes and ₹ 10 notes. The total money value of the notes is ₹ 230. The number of ₹ 10 notes that Raju has, is **[2 Marks]**
(a) 5 (b) 6
(c) 9 (d) 10

64. There are eight bags of rice looking alike, seven of which have equal weight and one is slightly heavier. The weighing balance is of unlimited capacity. Using this balance, the minimum number of weighings required to identify the heavier bag is **[2 Marks]**
(a) 2 (b) 3
(c) 4 (d) 8

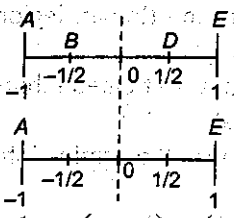
65. The data given in the following table summarizes the monthly budget of an average household.

Category	Amount (in ₹)
Food	4000
Clothing	1200
Rent	2000
Savings	1500
Other expenses	1800

The approximate percentage of the monthly budget not spent on savings is **[2 Marks]**
(a) 10% (b) 14%
(c) 81% (d) 86%

Answers with Explanations

1. (b) X and Y are two independent variables distributed in $[-1, 1]$ as shown in figure.



$$P(X,Y) < \frac{1}{2} = P\left(X < \frac{1}{2}\right) \cdot P\left(Y < \frac{1}{2}\right)$$

But $$P\left(X < \frac{1}{2}\right) = \frac{3}{4}$$

$$P\left(Y < \frac{1}{2}\right) = \frac{3}{4}$$

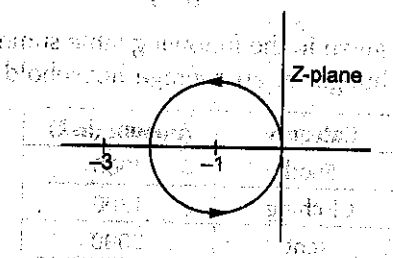
$$P(X,Y) < \frac{1}{2} = \frac{9}{16}$$

2. (a) $$x = \sqrt{-1} = i$$

$$= \cos\frac{\pi}{2} + i\sin\frac{\pi}{2} = e^{\left(i\frac{\pi}{2}\right)}$$

$$\therefore \quad x^x = e^{\left(i\frac{\pi}{2}\right)i} = e^{i^2\frac{\pi}{2}} = e^{-\frac{\pi}{2}}$$

3. (c) $$f(z) = \frac{1}{z+1} - \frac{2}{z+3}$$



$z = -1, -3$ are singular points but only $z = -1$ falls within the contour

$$\text{Res}(-1) = 2\pi j \ [\text{Res} f(-1)] = 2\pi j$$

$$\therefore \quad \frac{1}{2\pi j}\oint_C f(z)dz = 1$$

4. (d) $$t\frac{dx}{dt} + x = t$$

or $$\frac{dx}{dt} + \frac{x}{t} = 1$$

or $$\frac{dx}{dt} + Px = Q$$

where, P and Q are functions of t.

$$\text{IF} = e^{\int P\,dt}$$

$$= e^{\int \frac{1}{t}dt} = e^{\log_e t}$$

$$\text{IF} = t$$

Solution can be given by this equation

$$x \cdot \text{IF} = \int Q\,\text{IF} + C$$

$$xt = \int 1\,t + C$$

$$xt = \frac{t^2}{2} + C$$

Using initial condition, $t = 1$ and $x = 0.5$

$$0.5 = \frac{1}{2} + C$$

$$C = 0$$

$$\therefore \quad x = \left(\frac{t^2/2}{t}\right)$$

$$x = \frac{t}{2}$$

5. (d) $$L\{f(t)\} = \frac{1}{s^2 + s + 1}$$

From differential property of Laplace transform

$$L\{tf(t)\} = -F'(s)$$

$$= -\frac{d}{ds}\left[\frac{1}{s^2 + s + 1}\right]$$

$$= \frac{-(s^2 + s + 1)\cdot 0 - 1(2s+1)}{(s^2 + s + 1)^2}$$

$$L\{tf(t)\} = \frac{2s+1}{(s^2 + s + 1)^2}$$

6. (b) $$A = \begin{bmatrix} -5 & -3 \\ 2 & 0 \end{bmatrix}$$

Characteristic equation $|\lambda I - A| = 0$

$$\begin{vmatrix} \lambda + 5 & 3 \\ -2 & \lambda \end{vmatrix} = 0$$

$$\Rightarrow \quad \lambda^2 + 5\lambda + 6 = 0$$

From, Caylay-Hamilton theorem, A satisfies its own characteristic equation.

$$\therefore \quad A^2 + 5A + 6 = 0$$

$$\Rightarrow \quad A^2 = -(5A + 6I)$$

Now, $A^3 = A^2 \cdot A = -(5A + 6I) \cdot A$

$$= -5A^2 - 6AI$$

$$= -5[-(5A + 6I)] - 6A$$

$$= 25A + 30I - 6A = 19A + 30I$$

7. (b) $f(x) = x^3 - 9x^2 + 24x + 5$

$f'(x) = 3x^2 - 18x + 24$

For extrema, $f'(x) = 0 \Rightarrow x^2 - 6x + 8 = 0$

$\Rightarrow (x-2)(x-4) = 0$

$\Rightarrow x = 2, 4$

$f''(x) = 6x - 18,$

$\Rightarrow$ -6 for $x = 2 \rightarrow$ maximum

$= 6$ for $x = 4 \rightarrow$ not maximum

Thus, maximum value in [1, 6] is

$f(x)|_{x=2} = 2^3 - (9 \cdot 2^2) + 24 \cdot 2 + 5 = 25$

8. (c) The 'favourable outcome' *i.e.*, head appears for first time often even number of trials can be accomplished as follows :

$\{H\}, \{T, T, H\}, \{T, T, T, T, H\}, \ldots$

Thus, the probability of the favourable event is obtained as

$$\left(\frac{1}{2}\right) + \left(\frac{1}{2}\cdot\frac{1}{2}\cdot\frac{1}{2}\right) + \left(\frac{1}{2}\cdot\frac{1}{2}\cdot\frac{1}{2}\cdot\frac{1}{2}\cdot\frac{1}{2}\right) + \ldots$$

$$= \frac{1}{2} + \left(\frac{1}{2}\right)^3 + \left(\frac{1}{2}\right)^5 + \ldots$$

$$= \frac{1/2}{1-(1/2)^2} = \frac{1/2}{1-1/4} = \frac{1/2}{3/4} = \frac{2}{3}$$

[$\because$ From expression for sum of infinite GP

$$a + ar + ar^2 + \ldots = \frac{a}{1-r};\ |r| \le 1]$$

9. (a) $|A| = Kr^n$; K is constant

In spherical coordinate,

$$\nabla \cdot \vec{\mathbf{A}} = \frac{1}{r^2}\frac{\partial}{\partial r}(r^2 A_r) + \frac{1}{r\sin\theta}\frac{\partial}{\partial\theta}\sin\theta\, A_\theta + \frac{1}{r\sin\theta}\frac{\partial A_\phi}{\partial\phi}$$

Since, the direction of vector is radially outward.

Hence, the vector $\vec{\mathbf{A}}$ is the function of r only.

$$\nabla \cdot \vec{\mathbf{A}} = \frac{1}{r^2}\frac{\partial}{\partial r}(r^2 A_r);$$

$$\nabla \cdot \vec{\mathbf{A}} = \frac{1}{r^2}\cdot\frac{\partial}{\partial r}(r^2 A_r) = \frac{1}{r^2}\frac{\partial}{\partial r}(K r^{n+2})$$

$$= \frac{K}{r^2}(n+2)\, r^{n+1}$$

$$= K(n+2)\cdot r^{n-1}$$

$\therefore$ For $\nabla \cdot \vec{\mathbf{A}} = 0$ $\quad (n+2) = 0 \Rightarrow n = -2$

10. (d) Taking Laplace,

$$[s^2Y(s) - sY(0) - Y'(0)] + 2[sY(s) - Y(0)] + Y(s) = 1 \quad \ldots (i)$$

$Y(0) = -2, Y'(0) = 0$

$\therefore$ Putting the values in Eq. (i),

$$s^2Y(s) + 2s + 2sY(s) + 4 + Y(s) = 1$$

$$\Rightarrow Y(s) = \frac{-3-2s}{(s+1)^2} = -\frac{2}{s+1} - \frac{1}{(s+1)^2}$$

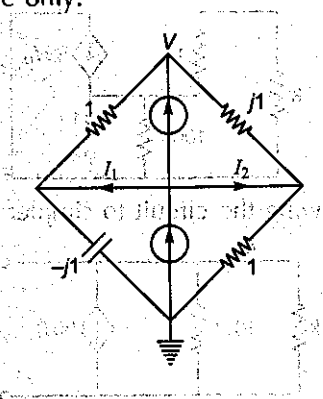
$\therefore$ Taking inverse Laplace,

$$y(t) = -2e^t - te^{-t}$$

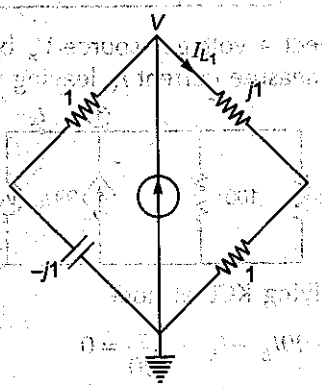
$$\therefore \quad \frac{dy(t)}{dt} = e^{-t} + te^{-t}$$

$$\therefore \quad \left.\frac{dy}{dt}\right|_{t=0^+} = 1$$

11. (c) Apply superposition theorem, taking current source only.



$I_1 = I_2 = 0$



Applying KCL,

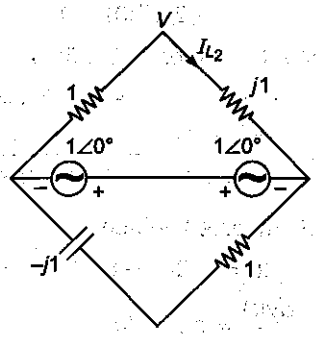
$$\frac{V}{1+j1} + \frac{V}{1-j1} = 1$$

or $\quad V = 1$

$\therefore \quad I_{L_1} = \frac{1}{1+j1}$

Taking voltage source only,

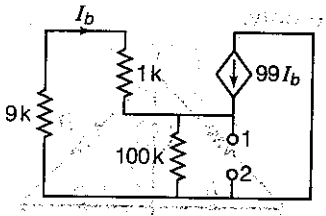
$$I_{L_2} = 0$$



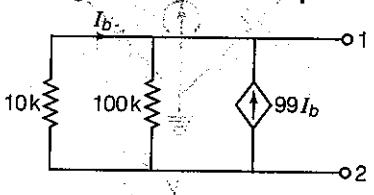
$\therefore$ Hence total inductor current $I_L = I_{L_1} + I_{L_2}$

$$= \frac{1}{1+j1}$$

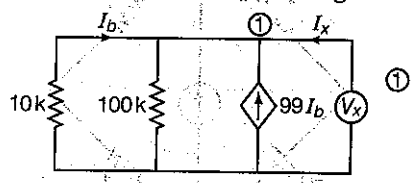
12. (a)



Re-drawing the circuit to simpler form,



Connect a voltage source V_x between 1 and 2 and measure current I_x leaving through source.



Applying KCL at node

$$-I_x - 99I_b - I_b + \frac{V_x}{100} = 0 \qquad \ldots(i)$$

Also voltage at node 1 is V_x.

$$\therefore \quad I_b = \frac{-V_x}{10} \Rightarrow I_b = \frac{-V_x}{10} \qquad \ldots(ii)$$

Substituting I_b from Eq. (ii) in Eq. (i), we get

$$-I_x - 100\,I_b + \frac{V_x}{100} = 0$$

$$\Rightarrow \quad -I_x + \frac{10\,V_x}{1} + \frac{Vx}{100} = 0$$

$$\therefore \quad \frac{V_x}{I_x} = 50\ \Omega$$

13. (d) $I_1(t) = I_m \sin(\omega t - \phi_1)$

$$I_2(t) = I_m \cos(\omega t - \phi_2)$$

or $$I_1(t) = I_m \cos\left(\frac{\pi}{2} - \omega t - \phi_1\right)$$

or $$I_1(t) = I_m \cos\left(\omega t - \frac{\pi}{2} + \phi_1\right) \qquad \ldots(i)$$

$$I_2(t) = I_m \cos(\omega t - \phi_2) \qquad \ldots(ii)$$

Equating Eqs. (i) and (ii),

These currents are balanced if $\phi_1 = \left(\frac{\pi}{2} + \phi_2\right)$

14. (b) $Z = (4 - j3)\ \Omega$

$$I = 5\cos(100\pi t + 100)$$

$$P = I^2R$$

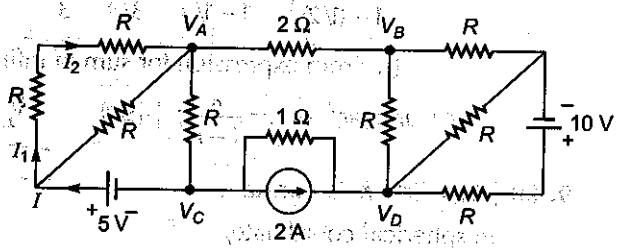
$$P = I^2 \cdot 4$$

Now, P is the average power and average value of $\cos^2\theta = \frac{1}{2}$

$$\therefore \quad P = 5^2 \times \frac{1}{2} \times 4$$

$$= 50\ \text{W}$$

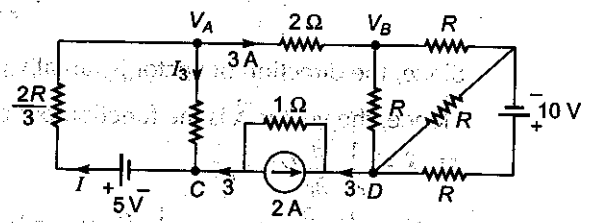
15. (a)



Since, $V_A - V_B = 6$ V and $R_{AB} = 2\ \Omega$

$$I_{AB} = 3\ \text{A}$$

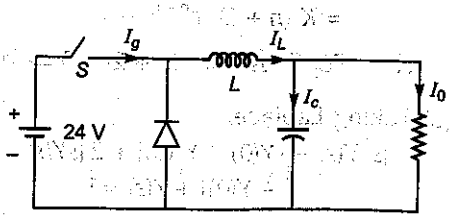
The circuit can be simplified as follows :

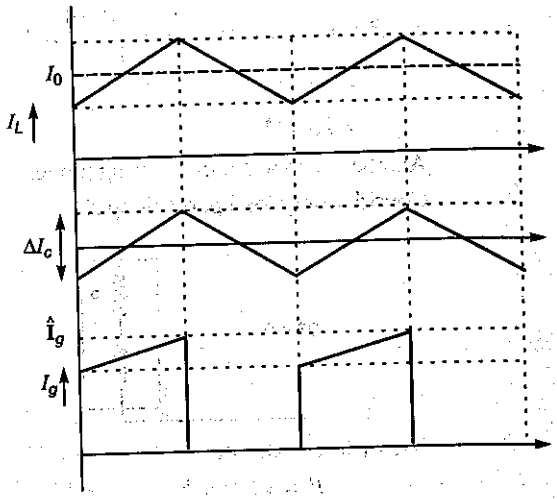


Current in the 1 Ω resistor is parallel with the 2 A source is given from KCL as 5A(3 + 2).

Thus, $V_{CD} = -5.1 = -5$ V

16. (c)





$$I_c = I_L - I_0$$
$$I_{L\,av} = I_0 \qquad \left[\because \int_0^{Ts} I_c\, dt = 0\right]$$

$I_g = I_L$ when S is On
$= 0$ when S if Off

$$\hat{I}_g = \hat{I}_L$$
$$= I_0 + \frac{\Delta I_L}{2} = I_0 + \frac{\Delta I_c}{2}$$
$$= 5 + 0.8 = 5.8 \text{ A}$$

17. (a)

$$P = VI$$
$$= (3 + IR)\cdot I$$

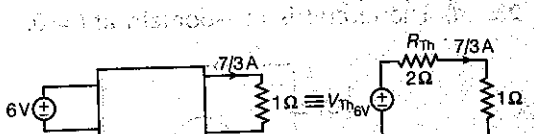
But $$I = \frac{10-3}{2+R} = \frac{7}{2+R}$$

$$\therefore \quad P = \left(3 + \frac{7R}{2+R}\right)\cdot\frac{7}{2+R}$$
$$= \frac{42 + 21R + 49R}{(2+R)^2} = \frac{42 + 70R}{(2+R)^2}$$

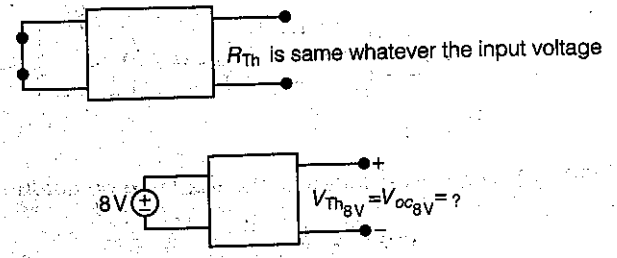
For maximum power transfer,

$$\frac{dP}{dR} = 0$$
$$0 = \frac{(2+R)^2 \cdot 70 - (42 + 70R)\,2(2+R)}{(2+R)^2}$$
$$0 = 5R^2 + 6R - 8$$
$$\Rightarrow R = 0.8\ \Omega$$

18. (c)



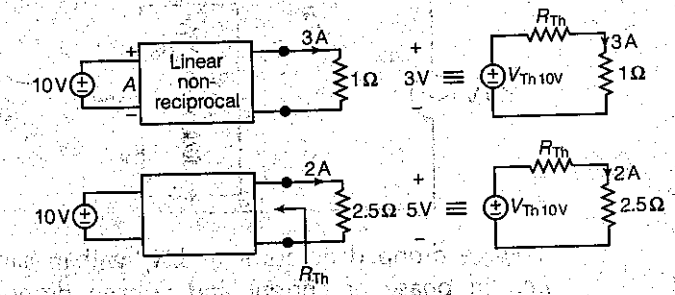
$$\therefore \quad V_{Th\,6V} = (R_{Th} + 1)\frac{7}{3} = 7 \text{ V}$$



As the given two port network is linear non-reciprocal we can express output at port 2 as a linear function of output.

i.e., $$V_{OC} = aV_{in} + b$$
$$9 = 10a + b \qquad \ldots(i)$$
$$7 = 6a + b \qquad \ldots(ii)$$

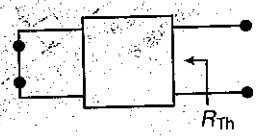
From Eqs. (i) and (ii),



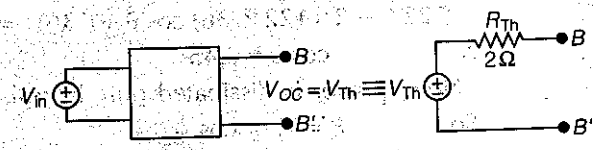
$$a = 0.5,\ b = 4$$
$$\therefore \quad V_{OC_{8V}} = 0.5\,(8) + 4 = 8 \text{ V}$$

19. (c)

To find R_{Th}



To find $V_{Th} = V_{OC}$

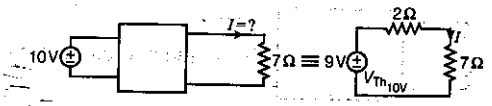


From figure,

$$V_{Th\,10\,V} = (R_{Th} + 1)3 \qquad \ldots(i)$$
$$V_{Th\,10\,V} = (R_{Th} + 2.5)\,2 \qquad \ldots(ii)$$

From Eqs. (i) and (ii),

$$V_{Th_{10\,V}} = V_{OC_{10\,V}} = 9 \text{ V}$$
$$R_{Th} = 2\ \Omega$$



$\Rightarrow \quad I = 1\text{ A}$

20. (a) The power factor of the load can be calculated as

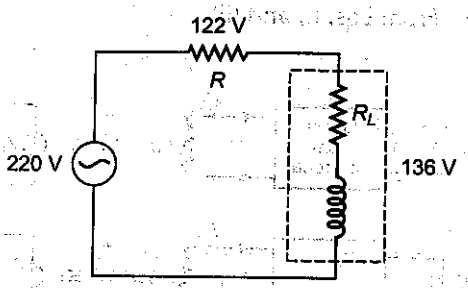
$$\cos\theta = \frac{V_1^2 - V_2^2 - V_3^2}{2V_2 \cdot V_3}$$

Putting the given value,

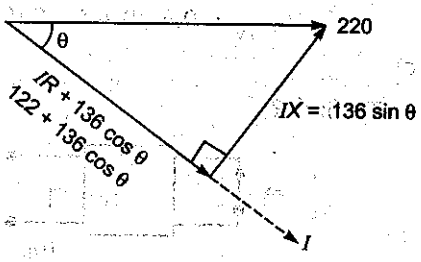
$$\cos\theta = \frac{(220)^2 - (122)^2 - (136)^2}{2\times 122\times 36}$$

$$= 0.45$$

21. (b)



Voltage dropped across R and R_L will in same *i.e.,* in phase of current and voltage dropped across inductor will be 90 to current phasor.



From phasor diagram,

$$(122 + 136\cos\theta)^2 + (136\sin\theta)^2 = (220)^2$$

$$(122)^2 + 2\times 122\times (36)\cos\theta + (136)^2 = (220)^2$$

$$\therefore \quad \cos\theta = -0.45$$

Now, power is dissipated only in resistance

So, $\quad P = V_R I_R \cos\phi$

$$P = 136\cos\phi \times \frac{136\cos\phi}{5}\times 1$$

[$\because$ consider only resistance voltage and resistance current]

Here, I took $\cos\phi = 1$ because current in resistance and voltage across the resistance are in same phase (see phasor diagram).
(voltage across R_L is $136\cos\phi$).

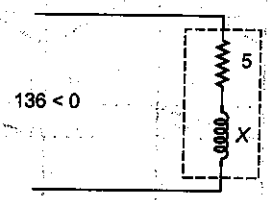
$$\therefore \quad P = 136\times 0.45\times\frac{136\times 0.45}{5}\times 1$$

$$= 749.2$$

$$= 750\text{ W}$$

Another method for calculating power

Consider only load part (along X)



Power in resistance

$$P = VI\cos\phi$$

$$= 136\times\frac{136}{\sqrt{5^2 + X^2}}\times\cos\phi \qquad \ldots(i)$$

We have calculated power factor as 0.45.

$$\therefore \quad 0.45 = \frac{5}{\sqrt{5^2 + X^2}}$$

$$\therefore \quad \frac{1}{\sqrt{5^2 + X^2}} = \frac{0.45}{5} \qquad \ldots(ii)$$

Putting the value of $\frac{1}{\sqrt{5^2 + X^2}}$ in Eq. (i),

$$P = 136\times\frac{136}{5}\times 0.45\times 0.45$$

$$= 749.2$$

$$\cong 750\text{ W}$$

22. (c) $x[n] = \left(\frac{1}{3}\right)^{|n|} - \left(\frac{1}{2}\right)^n U[n]$

$$= \left(\frac{1}{3}\right)^n U[n] + \left(\frac{1}{3}\right)^{-n} U[-n] - \left(\frac{1}{2}\right)^n U[n]$$

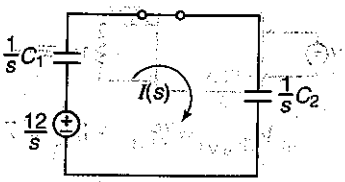
$$\left(\frac{1}{3}\right)^n U(n) + (3)^n U(-n) - \left(\frac{1}{2}\right)^n Un$$

$$\downarrow \qquad \downarrow \qquad \downarrow$$

$$\text{ROC}|z| > \frac{1}{3} \qquad \text{ROC}|z| < 3 \qquad \text{ROC}|z| > \frac{1}{2}$$

$$\therefore \text{ Common area } = \frac{1}{2} < |z| < 3$$

23. (d) The circuit is in s-domain at $t = 0$.



Applying KVL in the loop,

$$-\frac{12}{s}-I(s)\left[\frac{1}{sC_1}+\frac{1}{sC_2}\right]=0$$

$$I(s)=\frac{(12)(C_1C_2)}{C_1+C_2}$$

Taking inverse Laplace transform,

$$i(t)=\frac{12C_1C_2}{C_1+C_2}\,\delta(t)$$

Hence, the current is an impulse function *i.e.*, the capacitor resists an abrupt change in voltage and a discontinuous change in voltage requires an infinite current.

24. (a) $h[n]=\left(\frac{1}{2}\right)^n u[n]$

$y[0]=1$

$y[1]=\frac{1}{2}$

But $y[n]=g[n]*h[n]$

$$y[n]=\sum_{m\to\infty} g[m]\cdot h[n-m]$$

$$y[0]=\frac{1}{2}\times 0+\frac{1}{2}\times 0+\frac{1}{2}g[0]$$

$\therefore\quad g[0]=1$

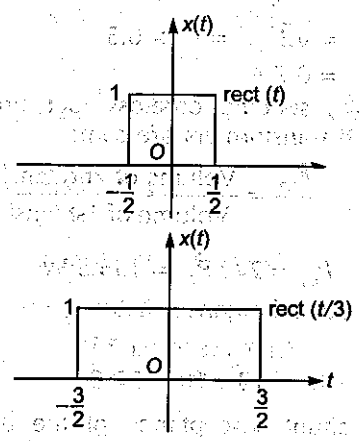
$$y[1]=\frac{1}{2}=\frac{1}{2}g[0]+g[1]$$

$\therefore\quad g[1]=0$

25. (d) Given, $H(j\omega)=\dfrac{2\cos\omega\cdot\sin 2\omega}{\omega}$

$$=\frac{\sin(2\omega+\omega)+\sin(2\omega-\omega)}{\omega}$$

$$=\frac{\sin 3\omega}{\omega}+\frac{\sin\omega}{\omega}$$



$$=3\cdot\frac{\sin 3\omega}{3\omega}+\frac{\sin\omega}{\omega}$$

$$=3\,\text{sinc}\,(3\omega)+\text{sinc}\,\omega\ (\because \text{sinc functions})$$

$h(t)=\text{IFT}\{H(j\omega)\}$

$=\text{rect}\,[t/3]+\text{rect}\,[t]$

$\therefore\quad n(0)=1+1=2$ (Am)

26. (b) Input $x(t)$, output $y(t)$ of the system is given by

$$y(t)=\int_{-\infty}^{t} x(\tau)\cos(3\tau)\,d\tau$$

(i) Test for stability

Consider a bounded input $x(t)=\cos 3t$

$$y(t)=\int_{-\infty}^{t}\cos^2(3\tau)\,d\tau$$

Thus, $y(t)$ is bounded, hence with BIBO concept system is stable.

(ii) Test for time invariance,

Consider $x(t)=\delta(t)$

$$y(t)=\int_{-\infty}^{t}\delta(\tau)\cos(3\tau)\,d\tau$$

$$y(t)=\delta(t)\cos 0°=\delta(t)$$

Time shifted input $x\left(t-\frac{\pi}{6}\right)=\delta\left(t-\frac{\pi}{6}\right)$

$$y'(t)=\int_{-\infty}^{t}\delta\left(\tau-\frac{\pi}{6}\right)\cos(3\tau)\,d\tau$$

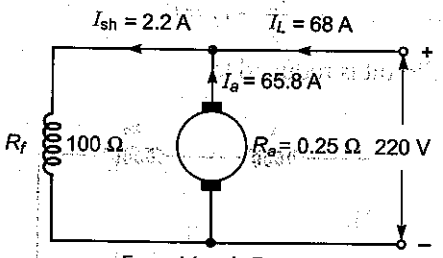
$$y'(t)=4(t)\cos\frac{3\times\pi}{6}=0$$

$y(t)\neq y'(t)$

System is not time invariant.

27. (d) The slip of an induction motor normally does not depend upon core-loss component.

28. (d) We know that



$$E_a=V-I_aR_a$$

or $$W_m=\frac{V-I_aR_a}{k\phi}$$

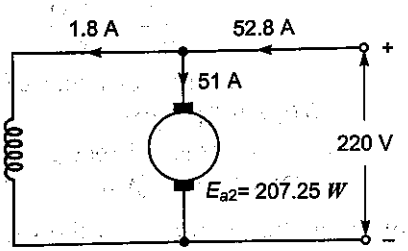
Case I

$$I_a=68-2.2=65.8\text{ A}$$

$$E_{a_1}=220-65.8\times 0.25$$

$$=203.55\text{ V}$$

$$k\phi_1=\frac{V-I_aR_a}{W_{m_1}}=\frac{203.55}{1000}=0.20355$$



Case II

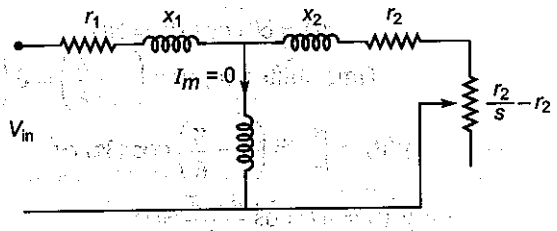
$$k\phi_2 = \frac{207.25}{1600} = 0.1295$$

$\therefore$ Percentage change in field flux

$$= \frac{0.20355 - 0.1295}{0.23555} \times 100$$

$$= 36.36\% \text{ decrease}$$

29. (b) Neglecting stator resistance and magnetizing current



Since, losses are neglected,

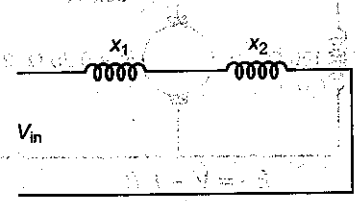
$$r_1 = r_2 = 0$$

Magnetizing current is neglected so that branch is open circuited.

$\therefore$ Rotor is blocked $\Delta\, s = 1$

$\therefore$ $$\frac{r_2}{s} - r_2 = 0$$

Circuit is reduced to



When frequency is increased

$$x_1 \to x_1 \times \frac{57}{50}$$

$$x_2 \to x_2 \times \frac{57}{50}$$

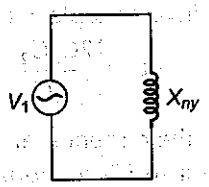
$$I = \frac{V_{in}}{x_1 + x_2}$$

$$I_1 = \frac{230}{x_1 + x_2} = 50 \text{ A}$$

$$I_2 = \frac{236}{\frac{57}{50}x_1 + \frac{57}{50}x_2} = \frac{50}{57} \times \frac{236}{x_1 + x_2}$$

$\therefore$ $$\frac{I_2}{I_1} = \frac{236}{230} \times \frac{50}{57} = 45.0 \text{ A}$$

30. (b) Equivalent circuit of 1 Ω transfer



We neglect coreloss component resistance as that value is very large.

For 2nd transformer, linear dimension is $\sqrt{2}$ times Ist one.

Now, $$L = \frac{N^2}{R_e} = \frac{N^2 \mu a}{l}$$

where,

L = inductance, N = number of turns

a = cross-sectional area of wire

l = length of flux path

$$\frac{L_{m_1}}{L_{m_2}} = \frac{N_1^2 \mu a_1}{l_1} \times \frac{l_2}{N_2^2 \mu a_2}$$

$$= \frac{a_1 l_2}{a_2 l_1} \qquad (N_1 = N_2)$$

$$= \frac{a_1 \times \sqrt{2} l_1}{2a_1 \times l_1} = \frac{1}{\sqrt{2}}$$

or $$L_{m_2} = \sqrt{2}\, L_{m_1}$$

$\therefore$ $$X_{m_1} = \sqrt{2}\, X_{m_1}$$

$\therefore$ I_{m_2} = Magnetizing current of second transformer

$$= \frac{V_2}{X_{m_2}} = \frac{2V_1}{\sqrt{2}\, X_{m_1}}$$

$$= \sqrt{2}\, I_{m_1} = \sqrt{2} \times 0.5$$

$$= 0.7 \text{ A}$$

Now, specific coreloss (per unit volume) of both transformers are same

$\therefore$ $$\frac{P_{c_2}}{P_{c_1}} = \frac{\text{Volume of 2nd tansformer}}{\text{Volume of 1st tansformer}} = 2\sqrt{2}$$

or, $$P_{c_2} = 2\sqrt{2}\, P_{c_1} = 155.5 \text{ W}$$

31. (b) For bus admittance matrix

$$y_{11} + y_{12} + y_{13} = 0$$

$$-13 + 10 + 5 \neq 0$$

So, shunt susceptance of the line connecting buses 1 and 2 is $+j2$.

32. (a) $P_{loss\,(pu)} = 0.5P_{G_1}^2$ (pu)

$$P_{G_1} + P_{G_2} - P_L = P_D$$
$$P_{G_1} + P_{G_2} - 0.5P_{G_1}^2 = 40 \quad \ldots(i)$$

Option (a) satisfying Eq. (i). Hence, correct choice is (a).

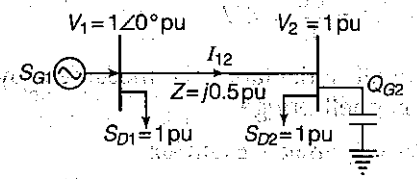
33. (c) Here,

$$I_{positive} = -(I_{negative} + I_{zero})$$
$$j1.5 = -(j0.5 - j1)$$
$$j1.5 = j1.5$$

So, fault is L-L-G.

34. (b) $\because$ Line is lossless,

$\therefore \quad S_{G_1} = S_{D_1} + S_{D_2} = 2$ pu



Power transfer from bus (1) to bus (2) = 1 pu

$$\therefore \quad 1 = \frac{|V_1||V_2|}{X_{12}} \sin(\theta_1 - \theta_2)$$

$$\Rightarrow \quad \theta_2 = -30°$$

$$\therefore \quad V_2 = 1\angle -30°$$

Now, $I_{12} = \dfrac{(1) - (1\angle -30°)}{j0.5} = 1 - j0.268$

Current in $S_{D_2} = 1\angle -30°$

$\therefore$ Current in $Q_{G_2} = (1\angle -30°) - (1 - j0.268)$

$$= 0.268\angle -120°$$

$\therefore$ VAR rating of capacitor $= |V_2||I_Q| \sin(\alpha)$

$$= (1)(0.268) \cdot \sin(+90°)$$
$$= 0.268$$

35. (d) For 3-ϕ dead short circuit fault,

The critical clearing angle

$$\delta_c = \cos^{-1}\left[\frac{P_s(\delta_m - \delta_0) + P_{mz}\cos\delta_m}{P_{mz}}\right]$$

P_s = steady state power = 0.5 pu

$\delta_m = 180 - \delta_0$

P_{mz} = Maximum power output after fault

$$= \frac{V_1 V_2}{X} = 1\text{ pu}$$

But $\quad P_s = P_{m_1}\sin\delta_0 = \dfrac{V_1 V_2}{X}\sin\delta_0$

Here, $\quad V_1 = 1.5,\ V_2 = 0.5$

$$X = X_L + X_g = 0.5 + 1 = 1.5$$

$$\therefore \quad 0.5 = \frac{1.5}{1.5}\sin\delta_0 \Rightarrow \delta_0 = 30°$$

$$\therefore \quad \delta_m = 180 - 30 = 150°$$

$$\delta_c = \cos^{-1}\left[\frac{0.5(150-30)\times\frac{\pi}{180} + 1\cdot\cos 150°}{1}\right]$$

$$\delta_c = 79.6°$$

36. (c) $G(s) = \dfrac{(s^2+9)(s+2)}{(s+1)(s+3)(s+4)}$

The system is excited by $(\sin\omega t)$.

$\therefore$ The sinusoidal transfer function is

$$G(j\omega) = \frac{(j^2\omega^2 + 9)(j\omega + 2)}{(j\omega+1)(j\omega+3)(j\omega+4)}$$
$$= \frac{(-\omega^2 + 9)(j\omega + 2)}{(j\omega+1)(j\omega+3)(j\omega+4)}$$

$\therefore G(j\omega)$ is zero for $\omega = 3$ rad/s

37. (d) $\quad \dot{X} = AX + B$

$$Y = CX$$

$$A = \begin{bmatrix} 0 & a_1 & 0 \\ 0 & 0 & a_2 \\ a_3 & 0 & 0 \end{bmatrix} \quad B = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

Controllability matrix should have full rank to make system controllable.

$$M = [B \quad AB \quad A^2B] = \begin{bmatrix} 0 & 0 & a_1a_2 \\ 0 & a_2 & 0 \\ 1 & 0 & 0 \end{bmatrix}$$

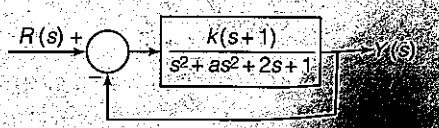
So, $\quad a_2 \neq 0$

$$a_1 a_2 \neq 0$$

$$\Rightarrow \quad a_1 \neq 0$$

$$\therefore \quad a_1 \neq 0,\ a_2 \neq 0,\ a_3 \neq 0$$

38. (a)



$$G(s) = \frac{k(s+1)}{s^3 + as^2 + 2s + 1}$$

$$1 + G(s)H(s) = 1 + \frac{k(s+1)}{s^3 + as^2 + 2s}$$

$$= \frac{s^3 + as^2 + 2s}{s^3 + as^2}$$

$$G_{C_1}(s) = \frac{G(s)}{1 + G(s)\,H(s)}$$

For oscillation, $1 + G(s)H(s)| = 0$

$s = j\omega$, where ω is the oscillation frequency in rad/s.

Also, $1 + G(s)\,H(s) = \dfrac{s^3 + as^2 + (k+2)\,s + (k+1)}{s^3 + as^2 + 2s + 1}$

$$\Rightarrow \quad 1 + G(j\omega)\,H(j\omega) = \frac{-j\omega^3 - a\omega^2 + (k+2)\,j\omega + (k+1)}{-j\omega^3 - a\omega^2 + 2j\omega + 1}$$

$$\therefore \quad 1 + G(j\omega)\,H(j\omega) = 0$$

$$\Rightarrow -j\omega^3 + j\omega(k+2) + (k+1-a\omega^2) = 0$$

Equating real and imaginary parts,

$$\omega(k+2) - \omega^3 = 0$$

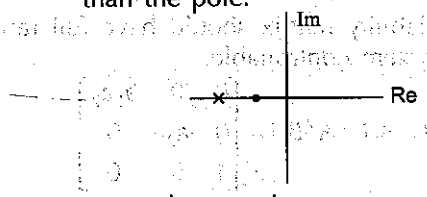
$$\Rightarrow \quad k + 2 = \omega^2$$

$$\Rightarrow \quad k = 2 \qquad [\because \omega = 2\,\text{rad/s}]$$

$$\Rightarrow \quad a = \frac{k+1}{\omega^2} = \frac{2+1}{(2)^2} = 0.75$$

39. (a) $$G_c(s) = \frac{s+a}{s+b}$$

In a lead compensator zero is near the origin than the pole.



In another words,

$$\angle G_c(s) = \tan^{-1}\frac{\omega}{a} - \tan^{-1}\frac{\omega}{b} > 0$$

$$\Rightarrow \quad a < b$$

So, $a = 1, b = 2$

40. (a) $$G_c(s) = \frac{s+1}{s+2}$$

Phase of $G_c(s)$ can be given from the expression

$$\angle G_c(s) = \phi = \tan^{-1}\frac{\omega}{1} - \tan^{-1}\frac{\omega}{2}$$

$$= \tan^{-1}\frac{\omega - \frac{\omega}{2}}{1 + \omega\cdot\frac{\omega}{2}}$$

Let $\tan\phi = y$

$$y = \frac{\omega}{2 + \omega^2}$$

For y to be maximum

$$\frac{dy}{d\omega} = 0$$

$$0 = \frac{(2+\omega^2)\,1 - \omega\,(2\,\omega)}{(2+\omega^2)^2}$$

$$2 + \omega^2 = 2\omega^2$$

$$\omega = \sqrt{2}\ \text{rad/s}$$

41. (a) PMMC type of instrument measure average value

$$= \frac{\text{Area of graph}}{\text{Total time}}$$

$$= \frac{\frac{1}{2}\times 10 \times 10 - 5 \times 2 + 8 \times 5}{20}$$

$$= \frac{80}{20} = 4\,\text{V}$$

42. (a) Mutual inductance is measured by Heaviside Compbell bridge.

43. (c) $V(t) = E_1 \sin\omega t + E_3 \sin 3\omega t$

$I(t) = I_1 \sin(\omega t - \phi_1) + I_3 \sin(\omega t - \phi_3) + I_5 \sin(5\omega t)$

Average power due to fundamental component

$$P_1 = E_{1\text{rms}} \times I_{1\text{rms}} \cdot \cos\phi_1$$

$$= \frac{E_1}{\sqrt{2}} \times \frac{I_1}{\sqrt{2}}\cos\phi_1 = \frac{E_1 I_1}{2}\cos\phi_1$$

Average power due to third harmonics

$$P_3 = \frac{E_3}{\sqrt{2}} \times \frac{I_2}{\sqrt{2}}\cos\phi_3$$

$$= \frac{E_3 I_3}{2}\cos\phi_3$$

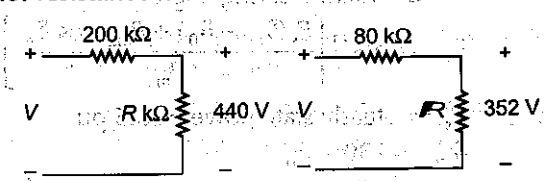
Average power due to fifth harmonics, $P_5 = 0$

[$\because$ There is no fifth harmonic voltage]

$\therefore$ Total power $P = P_1 + P_3 + P_5$

$$= \frac{E_1 I_1}{2}\cos\phi_1 + \frac{E_3 I_3}{2}\cos\phi_3$$

44. (d) Let resistance of voltmeter be R kΩ.

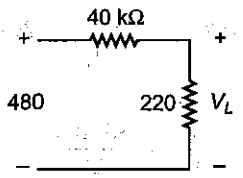


$$V = \left(\frac{440}{R}\right)20 + 440 \qquad \ldots(i)$$

$$V = 352 + \left(\frac{352}{R}\right)80 \qquad \ldots(ii)$$

Solving, $V = 480$ V

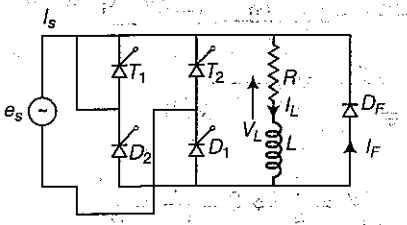
$R = 220\ \Omega$



$$V_L = \frac{480}{40 + 220} \times 220$$

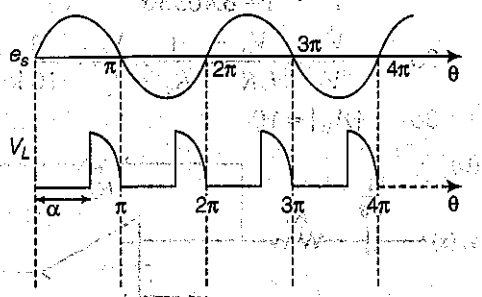
$$= 406 \text{ V}$$

45. (d)



Half controlled single phase bridge rectifier supplying an *RL* load

The freewheel diode is connected across inductive load, the load voltage attempt to reverse this diode is reverse biased and the inductive stored energy circulates the load current in the closed path of the load and D_F. The advantage of this method is that at zero main voltage the conducting thyristor is turn off and reduces the thyristor duty cycle.



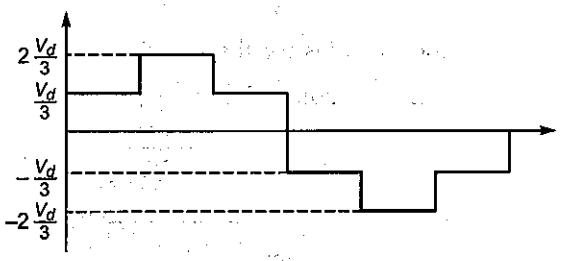
Hence, the fraction of cycle for which freewheeling diode conducts $= \alpha / \pi$

46. (b) The holding current value I_h is linked to the latching current value I_l and I_l will always be higher than the I_h value.

Ratio of latching current I_l to the holding current I_h depends upon the sensitivity and rating of the device.

Ratio	Sensitive Thyristor (I < 6 A)	Medium Power Thyristor (I > 6 A < 60 A)	High Power Thyristor (I > 60 A < 300 A)
$\frac{I_l}{I_h}$	1.1 to 1.5	1.5 to 2.0	2.0 to 5.0

47. (b) Load phase voltage waveform



$$V_{rms} = \sqrt{\frac{2}{9}} V_d$$

$$= \sqrt{\frac{2}{9}} \times 300 = 141.4 \text{ V}$$

48. (d) Power consumed by 3 phase load

$$P = 3 \times \frac{(V_{ph\,rms})^2}{R} = 3 \times \frac{(141.4)^2}{20}$$

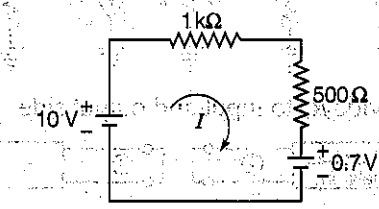
$$= 3 \times \frac{(\sqrt{2} \times 100)^2}{20} = 3 \text{ kW}$$

49. (d) For $V \geq 0.7$,

$$I = \frac{V - 0.7}{500}$$

$$\frac{dV}{dI} = 500\,\Omega = r_d$$

Redrawing the circuit,

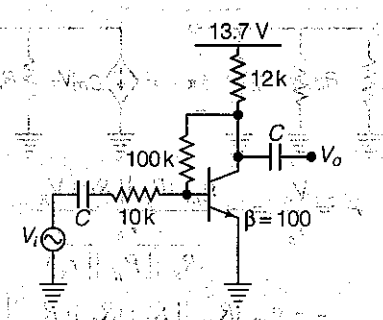


Applying KVL,

$$10 - 1000I - 500I - 0.7 = 0$$

$$I = \frac{9.3}{1500} = 6.2 \text{ mA}$$

50. (d)

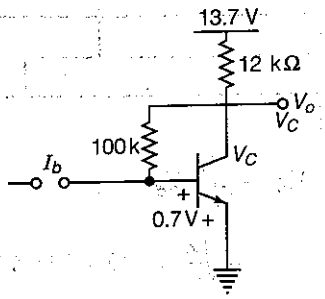


This is a shunt-shunt feedback amplifier. Output voltage is sampled and current is feedback.

$$I_c = \beta I_b,\ r_\pi = \frac{V_T}{I_b},\ g_m = \frac{\beta}{r_\pi},\ I_c = g_m V_\pi$$

Find DC bias for the circuit.

At DC, circuit reduces to



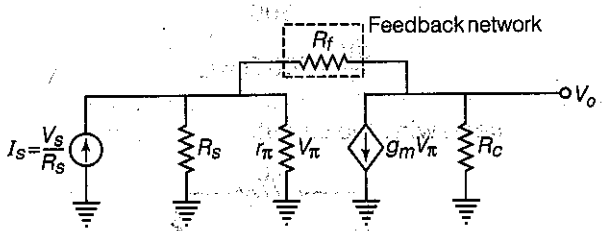
$$V_C = 100\, I_b + 0.7 \qquad \ldots(i)$$

$$\frac{13.7 - V_C}{12} = (\beta + 1)\, I_b \qquad \ldots(ii)$$

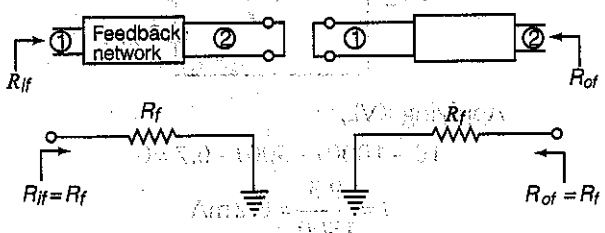
From Eqs. (i) and (ii),

$$\frac{13.7 - 100\, I_b - 0.7}{12} \simeq \beta I_b = 100\, I_b$$

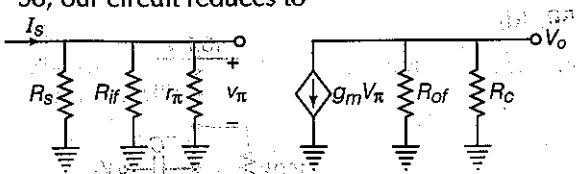
$$\Rightarrow \quad I_b = 0.01 \text{ mA} \quad \text{and} \quad I_C = \beta I_b = 1 \text{mA}$$



To divide R_f to input and output side



So, our circuit reduces to



$$A = \frac{V_o}{I_s} = -\frac{g_m (R_f \,||\, R_c)\, V_\pi}{\dfrac{V_\pi}{(R_s \,||\, R_{if} \,||\, r_\pi)}}$$

$$= -\, g_m\, (R_{of} \,||\, R_c)\, (R_s \,||\, R_{if} \,||\, r_\pi)$$

$$R_{of} = R_{if} = 100 \text{ k}\Omega$$

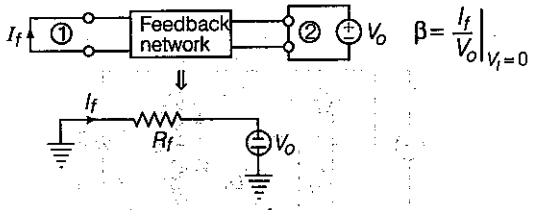
$$R_c = 12\,\text{k}\Omega,\ r_\pi = \frac{V_T}{I_b} = 2.5 \text{ k}\Omega$$

and $R_s = 10$ k

Also, $g_m = \dfrac{I_c}{V_T} = \dfrac{40 \text{ mA}}{V}$

$$A = -\,840.336 \text{ k}\Omega \qquad \ldots(i)$$

Note This is a trans-resistance amplifier. Now, feedback factor for shunt-shunt configuration is



$$\frac{V_o}{I_f} = \frac{1}{\beta} \Rightarrow \beta = \frac{I_f}{V_o} = \frac{-1}{R_f}$$

$$\frac{V_o}{I_f} = \frac{1}{\beta} \Rightarrow \beta = \frac{I_f}{V_o} = \frac{-1}{R_f}$$

So, here $\beta = \dfrac{-1}{100 \text{ k}\Omega}$ $\qquad \ldots(ii)$

So, gain after feedback

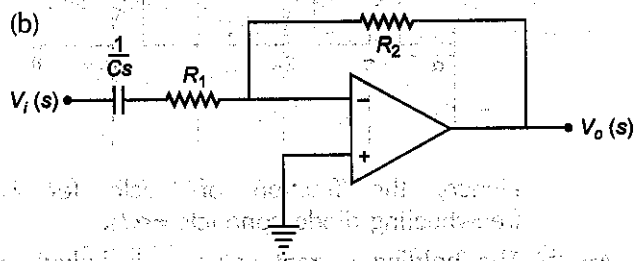
$$\frac{V_o}{I_s} = \frac{A}{1 + A\beta}$$

$$\Rightarrow \quad \frac{V_o}{I_s} = \frac{-840.336}{1 + 8.40336} \approx 10^5 \quad \text{(approx)}$$

So, $\dfrac{V_o}{V_I} = \dfrac{V_o}{I_s R_s} = \dfrac{1}{R_s} \times \dfrac{V_o}{I_s} = \dfrac{1}{10 \text{ k}} \times 10^5 = 10$

So, $|A_V| \approx 10$

51. (b)



$$\frac{V_i(s) - 0}{R_1 + \dfrac{1}{Cs}} = \frac{-V_o}{R_2}$$

or $$\frac{V_o(s)}{V_i(s)} = \frac{-R_2 Cs}{CsR_1 + 1} = \frac{-R_2 Cs}{\dfrac{s}{\left(\dfrac{1}{R_1 C}\right)} + 1}$$

$$\therefore \quad f_{B\,\text{dB}} = \frac{1}{R_1 C} \text{ rad/s}$$

At low frequency, $s \to 0$, $V_o \to 0$

So, high-pass filter with $f_{B\,dB} = \dfrac{1}{R_1 C}$ rad/s

2. (a) $f(X, Y, Z) = \Sigma(2, 3, 4, 5)$

	$\bar{Y}\bar{Z}$	$\bar{Y}Z$	YZ	$Y\bar{Z}$
X			1	1
$\bar{X}$	1	1		

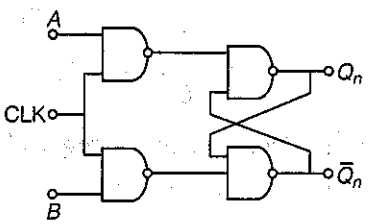
$\therefore$ Prime implecants are $\bar{X}Y$, $X\bar{Y}$.

3. (b)

A	B	$A > B$	$n(A >$)
00	00	0	3
01		1	
10		1	
11		1	
00	01	0	2
01		0	
10		1	
11		1	
00	10	0	1
01		0	
10		0	
11		1	
00	11	0	0
01		0	
10		0	
11		0	

Total number of combinations = 6

54. (c) In the circuit with the clock pulse, the value of output is not certain *i.e.*, will oscillate between 0 and 1. This situation is referred as race around condition.



The truth table

A	B	CLK	$Q_n + 1$
0	0	0	Q_n
0	0	1	Q_n
0	1	0	Q_n
0	1	1	0
1	0	0	Q_n
1	0	1	1
1	1	0	Q_n
1	1	1	Indeterminate

Hence, the race around condition occurs, when

CLK = 1 and $A = B = 1$

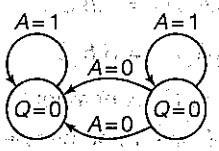
55. (d)

Flip-flop truth table		2×1 MUX truth table			
D	Q	X_0	X_1	A	Y
0	0	0	1	0	0
1	1	1	0	1	1

The overall system truth table can be written as

Q_n	X_1	X_0	A	$Y(D = Q_{n+1})$
0	0	1	0	1
0	0	1	1	0
1	1	0	0	0
1	1	0	1	1

So, the state diagram is



56. (b)

57. (d) Let $1.001 = x$

$x^{1259} = 3.52$, $x^{2062} = 7.85$

$x^{3321} = x^{1259} x^{2062} = 3.52 \times 7.85$

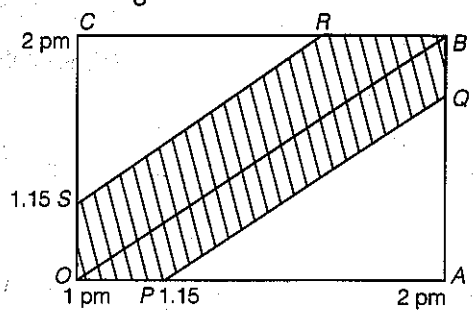
$= 27.64$

58. (c) If the tired soldier wanted to lie down, he should have taken the mattress out on the balcony.

59. (d)

60. (b) The meaning of Latitude is Freedom.

61. (c)



OB is the line when both A and B arrive at same time.

Total sample space = 60 × 60 = 3600

Favourable cases = Area of $OABC$ − 2 (Area of SRC)

$$= 3600 - 2 \times \left(\frac{1}{2} \times 45 \times 45\right) = 1575$$

Hence, required probability $= \dfrac{1575}{3600} = \dfrac{7}{16}$

62. (a)

63. (a) Suppose the number of ₹ 20 notes = x
and the number of ₹ 10 notes = y
According to the question,

$$20x + 10y = 230 \quad \ldots(i)$$

and $$x + y = 14 \quad \ldots(ii)$$

From Eqs. (i) and (ii), we get

$$x = 9 \text{ and } y = 5$$

∴ Number of ₹ 10 notes = 5

64. (a) Suppose category of bags are as follows :

$$A_1A_2A_3\,B_1B_2B_3\,C_1C_2$$

1st weighing A *vs* B

Case 1 $A_1A_2A_3 = B_1B_2B_3$

Case 2 $A_1A_2A_3 \neq B_1B_2B_3$

∴ Either C_1 or C_2 is heavier.

Either A or B would be heavier (say $A > B$)

2nd weighing

C_1 *vs* C_2; If $C_1 > C_2$, then C_1

If $C_1 < C_2$, then C_2

If $A_1 < A_2$, then A_2

A_1 *vs* A_2; If $A_1 = A_2$, then A_3

If $A_1 > A_2$, then A_1

65. (d) Total budget amount

$$= 4000 + 1200 + 2000 + 1500 + 1800 = ₹\,10500$$

Except savings, other expenditure

$$= ₹\,(10500 - 1500)$$

$$= ₹\,9000$$

Hence, required percentage

$$= \frac{9000 \times 100}{10500}$$

$$= 86\%$$

E 2012

10500

1

Engineering Mathematics

Syllabus

Linear Algebra *Matrix Algebra, Systems of linear equations, Eigen values and Eigen vectors.*
Calculus *Mean value theorems, Theorems of integral calculus, Evaluation of definite and improper integrals, Partial derivatives, Maxima and minima, Multiple integrals, Fourier series. Vector identities, Directional derivatives, Line, surface and volume integrals, Stoke's, Gauss's and Green's theorems.*
Differential Equations *First order equations (linear and non-linear), Higher order linear differential equations with constant coefficients, Method of variation of parameters, Cauchy's and Euler's equations, Initial and boundary value problems, Partial differential equations and variable separable method.*
Complex Variables *Analytic functions, Cauchy's integral theorem and integral formula, Taylor's and Laurent's series, Residue theorem, Solution integrals.*
Probability and Statistics *Sampling theorems, Conditional probability, Mean, median, mode and standard deviation, Random variables, Discrete and continuous distributions, Poisson, normal and binomial distributions, Correlation and regression analysis.*
Numerical Methods *Solutions of non-linear algebraic equations, Single and multi-step methods for differential equations.*
Transform Theory *Fourier transform, Laplace transform, Z-transform.*

GATE 2011 *(1 Mark Questions)*

▼ *Solution Points*

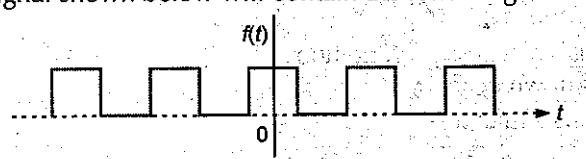
1. With K as a constant, the possible solution for the first order differential equation $\frac{dy}{dx} = e^{-3x}$ is

(a) $-\frac{1}{3}e^{-3x} + K$ (b) $-\frac{1}{3}e^{3x} + K$ (c) $-3e^{-3x} + K$ (d) $-3e^{-x} + K$

2. Roots of the algebraic equation $x^3 + x^2 + x + 1 = 0$ are

(a) $+1, +j, -j$ (b) $+1, -1, +1$ (c) $0, 0, 0$ (d) $-1, +j, -j$

3. The Fourier series expansion $f(t) = a_0 + \sum_{n=1}^{\infty} a_n \cos n\omega t + b_n \sin n\omega t$ of the periodic signal shown below will contain the following non-zero terms



(a) a_0 and b_n, $n = 1, 3, 5, \ldots \infty$ (b) a_0 and a_n, $n = 1, 2, 3, \ldots \infty$
(c) a_0, a_n and b_n, $n = 1, 2, 3, \ldots \infty$ (d) a_0 and a_n, $n = 1, 3, 5, \ldots \infty$

▼ *Solution Points*

GATE 2010

4. The value of the quantity P, where $P = \int_0^1 xe^x\,dx$, is equal to

(a) 0 (b) 1 (c) e (d) $1/e$

5. Divergence of the three-dimensional radial vector field $\vec{r}$ is

(a) 3 (b) $1/r$ (c) $\hat{i}+\hat{j}+\hat{k}$ (d) $3(\hat{i}+\hat{j}+\hat{k})$

6. The period of the signal $x(t) = 8\sin\left(0.8\pi t + \frac{\pi}{4}\right)$ is

(a) 0.4π s (b) 0.8π s (c) 1.25 s (d) 2.5 s

GATE 2009

7. The trace and determinant of a 2×2 matrix are known to be -2 and -35 respectively. Its eigen values are

(a) -30 and -5 (b) -37 and -1 (c) -7 and 5 (d) 17.5 and -2

GATE 2008

8. X is a uniformly distributed random variable that takes values between 0 and 1. The value of $E\{X^3\}$ will be

(a) 0 (b) 1/8 (c) 1/4 (d) 1/2

9. The characteristic equation of a (3×3) matrix P is defined as

$a(\lambda) = |\lambda I - P| = \lambda^3 + \lambda^2 + 2\lambda + I = 0$

If I denotes identity matrix, then the inverse of matrix P will be

(a) $(P^2 + P + 2I)$ (b) $(P^2 + P + I)$ (c) $-(P^2 + P + I)$ (d) $-(P^2 + P + 2I)$

10. If the rank of a (5×6) matrix Q is 4, then which one of the following statements is correct?

(a) Q will have four linearly independent rows and four linearly independent columns

(b) Q will have four linearly independent rows and five linearly independent columns

(c) QQ^T will be invertible

(d) Q^TQ will be invertible

GATE 2007

11. $x = [x_1 x_2 \ldots x_n]^T$ is an n-tuple non-zero vector. The $n\times n$ matrix $V = xx^T$

(a) has rank zero (b) has rank 1 (c) is orthogonal (d) has rank n

GATE 2005

12. In the matrix equation $px = q$, which of the following is a necessary condition for the existence of at least one solution for the unknown vector x?

(a) Augmented matrix $[pq]$ must have the same rank as matrix p

(b) Vector q must have only non-zero elements

(c) Matrix p must be singular

(d) Matrix p must be square

▼ *Solution Points*

13. If P and Q are two random events, then which of the following is true?
(a) Independence of P and Q implies that probability $(P \cap Q) = 0$
(b) Probability $(P \cup Q) \geq$ Probability (P) + Probability (Q)
(c) If P and Q are mutually exclusive, then they must be independent
(d) Probability $(P \cap Q) \leq$ Probability (P).

14. If $S = \int_1^\infty x^{-3}\, dx$, then S has the value
(a) $\frac{-1}{3}$ (b) $\frac{1}{4}$ (c) $\frac{1}{2}$ (d) 1

15. The solution of the first order differential equation $x'(t) = -3x(t), x(0) = x_0$ is
(a) $x(t) = x_0 e^{-3t}$ (b) $x(t) = x_0 e^{-3}$ (c) $x(t) = x_0 e^{-1/3}$ (d) $x(t) = x_0 e^{-t}$

GATE 2011

(2 Marks Questions)

16. The two vectors [1, 1, 1] and $[1, a, a^2]$, where $a = \left(-\frac{1}{2} + j\frac{\sqrt{3}}{2}\right)$ are
(a) orthonormal (b) orthogonal (c) parallel (d) collinear

17. The matrix $[A] = \begin{bmatrix} 2 & 1 \\ 4 & -1 \end{bmatrix}$ is decomposed into a product of a lower triangular matrix $[L]$ and an upper triangular matrix $[U]$. The properly decomposed $[L]$ and $[U]$ matrices respectively are

(a) $\begin{bmatrix} 1 & 0 \\ 4 & -1 \end{bmatrix}$ and $\begin{bmatrix} 1 & 1 \\ 0 & -2 \end{bmatrix}$ (b) $\begin{bmatrix} 2 & 0 \\ 4 & -1 \end{bmatrix}$ and $\begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$

(c) $\begin{bmatrix} 1 & 0 \\ 4 & 1 \end{bmatrix}$ and $\begin{bmatrix} 2 & 1 \\ 0 & -1 \end{bmatrix}$ (d) $\begin{bmatrix} 2 & 0 \\ 4 & -3 \end{bmatrix}$ and $\begin{bmatrix} 1 & 0.5 \\ 0 & 1 \end{bmatrix}$

18. The function $f(x) = 2x - x^2 + 3$ has
(a) a maximum at $x = 1$ and a minimum at $x = 5$
(b) a maximum at $x = 1$ and a minimum at $x = -5$
(c) only a maximum at $x = 1$
(d) only a minimum at $x = 1$

GATE 2010

19. At $t = 0$, the function $f(t) = \frac{\sin t}{t}$ has
(a) a minimum (b) a discontinuity
(c) a point of inflection (d) a maximum

20. A box contains 4 white balls and 3 red balls. In succession, two balls are randomly selected and removed from the box. Given that the first removed ball is white, the probability that the second removed ball is red is
(a) 1/3 (b) 3/7 (c) 1/2 (d) 4/7

▼ *Solution Points*

21. An eigen vector of $P=\begin{pmatrix}1 & 1 & 0\\ 0 & 2 & 2\\ 0 & 0 & 3\end{pmatrix}$ is

(a) $[-1\ \ 1\ \ 1]^T$ (b) $[1\ 2\ 1]^T$ (c) $[1\ -1\ 2]^T$ (d) $[2\ \ 1\ \ -1]^T$

22. For the differential equation $\frac{d^2x}{dt^2}+6\frac{dx}{dt}+8x=0$ with initial conditions $x(0)=1$ and $\left.\frac{dx}{dt}\right|_{t=0}=0$, the solution is

(a) $x(t)=2e^{-6t}-e^{-2t}$ (b) $x(t)=2e^{-2t}-e^{-4t}$
(c) $x(t)=-e^{-6t}+2e^{-4t}$ (d) $x(t)=e^{-2t}+2e^{-4t}$

23. For the set of equations

$$x_1+2x_2+x_3+4x_4=2$$
$$3x_1+6x_2+3x_3+12x_4=6$$

Which of the following statements is true?

(a) Only the trivial solution $x_1=x_2=x_3=x_4=0$ exists
(b) There are no solution
(c) A unique non-trivial solution exists
(d) Multiple non-trivial solutions exist

GATE 2009

24. $f(x, y)$ is a continuous function defined over $(x, y)\in[0,1]\times[0,1]$. Given the two constraints, $x>y^2$ and $y>x^2$, the volume under $f(x, y)$ is

(a) $\int_{y=0}^{y=1}\int_{x=y^2}^{x=\sqrt{y}} f(x,y)\,dx\,dy$ (b) $\int_{y=x^2}^{y=1}\int_{x=y^2}^{x=1} f(x,y)\,dx\,dy$

(c) $\int_{y=0}^{y=1}\int_{x=0}^{x=1} f(x,y)\,dx\,dy$ (d) $\int_{y=0}^{y=\sqrt{x}}\int_{x=0}^{x=\sqrt{y}} f(x,y)\,dx\,dy$

25. Assume for simplicity that N people, all born in April (a month of 30 days), are collected in a room. Consider, the event of at least two people in the room being born on the same date of the month, even if in different years, *e.g.*, 1980 and 1985. What is the smallest N so that the probability of this event exceeds 0.5?

(a) 20 (b) 7 (c) 15 (d) 16

26. A cubic polynomial with real coefficients

(a) can possibly have no extrema and no zero crossings
(b) may have upto three extrema and upto 2 zero crossings
(c) cannot have more than two extrema and more than three zero crossings
(d) will always have an equal number of extrema and zero crossings

27. Let $x^2-117=0$. The iterative steps for the solution using Newton-Raphson's method is given by

(a) $x_{k+1}=\frac{1}{2}\left(x_k+\frac{117}{x_k}\right)$ (b) $x_{k+1}=x_k-\frac{117}{x_k}$

(c) $x_{k+1}=x_k-\frac{x_k}{117}$ (d) $x_{k+1}=x_k-\frac{1}{2}\left(x_k+\frac{117}{x_k}\right)$

▼ *Solution Points*

28. $F(x, y) = (x^2 + xy)\,\hat{\mathbf{a}}_x + (y^2 + xy)\,\hat{\mathbf{a}}_y$. It's line integral over the straight line from $(x, y) = (0, 2)$ to $(x, y) = (2, 0)$ evaluates to

(a) -8 (b) 4 (c) 8 (d) 0

GATE 2008

29. Consider function $f(x) = (x^2 - 4)^2$, where x is a real number. Then, the function has

(a) only one minimum (b) two minima

(c) three minima (d) three maxima

30. Equation $e^x - 1 = 0$ is required to be solved using Newton's method with an initial guess $x_0 = -1$. Then, after one step of Newton's method, estimate x_1 of the solution will be given by

(a) 0.71828 (b) 0.36784 (c) 0.20587 (d) 0.00000

31. A is $m \times n$ full rank matrix with $m > n$ and I is an identity matrix. Let matrix $A' = (A^T A)^{-1} A^T$. Then, which one of the following statements is false ?

(a) $AA'A = A$ (b) $(AA')^2$ (c) $A'A = I$ (d) $AA'A = A$

32. A differential equation $dx/dt = e^{-2t}\,u(t)$ has to be solved using trapezoidal rule of integration with a step size $h = 0.01$ s. Function $u(t)$ indicates a unit step function. If $x(0^-) = 0$, then value of x at $t = 0.01$ s will be given by

(a) 0.00099 (b) 0.00495 (c) 0.0099 (d) 0.0198

33. Let P be a 2×2 real orthogonal matrix and $\vec{\mathbf{x}}$ is a real vector $[x_1, x_2]^T$ with length $\|\vec{\mathbf{x}}\| = (x_1^2 + x_2^2)^{1/2}$. Then, which one of the following statements is correct?

(a) $\|P\vec{\mathbf{x}}\| \le \|\vec{\mathbf{x}}\|$, where at least one vector satisfies $\|P\vec{\mathbf{x}}\| < \|\vec{\mathbf{x}}\|$

(b) $\|P\vec{\mathbf{x}}\| \le \|\vec{\mathbf{x}}\|$, for all vectors $\vec{\mathbf{x}}$

(c) $\|P\vec{\mathbf{x}}\| \ge \|\vec{\mathbf{x}}\|$, where at least one vector satisfies $\|P\vec{\mathbf{x}}\| > \|\vec{\mathbf{x}}\|$

(d) No relationship can be established between $\|\vec{\mathbf{x}}\|$ and $\|P\vec{\mathbf{x}}\|$

GATE 2007

34. The differential equation $\dfrac{dx}{dt} = \dfrac{1 - x}{\tau}$ is discretised using Euler's numerical integration method with a time step $\Delta T > 0$. What is the maximum permissible value of ΔT to ensure stability of the solution of the corresponding discrete time equation?

(a) 1 (b) $\tau/2$ (c) t (d) 2τ

35. The value of $\oint_C \dfrac{dz}{(1 + z^2)}$, where C is the contour $|z - i/2| = 1$ is

(a) $2\pi i$ (b) π (c) $\tan^{-1} z$ (d) $\pi i \tan^{-1} z$

▼ *Solution Points*

36. The integral $\frac{1}{2\pi}\int_0^{2\pi} \sin(t-\tau)\cos\tau\, d\tau$ equals

(a) $\sin t \cos t$ (b) 0 (c) $(1/2)\cos t$ (d) $(1/2)\sin t$

37. A loaded dice has following probability distribution of occurrences

Dice value	1	2	3	4	5	6
Probability	1/4	1/8	1/8	1/8	1/8	1/4

If three identical dice as the above are thrown, the probability of occurrence of values 1, 5 and 6 on the three dice is

(a) same as that of occurrence of 3, 4, 5
(b) same as that of occurrence of 1, 2, 5
(c) 1/128
(d) 5/8

38. Let x and y be two vectors in a 3-dimensional space and $< x, y >$ denote their dot product. Then, the determinant

$$\det\begin{bmatrix} < x, x > & < x, y > \\ < y, x > & < y, y > \end{bmatrix}$$

(a) is zero when x and y are linearly independent
(b) is positive when x and y are linearly independent
(c) is non-zero for all non-zero x and y
(d) is zero only when either x or y is zero

39. The linear operation $L(x)$ is defined by the cross product $L(x) = bX_k$, where $b = [0\ 1\ 0]^T$ and $x = [x_1\ x_2\ x_3]^T$ are three-dimensional vectors. The 3×3 matrix M of this operation satisfies

$$L(x) = M\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

Then, the eigen values of M are

(a) $0, +1, -1$ (b) $1, -1, 1$ (c) $i, -i, 1$ (d) $i, -i, 0$

Statements for Linked Answer Questions 40 and 41

Caylay-Hamilton theorem states that a square matrix satisfies its own characteristic equation. Consider a matrix,

$$A = \begin{bmatrix} -3 & 2 \\ -1 & 0 \end{bmatrix}$$

40. A satisfies the relation

(a) $A + 3I + 2A^{-1} = 0$ (b) $A^2 + 2A + 2I = 0$
(c) $(A + I)(A + 2I)$ (d) $\exp(A) = 0$

41. A^9 equals

(a) $511A + 510$ (b) $309A + 104$
(c) $154A + 155$ (d) $\exp(9A)$

▼ *Solution Points*

GATE 2006

42. The expression $V = \int_0^H \pi R^2 (1 - h/H)^2 \, dh$ for the volume of a cone is equal to

(a) $\int_0^R \pi R^2 (1 - h/H)^2 \, dr$ (b) $\int_0^R \pi R^2 (1 - h/H)^2 \, dh$

(c) $\int_0^H 2\pi r H (1 - r/R) \, dh$ (d) $\int_0^R 4\pi r H \left(1 - \frac{r}{R}\right)^2 dr$

43. A surface $S(x, y) = 2x + 5y - 3$ is integrated once over a path consisting of the points that satisfy $(x + 1)2 + (y - 1)2 = \sqrt{2}$. The integral evaluates to

(a) $17\sqrt{2}$ (b) $17/\sqrt{2}$ (c) $\sqrt{2}/17$ (d) 0

44. Two fair dice are rolled and the sum r of the numbers turned up is considered

(a) $\Pr(r > 6) = \frac{1}{6}$ (b) $\Pr(r/3 \text{ is an integer}) = \frac{5}{6}$

(c) $\Pr(r = 8 \mid r/4 \text{ is an integer}) = \frac{5}{9}$ (d) $\Pr(r = 6 \mid r/5 \text{ is an integer}) = \frac{1}{18}$

Statements for Linked Answer Questions 45 and 46

$$P = \begin{bmatrix} -10 \\ 1 \\ 3 \end{bmatrix}^T, Q = \begin{bmatrix} -2 \\ -5 \\ 9 \end{bmatrix}^T, R = \begin{bmatrix} 2 \\ -7 \\ 12 \end{bmatrix}^T \text{ are three vectors.}$$

45. An orthogonal set of vectors having a span that contains P, Q, R is

(a) $\begin{bmatrix} -6 \\ -3 \\ -6 \end{bmatrix} \begin{bmatrix} 4 \\ -2 \\ 3 \end{bmatrix}$ (b) $\begin{bmatrix} -4 \\ 2 \\ 4 \end{bmatrix} \begin{bmatrix} 5 \\ 7 \\ -11 \end{bmatrix} \begin{bmatrix} 8 \\ 2 \\ -3 \end{bmatrix}$

(c) $\begin{bmatrix} 6 \\ 7 \\ -1 \end{bmatrix} \begin{bmatrix} -3 \\ 2 \\ -2 \end{bmatrix} \begin{bmatrix} 3 \\ 9 \\ -4 \end{bmatrix}$ (d) $\begin{bmatrix} 4 \\ 3 \\ 11 \end{bmatrix} \begin{bmatrix} 1 \\ 31 \\ 3 \end{bmatrix} \begin{bmatrix} 5 \\ 3 \\ 4 \end{bmatrix}$

46. Which of the following vectors is linearly dependent upon the solution to the previous problem?

(a) $\begin{bmatrix} 8 \\ 9 \\ 3 \end{bmatrix}$ (b) $\begin{bmatrix} -2 \\ -17 \\ 30 \end{bmatrix}$ (c) $\begin{bmatrix} 4 \\ 4 \\ 5 \end{bmatrix}$ (d) $\begin{bmatrix} 13 \\ 2 \\ -3 \end{bmatrix}$

GATE 2005

47. For the matrix $P = \begin{bmatrix} 3 & -2 & 2 \\ 0 & -2 & 1 \\ 0 & 0 & 1 \end{bmatrix}$, one of the eigen values is equal to -2. Which of the following is an eigen vector?

(a) $\begin{bmatrix} 3 \\ -2 \\ 1 \end{bmatrix}$ (b) $\begin{bmatrix} -3 \\ 2 \\ -1 \end{bmatrix}$ (c) $\begin{bmatrix} 1 \\ -2 \\ 3 \end{bmatrix}$ (d) $\begin{bmatrix} 2 \\ 5 \\ 0 \end{bmatrix}$

▼ *Solution Points*

48. If $R=\begin{bmatrix}1 & 0 & -1\\ 2 & 1 & -1\\ 2 & 3 & 2\end{bmatrix}$, then top row of R^{-1} is

(a) [5 6 4] (b) [5 −3 1] (c) [2 0 −1] (d) [2 −1 1/2]

49. A fair coin is tossed three times in succession. If the first toss produces a head, then the probability of getting exactly two heads in three tosses is

(a) $\frac{1}{8}$ (b) $\frac{1}{2}$ (c) $\frac{3}{8}$ (d) $\frac{3}{4}$

50. For the function $f(x)=x^2e^{-x}$, the maximum occurs when x is equal to

(a) 2 (b) 1 (c) 0 (d) −1

51. For the scalar field $u=\frac{x^2}{2}+\frac{y^3}{3}$, magnitude of the gradient at the point (1, 3) is

(a) $\sqrt{\frac{13}{9}}$ (b) $\sqrt{\frac{9}{2}}$ (c) $\sqrt{5}$ (d) $\frac{9}{2}$

52. For the equation $x''(t)+3x'(t)+2x(t)=5$, the solution $x(t)$ approaches which of the following values as $t\to\infty$?

(a) 0 (b) $\frac{5}{2}$ (c) 5 (d) 10

Answers with Explanations

1. (a) $\dfrac{dy}{dx} = e^{-3x}$

$dy = e^{-3x}dx$

Taking integration on both sides,

$$y = \frac{e^{-3x}}{-3} + K = -\frac{1}{3}e^{-3x} + K$$

2. (d) $x^3 + x^2 + x + 1 = 0$

$(x^2 + 1)(x + 1) = 0$

$x^2 + 1 = 0 \Rightarrow x = \pm j$

$x + 1 = 0 \Rightarrow x = -1$

3. (d) The given signal satisfies even symmetry so $b_n = 0$ and it also satisfies half wave symmetry so, it will contain only odd harmonics.

4. (b) $P = \int_0^1 xe^x\,dx$

$= [x \int e^x\,dx]_0^1 - \int_0^1 \left\{\dfrac{d}{dx}(x)\int e^x\,dx\right\}dx$

$= [x \cdot e^x]_0^1 - \int_0^1 1 \cdot e^x\,dx$

$= (1 \cdot e - 0) - [e^x]_0^1$

$= e - e + e^0 = 1$

5. (a) The radial vector in 3-D space is

$$\vec{\mathbf{r}} = x\hat{\mathbf{i}} + y\hat{\mathbf{j}} + z\hat{\mathbf{k}}$$

Now, div $\vec{\mathbf{r}} = \nabla \cdot \vec{\mathbf{r}}$

$$= \left(\hat{\mathbf{i}}\frac{\partial}{\partial x} + \hat{\mathbf{j}}\frac{\partial}{\partial y} + \hat{\mathbf{k}}\frac{\partial}{\partial z}\right)(x\hat{\mathbf{i}} + y\hat{\mathbf{j}} + z\hat{\mathbf{k}})$$

$$= \frac{\partial}{\partial x}(x) + \frac{\partial}{\partial y}(y) + \frac{\partial}{\partial z}(z)$$

$$= 1 + 1 + 1 = 3$$

6. (d) The given signal

$$x(t) = 8\sin\left(0.8\pi t + \frac{\pi}{4}\right)$$

We know that, imperiodic function

$f(t + T) = f(t)$, where T = period

Then, $x(t + T) = x(t)$

$$\Rightarrow 8\sin\left\{0.8\,\pi(t + T) + \frac{\pi}{4}\right\} = 8\sin\left(0.8\,\pi t + \frac{\pi}{4}\right)$$

$$\Rightarrow \sin\left\{0.8\,\pi(t + T) + \frac{\pi}{4}\right\} = \sin\left\{2\pi + \left(0.8\,\pi t + \frac{\pi}{4}\right)\right\}$$

{$\because$ $\sin(2\pi + \theta) = \sin\theta$, period of $\sin\theta$ is 2π}

$$\Rightarrow 0.8\,\pi(t + T) + \frac{\pi}{4} = 2\pi + 0.8\,\pi t + \frac{\pi}{4}$$

$$\Rightarrow 0.8\,\pi t + 0.8\,T\pi = 2\pi + 0.8\,\pi t$$

$$\Rightarrow T = \frac{2\pi}{0.8\,\pi} \Rightarrow \frac{2}{0.8}$$

$$\Rightarrow T(\text{period}) = 2.5\text{ s}$$

7. (c) Given, trace of 2×2 matrix $= -2$

and determinant $= -35$

Let $A = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}_{2\times 2}$

then, trace of matrix

A = Addition of principal diagonal elements

$-2 = a_{11} + a_{22}$ ...(i)

and $|A| = a_{11} \cdot a_{22} - a_{12} \cdot a_{21} = -35$ ...(ii)

Also the eigen values of $[A]$

= Trace of matrix A

$\lambda_1 + \lambda_2 = -2$ ...(iii)

So, option (c) will satisfy Eq. (iii).

Hence, eigen values are -7 and 5.

8. (c) Since, X is a uniformly distributed random variable between $(0, 1) = (a, b)$

Then, probability density function

$$f(x) = \frac{1}{(b - a)} \Rightarrow \frac{1}{(1 - 0)} \Rightarrow 1$$

So, $f(x) = \begin{cases} 1, & 0 < x < 1 \\ 0, & \text{other value of } x \end{cases}$

So, $E(x^3) = \int_{x=0}^{1} x^3 \cdot f(x)\,dx$

$$= \int_0^1 x^3 \cdot 1\,dx$$

$$= \left[\frac{x^4}{4}\right]_0^1$$

$$= \frac{1}{4}(1 - 0)$$

$$= \frac{1}{4}$$

9. (d) Given, the characteristic equation is

$\lambda^3 + \lambda^2 + 2\lambda + I = 0$ ...(i)

We know that,

By Caylay-Hamilton theorem Every square matrix satisfies its characteristic equation.

Then, from Eq. (i), putting $(\lambda = P)$,

$$P^3 + P^2 + 2P + I = 0$$

Operating (P^{-1}) on both sides,

$$(P^{-1}P^3) + (P^{-1}P^2) + 2(P^{-1}P) + (P^{-1}I) = (0 \cdot P^{-1})$$

$$\Rightarrow \quad P^2 + P + 2I + P^{-1} = 0$$

$$\Rightarrow \quad P^{-1} = -(P^2 + P + 2I)$$

10. (a) Given, $f([Q]_{5\times 6}) = 4$ {$I \rightarrow$ denotes Rank of}

Then, [Q] must have four linearly independent rows and four linearly independent columns.

Becuase the matrix Q of order (4×4) will be non-singular matrix *i.e.*, $[Q]_{4\times 4}$

$$\Rightarrow \quad |Q|_{4\times 4} \neq 0$$

11. (a) Given, $x = [x_1\ x_2 \ldots x_n]^T$

Since, x is an n-tuple non-zero vector, that is, x is a non-singular matrix of order n, so its rank should be n.

i.e., $\quad I(x) = n$

The vector (x^T) also have rank (n) because transpose of any matrix does not altered its rank.

Then, matrix $[v = xT']$ must have the rank n, *i.e.*, $I(v) = n$, because resultant of the multiplication of two same rank matrices also has the same rank as the rank of multiplicative matrix.

12. (a) The matrix equation $px = q$ will have solutions if it is consistent.

So, for consistency of matrix equation is

Rank of $[pq]$ = Rank of $[p]$

where, $[pq] \rightarrow$ Augmented matrix

13. (d) Given, P and Q are two random events, then

(a) Independence of P and Q,

Prob $(P \cap Q)$ = Prob (P) · Prob (Q)

If two events P and Q are mutually disjoint, then

Prob $(P \cap Q) = 0$

So, option (a) is incorrect.

(b) For two events P and Q,

Prob

$(P \cup Q)$ = Prob (P) + Prob (Q) − Prob $(P \cap Q)$

Here, Prob $(P \cup Q) \leq$ Prob (P) + Prob (Q)

So, option (b) is incorrect.

(c) There is no relation between mutually exclusive and independent for two random variables P and Q.

So, option (c) is incorrect.

(d) And Prob $(P \cap Q) \leq$ Prob (P) which is true for every events.

So, option (d) is correct.

14. (c) $\quad S = \int_1^\infty x^{-3}\,dx$

$$= \left[\frac{x^{-2}}{-2}\right]_1^\infty = -\frac{1}{2}\left[\frac{1}{x^2}\right]_1^\infty$$

$$= \frac{1}{2}\left[\frac{1}{\infty} - \frac{1}{1}\right] = -\frac{1}{2}(0-1)$$

$$= \frac{1}{2}$$

15. (a) $\quad x'(t) = -3x(t) \Rightarrow \frac{dx}{dt} = -3x$

$$\Rightarrow \quad \int \frac{dx}{x} = -\int 3\,dt \qquad \text{(on integrating)}$$

$$\Rightarrow \quad \log x = -3t + \log c$$

$$\Rightarrow \quad x = ce^{-3t} \qquad \ldots(i)$$

But at $(t = 0)$ and $(x = x_0)$,

$$\Rightarrow \quad x_0 = ce^0 \Rightarrow (c = x_0)$$

Then, from Eq. (i),

$$x = x_0 e^{-3t}$$

or $\quad x(t) = x_0 e^{-3t}$

16. (b) These two vectors is orthogonal because the dot product of these two vectors is zero.

17. (d) $\quad [A] = [L]\,[U]$

$$= \begin{bmatrix} 2 & 0 \\ 4 & -3 \end{bmatrix}\begin{bmatrix} 1 & 0.5 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 2+0 & 1+0 \\ 4+0 & 2-3 \end{bmatrix}$$

$$= \begin{bmatrix} 2 & 1 \\ 4 & -1 \end{bmatrix}$$

18. (c) $\quad f(x) = 2x - x^2 + 3$

$$f'(x) = 2 - 2x = 0$$

$$\therefore \quad x = 1$$

at $\quad x = 1,\ f''(x) = -2 < 0 \Rightarrow f''(x) < 0$

So, $f(x)$ having only a maximum at $x = 1$.

19. (d) Given, $\quad f(t) = \frac{\sin t}{t}$

At $t = 0$, first we will check continuity of the function.

$$\text{LHL} \quad f(0-h) = \lim_{h\to 0} \frac{\sin(0-h)}{(0-h)}$$

$$= \lim_{h\to 0} \frac{-\sin h}{-h} \qquad [\because \sin(-\theta) = -\sin\theta]$$

$$= 1 \qquad \left[\because \lim_{\theta\to 0} \frac{\sin\theta}{\theta} = 1\right]$$

$$\text{RHL} \quad f(0+h) = \lim_{h\to 0} \frac{\sin(0+h)}{(0+h)}$$

$$= \lim_{h\to 0} \frac{\sin h}{h} = 1$$

and $f(0) = 1$

since, LHL = RHL = $f(0)$

So, the function is continuous at $t = 0$.

Now, we check the function is maximum or minimum.

$$f'(t) = \frac{1}{t}\cos t - \frac{1}{t^2}\sin t$$

and $$f''(t) = -\frac{1}{t}\sin t - \frac{1}{t^2}\cos t - \frac{1}{t^2}\cos t + \frac{2\sin t}{t^3}$$

$$= -\frac{\sin t}{t} - \frac{2\cos t}{t^2} + \frac{2\sin t}{t^3}$$

For max or min value of $f(x)$,

$$f'(x) = 0$$

$$\frac{\cos t}{t} - \frac{\sin t}{t^2} = 0$$

$$\Rightarrow \quad \frac{\tan t}{t} = 1$$

Now, $$\lim_{t\to 0} f''(t) = -\lim_{t\to 0}\left(\frac{\sin t}{t}\right) + \lim_{t\to 0}\frac{(2\sin t - 2t\cos t)}{t^3} \quad \left[\therefore \left(\frac{0}{0}\right)\text{form}\right]$$

$$= -1 + \lim_{t\to 0}\frac{(2\cos t - 2\cos t + 2t\sin t)}{3t^2}$$

[$\because$ Using L hospital's rule]

$$= -1 + \lim_{t\to 0}\frac{2t\sin t}{3t}$$

$$= -1 + \frac{2}{3}\lim_{t\to 0}\left(\frac{\sin t}{t}\right)$$

$$= -1 + \frac{2}{3}\times 1 = -\frac{1}{3} < 0 \qquad \text{(Maxima)}$$

So, function $f(t)$ is maximum at $t = 0$.

20. (c) Probability (IInd ball is red/Ist ball is white)

$$= \frac{P(\text{IInd is red and Ist is white})}{P(\text{Ist ball is white})}$$

$$= \frac{P(\text{Ist ball is white and IInd ball is red})}{P(\text{Ist ball is white})}$$

$$= \frac{(\text{Probability of Ist ball is white}) \times (\text{Probability of IInd ball is red})}{(\text{Probability of Ist ball is white})}$$

= Probability of IInd ball is red

$$= \frac{3}{6} = \frac{1}{2}$$

21. (b) $$P = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 2 & 2 \\ 0 & 0 & 3 \end{bmatrix}$$

The characteristic equation of P is

$$|P - \lambda I| = 0$$

$$\begin{vmatrix} 1-\lambda & 1 & 0 \\ 0 & 2-\lambda & 2 \\ 0 & 0 & 3-\lambda \end{vmatrix} = 0$$

$$(1-\lambda)(2-\lambda)(3-\lambda) = 0$$

$(\lambda = 1, 2, 3)$ are the eigen values of P.

Let $x_1 = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$ be the eigen vector, corresponding to the eigen value $(\lambda = 3)$, then

Characteristic matrix $= (A - 3I)$

$$= \begin{bmatrix} 1-3 & 1 & 0 \\ 0 & 2-3 & 2 \\ 0 & 0 & 3-3 \end{bmatrix} = \begin{bmatrix} -2 & 1 & 0 \\ 0 & -1 & 2 \\ 0 & 0 & 0 \end{bmatrix}$$

Now, the matrix form is $(AX = 0)$

$$\begin{bmatrix} -2 & 1 & 0 \\ 0 & -1 & 2 \\ 0 & 0 & 0 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$-2x + y = 0 \qquad \ldots(i)$$

$$-y + 2z = 0 \qquad \ldots(ii)$$

Let $z = k$

From Eq. (ii), $y = 2k$

From Eq. (i), $x = k$

So, the eigen vector x_1 corresponding to the eigen value $(\lambda = 3)$ is $\begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix}$ or $[1 \;\; 2 \;\; 1]^T$.

22. (b) $\dfrac{d^2x}{dt^2} + 6\dfrac{dx}{dt} + 8x = 0$, Given $\begin{cases} t = 0 \text{ and } x = 1 \\ \dfrac{dx}{dt} = 0 \text{ at } t = 0 \end{cases}$

Auxiliary equation of $(D^2 + 6D + 8)x = 0$ is

$$m^2 + 6m + 8 = 0$$

$$(m+2)(m+4) = 0 \Rightarrow m = -2, -4$$

$$CF = c_1 e^{-2t} + c_2 e^{-4t}$$

and $PI = 0$

So, the complete solution is

$$x = CF + PI$$

$$x = c_1 e^{-2t} + c_2 e^{-4t} \qquad \ldots(i)$$

$$\frac{dx}{dt} = -2c_1e^{-2t} - 4c_2e^{-4t} \quad \ldots(ii)$$

At $t = 0$ and $x = 1$,

From Eq. (i), $1 = c_1 + c_2$ ...(iii)

At $\frac{dx}{dt} = 0$ and $t = 0$;

From Eq. (ii), $0 = -2c_1 - 4c_2$

$\Rightarrow$ $c_1 + 2c_2 = 0$ ...(iv)

$\Rightarrow$ $c_2 = -1$ and $c_1 = 2$

Hence, $x = 2e^{-2t} - e^{-4t}$

or $x(t) = 2e^{-2t} - e^{-4t}$

23. (d) Given system of equations

$$x_1 + 2x_2 + x_3 + 4x_4 = 2$$

$$3x_1 + 6x_2 + 3x_3 + 12x_4 = 6$$

which is the non-homogeneous system of equations.

The augmented matrix is

$$[A : B] = \begin{bmatrix} 1 & 2 & 1 & 4 & : & 2 \\ 3 & 6 & 3 & 12 & : & 6 \end{bmatrix}$$

Applying $R_3 \to R_3 - 3R_1$,

$$\sim \begin{bmatrix} 1 & 2 & 1 & 4 & : & 2 \\ 0 & 0 & 0 & 0 & : & 0 \end{bmatrix}$$

Here, Rank of $[A : B] = 1$

and Rank of $A = 1$

Since, $f[A : B] = f[A]$

So, the given system of equations is consistent.

But, the rank of $A <$ the number of unknowns

So, the system of non-homogeneous equation will have infinitely many solutions.

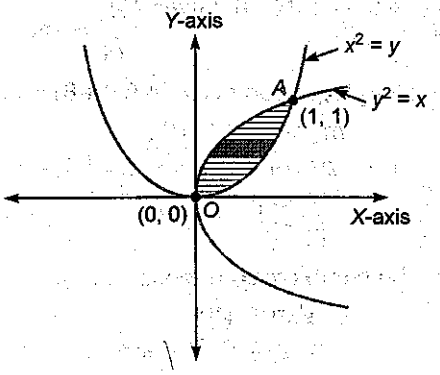
In other words, multiple non-trivial solutions exist.

24. (a) Given that

$f(x, y)$ is a continuous function.

and $(x, y) \in [0, 1] \times [0, 1]$

Then, the intersection point of the curves $x = y^2$ and $y = x^2$ is



$$y = y^4 \Rightarrow y(y^3 - 1) = 0$$

$$y(y - 1)(y^2 + 1 + y) = 0$$

$\Rightarrow$ $y = 0$ and 1

$\Rightarrow$ $x = 0$ and 1

So, points are (0, 0) and (1, 1).

So, the volume under $f(x, y)$ is

$$= \int_{y=0}^{y=1} \int_{x=y^2}^{x=\sqrt{y}} f(x, y)\, dy\, dx$$

or $$\int_{y=0}^{y=1} \int_{x=y^2}^{x=\sqrt{y}} f(x, y)\, dx\, dy$$

25. (b) Let N people in room.

So, number of events that at least two people in room born at same date N_{C_2}

$$\therefore \quad 0.5 \geq \frac{N_{C_2} \times N_{C_3} \times N_{C_4} \ldots N_{C_N}}{N!}$$

$\Rightarrow$ $N = 7$

26. (c) A cubic polynomial with real coefficients cannot have more than two extrema and more than three zero crossings.

27. (a) Let $f(x) \equiv x^2 - 117 = 0$

and $f'(x) = 2x$

Now by Newton-Raphson's method,

$$x_{k+1} = x_k - \frac{f(x_k)}{f'(x_k)}$$

$$x_{k+1} = x_k - \frac{(x_k^2 - 117)}{(2x_k)}$$

$$= \frac{(2x_k^2 - x_k^2 + 117)}{2x_k}$$

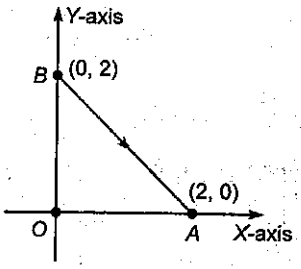
$$= \frac{(x_k^2 + 117)}{2x_k}$$

$\Rightarrow$ $$x_{k+1} = \frac{1}{2}\left(x_k + \frac{117}{x_k}\right)$$

28. (d) Given, $F(x, y) = (x^2 + xy)\,\hat{\mathbf{a}}_x + (y^2 + xy)\,\hat{\mathbf{a}}_y$

$$\begin{cases} \hat{\mathbf{a}}_x = \text{component of } x \text{ in } X\text{- axis} \\ \quad = \hat{\mathbf{i}} \\ \hat{\mathbf{a}}_y = \text{component of } y \text{ in } Y\text{- axis} \\ \quad = \hat{\mathbf{j}} \end{cases}$$

$\Rightarrow$ $F(x, y) = (x^2 + xy)\,\hat{\mathbf{i}} + (y^2 + xy)\,\hat{\mathbf{j}}$ ...(i)



The equation of line BA is

$$(y-2)=\frac{0-2}{2-0}(x-0)$$

$$y-2=-x$$

$$x+y=2 \qquad \ldots(ii)$$

$$dx=-dy$$

Let $\vec{r}$ be the position vector in 2-dimensional space then,

$$\vec{r}=x\hat{i}+y\hat{j}$$

$$d\vec{r}=dx\hat{i}+dy\hat{j}$$

Then, the line integral over the line BA is

$$\oint_C \vec{F}\cdot d\vec{r}$$

$$=\oint_C \{(x^2+xy)\hat{i}+(y^2+xy)\hat{j}\}\cdot(dx\hat{i}+dy\hat{j})$$

$$=\oint_C (x^2+xy)\,dx+(y^2+xy)\,dy$$

$$=\int_{x=0}^{x=2}\{x^2+x(2-x)\}\,dx+\{(2-x)^2+x(2-x)\}(-dx)$$

$$=\int_0^2(x^2+2x-x^2-4-x^2+4x-2x+x^2)\,dx$$

$$=\int_0^2 4(x-1)\,dx=\left[4\left(\frac{x^2}{2}-x\right)\right]_0^2$$

$$=4(2-2)=4\cdot 0=0$$

29. (b) Given function is $f(x)=(x^2-4)^2$ and $x\in R$

$$\Rightarrow \qquad f'(x)=2(x^2-4)(2x)$$

For maximum or minimum value of $f(x)$,

Putting $f'(x)=0$,

$$4x(x^2-4)=0$$

$$\Rightarrow \qquad x=0,2,-2$$

Now, $f''(x)=4\{(x^2-4)\cdot 1+x(2x)\}$

$$=4\{x^2-4+2x^2\}$$

$$=4(3x^2-4)$$

At $(x=0)$, $f''(0)=-16<0$, (Maxima)

At $(x=2)$, $f''(2)=32>0$, (Minima)

At $(x=-2)$, $f''(-2)=32>0$, (Minima)

Hence, the function $f(x)$ has only two minima.

30. (a) Let $f(x)\equiv e^x-1=0$

$$\Rightarrow \qquad f'(x)\equiv e^x$$

Now, by Newton-Raphson method,

$$x_{k+1}=x_k-\frac{f(x_k)}{f'(x_k)}$$

$$x_{k+1}=x_k-\frac{(e^{x_k}-1)}{(e^{x_k})}=\frac{(e^{x_k}\cdot x_k-e^{x_k}+1)}{e^{x_k}} \qquad \ldots(i)$$

But at $(k=0), x_0=-1$

$$x_1=\frac{(x_0\cdot e^{x_0}-e^{x_0}+1)}{e^{x_0}}=\frac{(-1\cdot e^{-1}-e^{-1}+1)}{e^{-1}}$$

$$=\frac{(-1/e-1/e+1)}{1/e}=\frac{(-2+e)}{1}$$

$$\Rightarrow \qquad x_1=0.71828$$

31. (b) Given $A^T=(A^TA)^{-1}A^T$

$$=A^{-1}(A^T)^{-1}A^T \qquad [\because AA^{-1}=I]$$

$$=A^{-1}(A^T)(A^T)^{-1}$$

$$=A^{-1}I=A^{-1} \qquad \ldots(i)$$

(a) LHS $=AA^TA$

$$=A[(A^TA)^{-1}A^T]\cdot A=(A\cdot A^{-1})A$$

$$=I\cdot A=A \qquad \text{(True)}$$

(b) $(AA^t)^2=[AA^{-1}]^2$ [From Eq. (i)]

$$=[I]^2=I \qquad \text{(False)}$$

$\Rightarrow$ It is not an orthogonal matrix.

(c) LHS $A^TA=A^{-1}A$ [Ffrom Eq. (i)]

$$=I \qquad \text{(True)}$$

$=$ orthogonal matrix

32. (c) The given differential equation

$$\frac{dx}{dt}=e^{-2t}u(t)$$

$$\Rightarrow \qquad x=\int e^{-2t}u(t)\,dt \qquad \text{(After integrating)}$$

Let $x=\int f(t)\,dt \Rightarrow [f(t)=e^{-2t}u(t)]$

where, $(u(t)=$ unit step function

that means $v(t)=1$

Given, step size $h=0.01$ s

Also, $x(0)=0$

Now, $f(0)=e^{-0}\cdot 1=1$

and $f(0.01)=e^{-2(0.01)}\cdot 1=e^{-0.02}$

Then, by trapezoidal rule,

$$x=\int f(t)\,dt=\frac{h}{2}[(f(0)+f(0.01))]$$

$$=\frac{0.01}{2}[1+e^{-0.02}]=\frac{0.01}{2}(1.98)$$

$$x=0.0099$$

33. (b) Let the real orthogonal matrix of order 2×2 is

$$P=\frac{1}{2\sqrt{2}}\begin{bmatrix}2 & -2\\ 2 & 2\end{bmatrix}$$

and $\quad \vec{\mathbf{x}}=[x_1,x_2]^T,\ |\vec{\mathbf{x}}|=x_1\hat{\mathbf{i}}+x_2\hat{\mathbf{j}}$

Length $||\vec{\mathbf{x}}||=\sqrt{x_1^2+x_2^2}$

Now, $(P\vec{\mathbf{x}})=\frac{1}{2\sqrt{2}}\begin{bmatrix}2 & -2\\ 2 & 2\end{bmatrix}\begin{bmatrix}x_1\\ x_2\end{bmatrix}$

$$(P\vec{\mathbf{x}})=\frac{1}{2\sqrt{2}}\begin{bmatrix}2x_1-2x_2\\ 2x_1+2x_2\end{bmatrix}$$

$$|P\vec{\mathbf{x}}|=\frac{1}{2\sqrt{2}}[(2x_1-2x_2)\hat{\mathbf{i}}+(2x_1+2x_2)\hat{\mathbf{j}}]$$

$$=\frac{1}{\sqrt{2}}[(x_1-x_2)\hat{\mathbf{i}}+(x_1+x_2)\hat{\mathbf{j}}]$$

$$||P\vec{\mathbf{x}}||=\frac{1}{\sqrt{2}}\sqrt{(x_1-x_2)^2+(x_1+x_2)^2}$$

$$=\frac{1}{\sqrt{2}}\sqrt{x_1^2+x_2^2-2x_1x_2+x_1^2+x_2^2+2x_1x_2}$$

$$=\frac{1}{\sqrt{2}}\cdot\sqrt{2}\sqrt{x_1^2+x_2^2}$$

$$=\sqrt{x_1^2+x_2^2}$$

$$=||\vec{\mathbf{x}}||$$

$||P\vec{\mathbf{x}}||=||\vec{\mathbf{x}}||$ for all $\vec{\mathbf{x}}$.

34. (d) The maximum permissible value of $\Delta T=2\tau$

35. (b) We have, $\oint_C \frac{1}{1+z^2}dz$ and $|z-i/2|=1$

$$\Rightarrow \oint_{|z-i/2|}\frac{dz}{1+z^2}=\oint_{|z-i/2|}\frac{dz}{(z+i)(z-i)}$$

$P(0,1)$ lies inside the circle $|z-i/2|=1$ and $P(0,-1)$ does not lie. Thus, by Cauchy's integral formula,

$$I=2\pi i\lim_{z\to i}(z-i)\cdot\frac{1}{(z+i)(z-i)}$$

$$I=2\pi i\frac{1}{2i}$$

$$I=\pi$$

36. (d) $\frac{1}{2\pi}\int_0^{2\pi}\sin(t-\tau)\cos\tau\,d\tau$

$$=\frac{1}{2\pi}\int_0^{2\pi}(\sin t\cos\tau-\cos t\sin\tau)\cdot\cos\tau\,d\tau$$

$$=\frac{1}{2\pi}\int_0^{2\pi}\sin t\cos^2\tau\,d\tau-\frac{1}{2\pi}\int_0^{2\pi}\cos t\sin\tau\cos\tau\,d\tau$$

$$=\frac{1}{4\pi}\sin t\int_0^{2\pi}(1+\cos 2\tau)\,d\tau-\frac{1}{4\pi}\cos t\int_0^{2\pi}\sin 2\tau\,d\tau$$

$$=\frac{1}{4\pi}\sin t\left[\tau+\frac{\sin 2\tau}{2}\right]_0^{2\pi}-\frac{1}{4\pi}\cos t\left[-\frac{1}{2}\cos 2\tau\right]_0^{2\pi}$$

$$=\frac{1}{4}\sin t\left\{2\pi+\frac{1}{2}\cdot 0-0\right\}-\frac{1}{4\pi}\cos t\left\{-\frac{1}{2}\cdot 1+\frac{1}{2}\cdot 1\right\}$$

$$=\frac{1}{4\pi}\sin t\,(2\pi)-\frac{1}{4\pi}\cos t\left(-\frac{1}{2}+\frac{1}{2}\right)$$

$$=\frac{1}{2}\sin t$$

37. (c) Probability of occurrence of values 1, 5 and 6 on three dice $=P(1)\times P(5)\times P(6)$

$$=\frac{1}{4}\times\frac{1}{8}\times\frac{1}{4}=\frac{1}{128}$$

Similarly, probability of occurence of values 3, 4 and 5 on three dice $=P(3)\times P(4)\times P(5)$

$$=\frac{1}{8}\times\frac{1}{8}\times\frac{1}{8}\Rightarrow\frac{1}{512}$$

and, probability of occurence of values 1, 2 and 5 on three dice $=P(1)\times P(2)\times P(5)$

$$=\frac{1}{4}\times\frac{1}{8}\times\frac{1}{8}$$

$$=\frac{1}{256}$$

38. (d) Let the vector x in 3-dimensional space is

$$\vec{\mathbf{x}}=x_1\hat{\mathbf{i}}+x_2\hat{\mathbf{j}}+x_3\hat{\mathbf{k}}$$

and, the vector y in 3-D space is

$$\vec{\mathbf{y}}=y_1\hat{\mathbf{i}}+y_2\hat{\mathbf{j}}+y_3\hat{\mathbf{k}}$$

where, $<x,y>=x\cdot y=<y_1x>$

$$=x_1y_1+x_2y_2+x_3y_3$$

Then, $<x,x>=x\cdot x=x_1^2+x_2^2+x_3^2$

$$<y,y>=y\cdot y=y_1^2+y_2^2+y_3^2$$

Then, $\begin{vmatrix}<x,x> & <x,y>\\ <y,x> & <y,y>\end{vmatrix}=\begin{vmatrix}\Sigma x_1^2 & \Sigma x_1y_1\\ \Sigma x_1y_1 & \Sigma y_1^2\end{vmatrix}$

$$=(\Sigma x_1^2)(\Sigma y_1^2)-(\Sigma x_1y_1)^2$$

Here, when either $\vec{\mathbf{x}}$ or $\vec{\mathbf{y}}$ is zero, then

$=0$ *i.e.*, $\begin{cases}\vec{\mathbf{x}}=0\hat{\mathbf{i}}+0\hat{\mathbf{j}}+0\hat{\mathbf{k}}\\ \text{or } \vec{\mathbf{y}}=0\hat{\mathbf{i}}+0\hat{\mathbf{j}}+0\hat{\mathbf{k}}\end{cases}$

39. (d) Since, $L(x) = M\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$

Then, the eigen values of M are $i, -i, 0$.

40. (a) Given, $A = \begin{bmatrix} -3 & 2 \\ -1 & 0 \end{bmatrix}$

The characteristic equation of A is

$$|A - \lambda I| = 0$$

$$\begin{vmatrix} (-3-\lambda) & 2 \\ -1 & (0-\lambda) \end{vmatrix} = 0 \Rightarrow (-3-\lambda)(-\lambda) + 2 = 0$$

$$\Rightarrow \quad 3\lambda + \lambda^2 + 2 = 0$$

$$\Rightarrow \quad \lambda^2 + 3\lambda + 2 = 0 \qquad \ldots(i)$$

Now, by Caylay-Hamilton theorem, "Every square matrix satisfies its characteristic equation.

Then, putting $\lambda = A$ in Eq. (i),

$$A^2 + 3A + 2 = 0 \qquad \ldots(ii)$$

Operating (A^{-1}) on both sides,

$$(A^{-1}A)\,A + 3(A^{-1}A) + 2A^{-1} = 0$$

$$[\because AA^{-1} = I,\ AI = A]$$

$$I \cdot A + 3I + 2A^{-1} = 0$$

$$A + 3I + 2A^{-1} = 0$$

41. (a) From Eq. (ii),

$$A^2 + 3A + 2 = 0$$

$$\Rightarrow \quad A^2 = -3A - 2$$

$$\Rightarrow \quad A^3 = -3A^2 - 2A$$

$$= -3(-3A - 2) - 2A$$

$$= 9A - 2A + 6$$

$$A^3 = 7A + 6$$

$$\Rightarrow A^4 = 7A^2 + 6A = 7(-3A - 2) + 6A \Rightarrow -15A - 14$$

$$\Rightarrow \quad A^5 = 7A^3 + 6A^2$$

$$= 7(7A + 6) + 6(-3A - 2)$$

$$= 49A + 42 - 18A - 12$$

$$A^5 = 31A + 30$$

$$\Rightarrow \quad A^6 = 31A^2 + 30A$$

$$\Rightarrow \quad A^7 = 31A^3 + 30A^2$$

$$\Rightarrow \quad A^8 = 31A^4 + 30A^3$$

$$\Rightarrow \quad A^9 = 31A^5 + 30A^4$$

$$= 31(31A + 30) + 30(-15A - 14)$$

$$= 961A + 930 - 450A - 420$$

$$A^9 = 511A + 510$$

42. (d) $V = \int_0^H \pi R^2 (1 - h/H)^2\, dh =$ Volume of a cone

$$V = \pi R^2 \int_0^H (1 - h/H)^2\, dh = \pi R^2 \left[\frac{(1-h/H)^3}{3} \times -H\right]_0^H$$

$$= -\frac{\pi R^2 H}{3}[0 - 1] = 1 + \frac{1}{3}\pi R^2 H \qquad \ldots(i)$$

(a) The integral does not contain the variable $\vec{r}$. So, we never get the volume of cone.

(b) Since, the integrand of this option and integrand of given integral is same but respective variable is different. So, we can't find the volume of cone.

(c) The integral does not contain the integral variable h. So, we never get the volume of cone.

(d) Now, we check last option

$$\int_0^R 4\pi r H\left(1 - \frac{r}{R}\right)^2 dr = 4\pi H \int_0^R r\left(1 + \frac{r^2}{R^2} - \frac{2r}{R}\right) dr$$

$$= 4\pi H \int_0^R \left(r + \frac{r^3}{R^2} - \frac{2r^2}{R}\right) dr$$

$$= 4\pi H \left[\frac{r^2}{2} + \frac{r^4}{4R^2} - \frac{2r^3}{3R}\right]_0^R$$

$$= 4\pi H \left\{\frac{R^2}{2} + \frac{R^2}{4} - \frac{2}{3}R^2\right\}$$

$$= 4\pi H \left\{\frac{3R^2}{4} - \frac{2}{3}R^2\right\}$$

$$= 4\pi H \left\{\frac{9R^2 - 8R^2}{12}\right\}$$

$$= 4\pi H \cdot \frac{R^2}{12}$$

$$= \frac{1}{3}\pi R^2 H$$

43. (d) The required integral $= 0$

44. (c) If two fair dice rolle the probability distribution of r, where r is the sum of the numbers on each dice is given by

r	2	3	4	5	6	7	8	9	10	11	12
$P(r)$	$\frac{1}{36}$	$\frac{2}{36}$	$\frac{3}{36}$	$\frac{4}{36}$	$\frac{5}{36}$	$\frac{6}{36}$	$\frac{5}{36}$	$\frac{4}{36}$	$\frac{3}{36}$	$\frac{2}{36}$	$\frac{1}{36}$

The above table has been obtained by taking all different ways of obtaining a particular sum.

Example A sum of 6 can be obtained by (1, 5), (2, 4), (3, 3), (4, 2) and (5, 1).

$$\therefore \quad P(x=6)=\frac{6}{36}$$

(a) Probability $(r>6)$ = Probability $(r\geq 7)$

$$=\frac{6}{36}+\frac{5}{36}+\frac{4}{36}+\frac{3}{36}+\frac{2}{36}+\frac{1}{36}$$

$$=\frac{21}{36}=\frac{7}{12}$$

So, option (a) is wrong.

(b) Probability ($r/3$ is an integer)

= Probability $(r=3)$ + Probability $(r=6)$ + Probability $(r=9)$ + Probability $(r=12)$

$$=\frac{2}{36}+\frac{5}{36}+\frac{4}{36}+\frac{1}{36}$$

$$=\frac{12}{36}=\frac{1}{3}$$

So, option (b) is wrong.

(c) Probability ($r/4$ is an integer)

= Probability $(r=4)$ + Probability $(r=8)$ + Probability $(r=12)$

$$=\frac{3}{36}+\frac{5}{36}+\frac{1}{36}=\frac{9}{36}=\frac{1}{4}$$

and probability $(r=8)=\frac{5}{36}$

Then, probability $(r=8 \mid r/4$ is an integer)

$$=\frac{5/36}{1/4}\Rightarrow\frac{5}{9}$$

So, option (c) is correct.

(d) Probability ($r/5$ is an integer)

= Probability $(r=5)$ + Probability $(r=10)$

$$=\frac{4}{36}+\frac{3}{36}=\frac{7}{36}$$

and probability $(r=6)=\frac{5}{36}$

Then, probability $(r=6 \mid r/5$ is an integer)

$$=\frac{5/36}{7/36}$$

$$=\frac{5}{7}$$

So, option (d) is also wrong.

45. (d) The orthogonal set of vectors are

$$\begin{bmatrix}4\\3\\11\end{bmatrix}\begin{bmatrix}1\\31\\3\end{bmatrix}\begin{bmatrix}5\\3\\4\end{bmatrix}$$

46. (d) Hence, the vector is linearly dependent upon the solution is

$$\begin{bmatrix}13\\2\\-3\end{bmatrix}$$

47. (d) $P=\begin{bmatrix}3&-2&2\\0&-2&1\\0&0&1\end{bmatrix}$

Also eigen value $\lambda=-2$

Let $X_1=[x_1\ x_2\ x_3]^T$ be eigen vector of the corresponding eigen value $\lambda=-2$ then, the characteristic matrix $=A-\lambda I$

$$=A+2I$$

$$=\begin{bmatrix}3+2&-2&2\\0&-2+2&1\\0&0&1+2\end{bmatrix}$$

$$=\begin{bmatrix}5&-2&2\\0&0&1\\0&0&2\end{bmatrix}\quad [\text{Apply } R_3\to R_3-3R_2]$$

$$\sim\begin{bmatrix}5&-2&2\\0&0&1\\0&0&0\end{bmatrix}$$

Here, the rank of $P=2$

Also, the rank of P is less than the number of unknowns, then the system will have infinitely many solutions, then the matrix form of given system is

$$AX_1=0$$

$$\begin{bmatrix}5&-2&2\\0&0&1\\0&0&0\end{bmatrix}\begin{bmatrix}x_1\\x_2\\x_3\end{bmatrix}=\begin{bmatrix}0\\0\\0\end{bmatrix}$$

$$5x_1-2x_2+2x_3=0$$

$$x_3=0$$

Let $x_2=k$

Then, $5x_1=2k$

$$x_1=\frac{2k}{5}$$

Then, the corresponding eigen vector X_1 is

$$x_1 : x_2 : x_3 = \frac{2k}{5} : k : 0$$

$$x_1 : x_2 : x_3 = \frac{2}{5} : 1 : 0$$

$$x_1 : x_2 : x_3 = 2 : 5 : 0$$

$$\Rightarrow \quad X_1 = [2, 5, 0]^T$$

48. (b) $R = \begin{bmatrix} 1 & 0 & -1 \\ 2 & 1 & -1 \\ 2 & 3 & 2 \end{bmatrix}$

$$R = \begin{vmatrix} 1 & 0 & -1 \\ 2 & 1 & -1 \\ 2 & 3 & 2 \end{vmatrix} = (1)\begin{vmatrix} 1 & -1 \\ 3 & 2 \end{vmatrix} - 0 + (-1)\begin{vmatrix} 2 & 1 \\ 2 & 3 \end{vmatrix}$$

$$= (2+3) - (6-2) = 5 - 4 = 1$$

Cofactors of R

$$x_{11} = (-1)^2 \begin{vmatrix} 1 & -1 \\ 3 & 2 \end{vmatrix} = (2+3) = 5$$

$$x_{12} = (-1)^3 \begin{vmatrix} 2 & -1 \\ 2 & 2 \end{vmatrix} = -(4+2) = -6$$

$$x_{13} = (-1)^4 \begin{vmatrix} 2 & 1 \\ 2 & 3 \end{vmatrix} = (6-2) = 4$$

$$x_{21} = (-1)^3 \begin{vmatrix} 0 & -1 \\ 3 & 2 \end{vmatrix} = -(0+3) \Rightarrow -3$$

$$x_{22} = (-1)^4 \begin{vmatrix} 1 & -1 \\ 2 & 2 \end{vmatrix} = (2+2) \Rightarrow 4$$

$$x_{23} = (-1)^5 \begin{vmatrix} 1 & 0 \\ 2 & 3 \end{vmatrix} = -(3-0) \Rightarrow -3$$

$$x_{31} = (-1)^4 \begin{vmatrix} 0 & -1 \\ 1 & -1 \end{vmatrix} = (0+1) \Rightarrow 1$$

$$x_{32} = (-1)^5 \begin{vmatrix} 1 & -1 \\ 2 & -1 \end{vmatrix} = -(-1+2) \Rightarrow -1$$

$$x_{33} = (-1)^6 \begin{vmatrix} 1 & 0 \\ 2 & 1 \end{vmatrix} = (1-0) \Rightarrow 1$$

Let $B = \begin{bmatrix} x_{11} & x_{12} & x_{13} \\ x_{21} & x_{22} & x_{23} \\ x_{31} & x_{32} & x_{33} \end{bmatrix} = \begin{bmatrix} 5 & -6 & 4 \\ -3 & 4 & -3 \\ 1 & -1 & 1 \end{bmatrix}$

So, $\text{Adj}\, R = B^T = \begin{bmatrix} 5 & -3 & 1 \\ -6 & 4 & -1 \\ 4 & -3 & 1 \end{bmatrix}$

Hence, $R^{-1} = \dfrac{\text{Adj}\, R}{|R|} = \begin{bmatrix} 5 & -3 & 1 \\ -6 & 4 & -1 \\ 4 & -3 & 1 \end{bmatrix}$

The upper row of R^{-1} is $[5 \ -3 \ 1]$,

49. (b) Total samples $= (H,T),(H,T),(H,T)$

Given, first toss produces a head,

Then, total chance becomes $= H,(H,T),(H,T)$

For exactly two heads in three tosses $= H,H,H$

So, probability $= \dfrac{\text{Total favourable events}}{\text{Total sample}}$

$$= \frac{3}{6} = \frac{1}{2}$$

50. (a) Given, $f(x) = x^2 e^{-x}$

$$f'(x) = x^2(-e^{-x}) + 2xe^{-x}$$

$$= e^{-x}(2x - x^2)$$

For max and min values of $f^{-1}(x)$,

$$f'(x) = 0$$

$$e^{-x}(2x - x^2) = 0$$

$$\Rightarrow \quad x(2-x) = 0 \quad \Rightarrow \quad x = 0, 2$$

$$f''(x) = e^{-x}(2-2x) - e^{-x}(2x - x^2)$$

At $x = 0$,

$$f''(0) = e^0(2-0) - e^0(0-0) \Rightarrow 2 > 0 \quad \text{(Minima)}$$

At $(x = 2)$,

$$f''(2) = e^{-2}(2-4) - e^{-2}(4-4)$$

$$= -2/e^2 - 0 = < 0 \qquad \text{(Maxima)}$$

So, the $f(x)$ is maximum at $x = 2$.

51. (c) Given, $u = \dfrac{x^2}{2} + \dfrac{y^2}{3}$ $\qquad [\text{At } P \to 1, 3]$

$$\text{grad} \quad u = \nabla u = \left(\hat{\mathbf{i}}\frac{\partial}{\partial x} + \hat{\mathbf{j}}\frac{\partial}{\partial y} + \hat{\mathbf{k}}\frac{\partial}{\partial z}\right)u$$

$$= \hat{\mathbf{i}}(x) + \hat{\mathbf{j}}\left(\frac{2}{3}y\right) + 0 = x\hat{\mathbf{i}} + \frac{2y}{3}\hat{\mathbf{j}}$$

$$|\text{grad}\, u|_{\text{at}\,(1,3)} = \hat{\mathbf{i}} + 2\hat{\mathbf{j}}$$

Then, $|\text{grad}\, u|_{\text{at}\,(1,3)} = \sqrt{1+4} = \sqrt{5}$

52. (b) Given, $x''(t) + 3x'(t) + 2x(t) = 5$

$$\Rightarrow \quad \frac{d^2x}{dt} + 3\frac{dx}{dt} + 2x = 5$$

$$\Rightarrow \quad (D^2 + 3D + 2)x = 5 \quad \text{where} \left(D = \frac{d}{dt}\right)$$

Auxiliary equation

$$m^2 + 3m + 2 = 0$$

$$(m+1)(m+2) = 0$$

$$m = -1, -2$$

Then, $CF = c_1 e^{-t} + c_2 e^{-2t}$

and $PI = \dfrac{5}{(D+1)(D+2)}$ $\left\{\because PI = \dfrac{Q}{F(D)}\right\}$

$$= 5(1+D)^{-1} \frac{1}{2}\left[1 + \frac{D}{2}\right]^{-1}$$

$$= \frac{5}{2}(1 - D + D^2 \ldots \infty)\left(1 - \frac{D}{2} + \frac{D^2}{4} \ldots \infty\right)$$

$$= \frac{5}{2}(1 \ldots\ldots)$$

{$\because$ containing the higher terms of D}

$$PI = \frac{5}{2}$$

So, the complete solution is

$$x = CF + PI$$

$$x = c_1 e^{-t} + c_2 e^{-2t} + 5/2 \quad \ldots(i)$$

The value of x at $t \to \infty$ is

$$x = c_1 e^{-\infty} + c_2 e^{-\infty} + 5/2$$

$$x = 0 + 0 + 5/2$$

$$x = 5/2$$

□

2

Electric Circuits and Fields

Syllabus

Electric Circuits and Fields *Network graph, KCL, KVL, Node and mesh analysis, Transient response of DC and AC networks, Sinusoidal steady-state analysis, Resonance, Basic filter concepts, Ideal current and voltage sources, Thevenin's, Norton's and Superposition and Maximum power transfer theorems, Two-port networks, Three-phase circuits, Gauss's theorem, Electric field and potential due to point, line, plane and spherical charge distributions, Ampere's and Biot-Savart's laws, Inductance, Dielectrics, Capacitance.*

GATE 2011

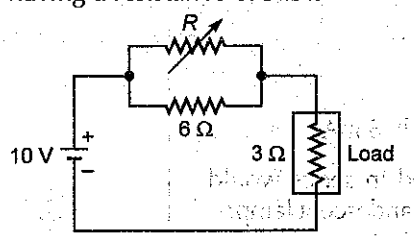
(1 Mark Questions)

▼ *Solution Points*

1. The voltage applied to a circuit is $100\sqrt{2}\cos(100\,\pi t)$ V and the circuit draws a current of $10\sqrt{2}\sin(100\pi t + \pi/4)$ A. Taking the voltage as the reference phasor, the phasor representation of the current in ampere is

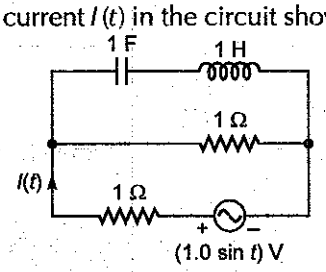
(a) $10\sqrt{2}\angle -\pi/4$ (b) $10\angle -\pi/4$ (c) $10\angle +\pi/4$ (d) $10\sqrt{2}\angle +\pi/4$

2. In the circuit given below, the value of R required for the transfer of maximum power to the load having a resistance of $3\,\Omega$ is



(a) zero (b) $3\,\Omega$ (c) $6\,\Omega$ (d) infinity

3. The rms value of the current $I(t)$ in the circuit shown below is



(a) $\frac{1}{2}$ A (b) $\frac{1}{\sqrt{2}}$ A (c) 1 A (d) $\sqrt{2}$ A

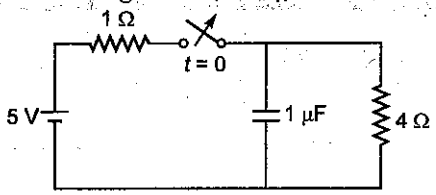
▼ *Solution Points*

4. A low-pass filter with a cut-off frequency of 30 Hz is cascaded with a high-pass filter with a cut-off frequency of 20 Hz. The resultant system of filters will function as

(a) an all-pass filter (b) an all-stop filter
(c) a band-stop (band-reject) filter (d) a band-pass filter

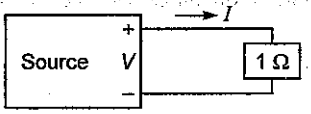
GATE 2010

5. The switch in the circuit has been closed for a long time. It is opened at $t = 0$. At $t = 0^+$, the current through the 1 μF capacitor is



(a) zero (b) 1 A (c) 1.25 A (d) 5 A

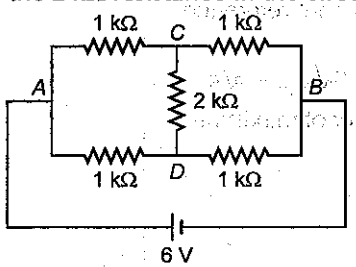
6. As shown in the figure, a 1Ω resistance is connected across a source that has a load line $V + I = 100$. The current through the resistance is



(a) 25 A (b) 50 A (c) 100 A (d) 200 A

GATE 2009

7. The current through the 2 kΩ resistance in the circuit shown is



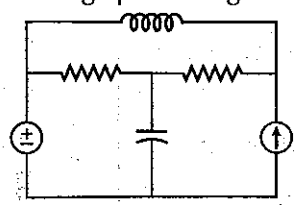
(a) zero (b) 1 mA (c) 2 mA (d) 6 mA

8. How many 220 W/220 V incandescent lamps connected in series would consume the same total power as a single 100 W/220 V incandescent lamp?

(a) Not possible (b) 4 (c) 3 (d) 2

GATE 2008

9. The number of chords in the graph of the given circuit will be

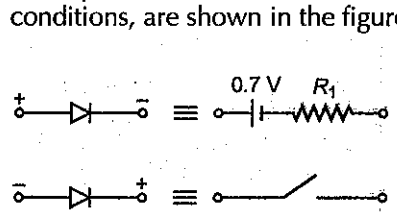


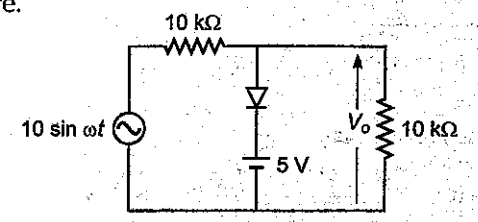
(a) 3 (b) 4 (c) 5 (d) 6

▼ *Solution Points*

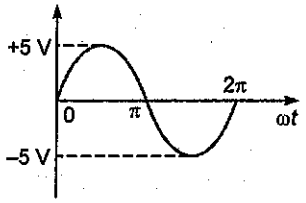
10. The Thevenin's equivalent of a circuit operating at $\omega = 5$ rad/s, has $V_{out} = 3.71\angle -15.9°$ V and $Z_0 = 2.38 - j\,0.667\,\Omega$. At this frequency, the minimal realization of the Thevenin's impedance will have a
(a) resistor and a capacitor and an inductor
(b) resistor and a capacitor
(c) resistor and an inductor
(d) capacitor and an inductor

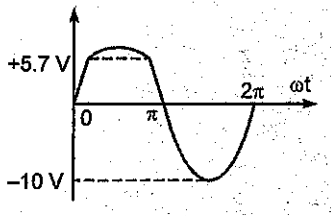
11. The equivalent circuits of a diode, during forward biased and reverse biased conditions, are shown in the figure.

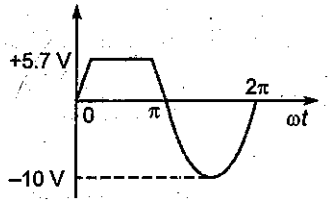


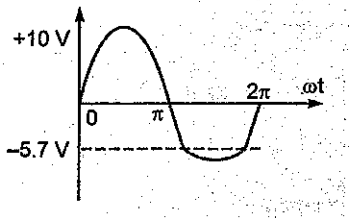


If such a diode is used in clipper circuit of figure given above, the output voltage (V_o) of the circuit will be

(a)


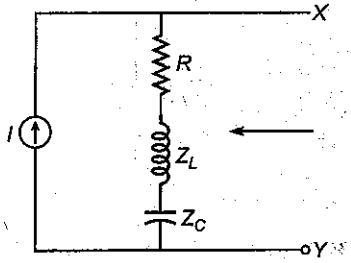
(b)


(c)


(d)


GATE 2006

12. In the figure the current source is $1\angle 0$ A, $R = -1\,\Omega$, the impedances are $Z_C = -j\,\Omega$ and $Z_L = -2j\,\Omega$. The Thevenin equivalent looking into the circuit across X-Y is



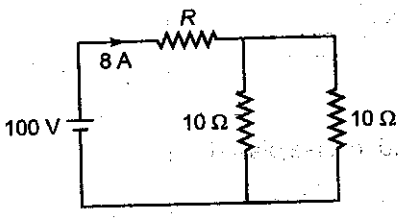
(a) $\sqrt{2}\angle 0$ V, $(1+2j)\,\Omega$
(b) $2\angle 45°$ V, $(1-2j)\,\Omega$
(c) $2\angle 45°$ V, $(1+j)\,\Omega$
(d) $\sqrt{2}\angle 45°$ V, $(1+j)\,\Omega$

13. Which of the following statements holds for the divergence of electric and magnetic flux densities?
(a) Both are zero
(b) These are zero for static densities but non-zero for time varying densities
(c) It is zero for the electric flux density
(d) It is zero for the magnetic flux density

▼ *Solution Points*

GATE 2005

14. In the figure given below the value of R will be

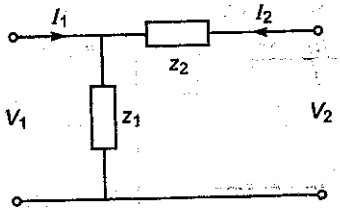


(a) 2.5 Ω (b) 5.0 Ω (c) 7.5 Ω (d) 10.0 Ω

15. The rms value of the voltage $V(t) = 3 + 4\cos(3t)$ is

(a) $\sqrt{17}$ V (b) 5 V (c) 7 V (d) $(3 + 2\sqrt{2})$ V

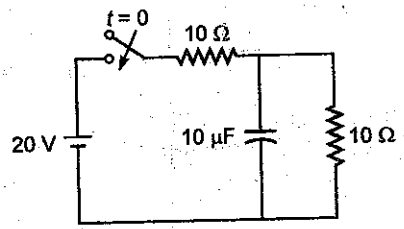
16. For the two-port network shown in the figure given below, the z-matrix is given by



(a) $\begin{bmatrix} z_1 & z_1 + z_2 \\ z_1 + z_2 & z_2 \end{bmatrix}$ (b) $\begin{bmatrix} z_1 & z_1 \\ z_1 + z_2 & z_2 \end{bmatrix}$

(c) $\begin{bmatrix} z_1 & z_1 \\ z_2 & z_1 + z_2 \end{bmatrix}$ (d) $\begin{bmatrix} z_1 & z_1 \\ z_1 & z_1 + z_2 \end{bmatrix}$

17. In the figure given below, the initial capacitor voltage is zero. The switch is closed at $t = 0$, the final steady state voltage across the capacitor is



(a) 20 V (b) 10 V (c) 5 V (d) zero

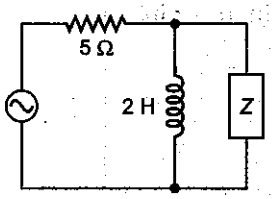
18. If E is the electric field intensity, then $\nabla \cdot (\nabla \times E)$ is equal to

(a) $\vec{E}$ (b) $|\vec{E}|$ (c) null vector (d) zero

GATE 2004

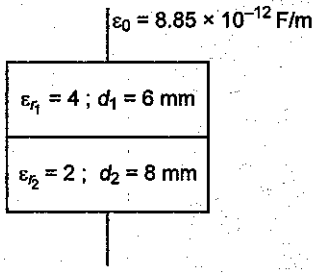
▼ *Solution Points*

19. The value of Z in figure which is most appropriate to cause parallel resonance at 500 Hz is



(a) 125.00 mH (b) 304.20 μF (c) 2.0 μF (d) 0.05 μF

20. A parallel plate capacitor is shown in figure given below. It is made of two square metal plates of 400 mm side. The 14 mm space between the plates is filled with two layers of dielectrics of $\varepsilon_r = 4,6$ mm thick and $\varepsilon_r = 2,8$ mm thick. Neglecting fringing of fields at the edges, the capacitance is



(a) 1298 pF (b) 944 pF (c) 354 pF (d) 257 pF

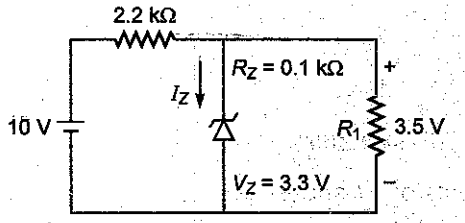
21. The inductance of a long solenoid of length 1000 mm wound uniformly with 3000 turns on a cylindrical paper tube of 60 mm diameter is

(a) 3.2 μH (b) 3.2 mH (c) 32.0 mH (d) 3.2 H

22. Total instantaneous power supplied by a 3-phase AC supply to a balanced *R-L* load is

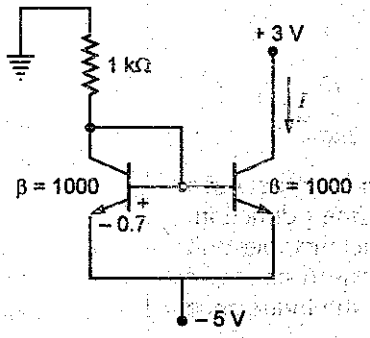
(a) zero (b) constant
(c) pulsating with zero average (d) pulsating with non-zero average

23. The current through the Zener diode in figure given below is



(a) 33 mA (b) 3.3 mA (c) 2 mA (d) zero

24. Two perfectly matched silicon transistors are connected as shown in figure given below. The value of the current I is

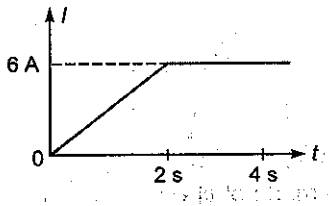


(a) zero (b) 2.3 mA (c) 4.3 mA (d) 7.3 mA

GATE 2003

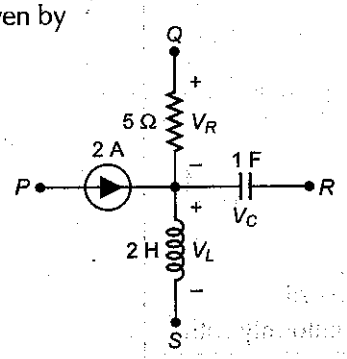
▼ *Solution Points*

25. Figure shows the waveform of the current passing through an inductor of resistance 1 Ω and inductance 2 H. The energy absorbed by the inductor in the first four seconds is



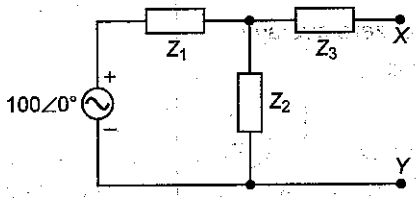
(a) 144 J (b) 98 J (c) 132 J (d) 168 J

26. A segment of a circuit is shown in figure given below $V_R = 5\,V$, $V_C = 4\sin 2t$. The voltage V_L is given by



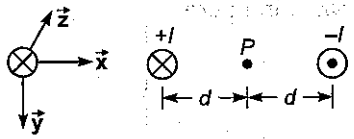
(a) $3 - 8\cos 2t$ (b) $32\sin 2t$ (c) $16\sin 2t$ (d) $16\cos 2t$

27. In the figure given below $Z_1 = 10\angle -60°$, $Z_2 = 10\angle 60°$, $Z_3 = 50\angle 53.13°$. Thevenin's impedance seen from X-Y is



(a) $56.6\angle 45°$ (b) $60\angle 30°$ (c) $70\angle 30°$ (d) $34.4\angle 65°$

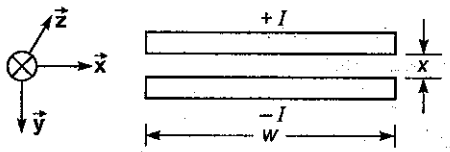
28. Two conductors are carrying forward and return currents of $+I$ and $-I$ as shown in figure given below. The magnetic field intensity $\vec{H}$ at point P is



(a) $\frac{I}{\pi d}\vec{y}$ (b) $\frac{I}{\pi d}\vec{x}$ (c) $\frac{I}{2\pi d}\vec{y}$ (d) $\frac{I}{2\pi d}\vec{x}$

29. Two infinite strips of width w metre in x-direction as shown in figure given below are carrying forward and return currents of $+I$ and $-I$ in the z-direction. The strips are separated by a distance of x metre. The inductance per unit length of the configuration is measured to be L H/m. If the distance of separation between the strips is now reduced to $x/2$ metre, the inductance per unit length of the configuration is

▼ *Solution Points*



(a) $2L$ H/m (b) $L/4$ H/m (c) $L/2$ H/m (d) $4L$ H/m

GATE 2002

30. A current impulse $5\delta(t)$ is forced through a capacitor C. The voltage $V_C(t)$ across the capacitor is given by

(a) $5t$ (b) $5u(t) - C$ (c) $\frac{5}{C}t$ (d) $\frac{5u(t)}{C}$

31. The graph of an electrical network has N nodes and B branches. The number of links L with respect to the choice of a tree, is given by

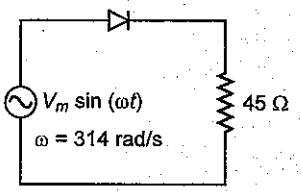
(a) $B - N + 1$ (b) $B + N$ (c) $N - B + 1$ (d) $N - 2B - 1$

32. Given a vector field $\vec{F}$, the divergence theorem states that

(a) $\int_S \vec{F} \cdot d\vec{S} = \int_V \vec{\nabla} \cdot \vec{F}\, dV$ (b) $-\int_S \vec{F} \cdot d\vec{S} = \int_V \vec{\nabla} \times \vec{F}\, dV$

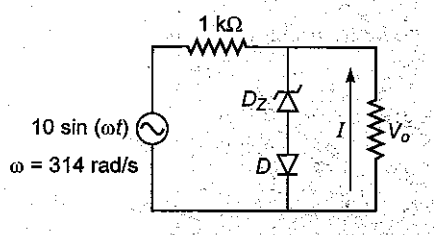
(c) $\int_S \vec{F} \times d\vec{S} = \int_V \vec{\nabla} \cdot \vec{F}\, dV$ (d) $\int_S \vec{F} \times d\vec{S} = \int_V \vec{\nabla} \cdot \vec{F}\, dV$

33. The forward resistance of the diode shown in figure given below is 5 Ω and the remaining parameters are same as those of an ideal diode. The DC component of the source current is



(a) $\frac{V_m}{50\,\pi}$ (b) $\frac{V_m}{60\,\pi\sqrt{2}}$ (c) $\frac{V_m}{100\pi\sqrt{2}}$ (d) $\frac{2V_m}{50\pi}$

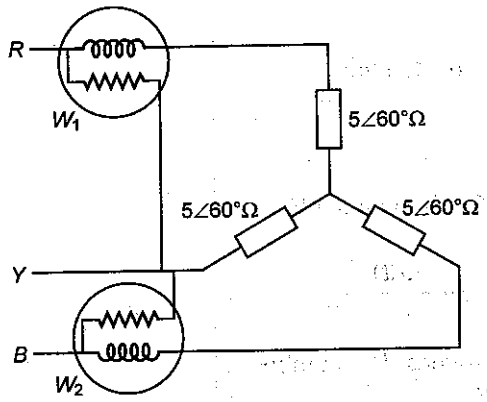
34. The cut-in voltage of both Zener diode D_z and diode D shown in figure given below is 0.7 V, while breakdown voltage of D_z is 3.3 V and reverse breakdown voltage of D is 50 V. The other parameters can be assumed to be the same as those of an ideal diode. The values of the peak output voltage (V_o) are



(a) 3.3 V in the positive half cycle and 1.4 V in the negative half cycle
(b) 4 V in the positive half cycle and 5 V in the negative half cycle
(c) 3.3 V in both positive and negative half cycles
(d) 4 V in both positive and negative half cycles

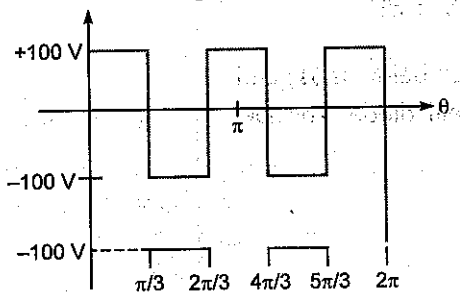
▼ *Solution Points*

35. The line-to-line input voltage to the 3-phase, 50 Hz, AC circuit shown in figure given below is 100 V rms. Assuming that the phase sequence is *RYB*, the wattmeters would read



(a) $W_1 = 886$ W and $W_2 = 886$ W
(b) $W_1 = 500$ W and $W_2 = 500$ W
(c) $W_1 =$ Zero and $W_2 = 1000$ W
(d) $W_1 = 250$ W and $W_2 = 750$ W

36. What is the rms value of the voltage waveform shown in figure given below?



(a) $200/\pi$ V (b) $100/\pi$ V (c) 200 V (d) 100 V

GATE 2001

37. In a series *R-L-C* circuit at resonance, the magnitude of the voltage developed across the capacitor
(a) is always zero
(b) can never be greater than the input voltage
(c) can be greater than the input voltage, however, it is 90° out of phase with the input voltage
(d) can be greater than the input voltage, and is in phase with the input voltage

38. Two incandescent light bulbs of 40 W and 60 W ratings are connected in series across the mains. Then
(a) the bulbs together consume 100 W
(b) the bulbs together consume 50 W
(c) the 60 W bulb glows brighter
(d) the 40 W bulb glows brighter

39. A unit step voltage is applied at $t = 0$ to a series *R-L* circuit with zero initial conditions.

▼ *Solution Points*

Which of the following statements is correct?
(a) It is possible for the current to the oscillatory
(b) The voltage across the resistor at $t = 0^+$ is zero
(c) The energy stored in the inductor in the steady state is zero
(d) The resistor current eventually falls to zero

40. Given two coupled inductors L_1 and L_2, their mutual inductance M satisfies

(a) $M = \sqrt{L_1^2 + L_2^2}$ (b) $M > \frac{(L_1 + L_2)}{2}$

(c) $M > \sqrt{L_1 L_2}$ (d) $M \leq \sqrt{L_1 L_2}$

41. A passive 2-port network is in a steady state. Compared to its input, the steady state output can never offer

(a) higher voltage (b) lower impedance
(c) greater power (d) better regulation

GATE 2000

42. An electron with velocity u is placed in an electric field E and magnetic field B. The force experienced by the electron is given by

(a) $-eE$ (b) $-eu \times B$
(c) $-e(u \times E + B)$ (d) $-e(E + u \times B)$

43. A voltage waveform $V(t) = 12t^2$ is applied across a 1H inductor for $t \geq 0$, with initial current through it being zero. The current through the inductor for $t \geq 0$ is given by

(a) $12\,t$ (b) $24t$ (c) $12t^3$ (d) $4t^3$

GATE 2011 *(2 Marks Questions)*

44. A capacitor is made with a polymeric dielectric having an ε_r of 2.26 and a dielectric breakdown strength of 50 kV/cm. The permittivity of free space is 8.85 pF/m. If the rectangular plates of the capacitor have a width of 20 cm and a length of 40 cm, then the maximum electric charge in the capacitor is

(a) $2\mu C$ (b) $4\mu C$ (c) $8\,\mu C$ (d) $10\,\mu C$

45. A zero mean random signal is uniformly distributed between limits $-a$ and $+a$ and its mean square value is equal to its variance. Then the rms value of the signal is

(a) $\frac{a}{\sqrt{3}}$ (b) $\frac{a}{\sqrt{2}}$ (c) $a\sqrt{2}$ (d) $a\sqrt{3}$

46. A lossy capacitor C_X, rated for operation at 5 kV, 50 Hz is represented by an equivalent circuit with an ideal capacitor C_P in parallel with a resistor R_P. The value of C_P is found to be 0.102 μF and the value of $R_P = 1.25\,M\Omega$. Then the power loss and tan δ of the lossy capacitor operating at the rated voltage, respectively, are

(a) 10 W and 0.0002 (b) 10 W and 0.0025
(c) 20 W and 0.025 (d) 20 W and 0.04

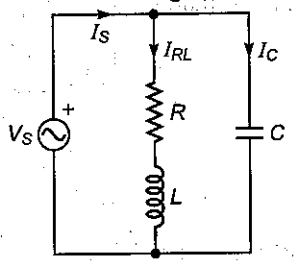
▼ *Solution Points*

Common Data for Questions 47 and 48
The input voltage given to a converter is
$V_i = 100\sqrt{2}\sin(100\pi t)$ V
The current drawn by the converter is
$I_i = [10\sqrt{2}\sin(100\pi t - \pi/3) + 5\sqrt{2}\sin(300\pi t + \pi/4) + 2\sqrt{2}\sin(500\pi t - \pi/6)]$ A

47. The active power drawn by the converter is
(a) 181 W (b) 500 W (c) 707 W (d) 887 W

48. The input power factor of the converter is
(a) 0.31 (b) 0.44 (c) 0.5 (d) 0.71

Common Data for Questions 49 and 50
An *R-L-C* circuit with relevant data is given below.



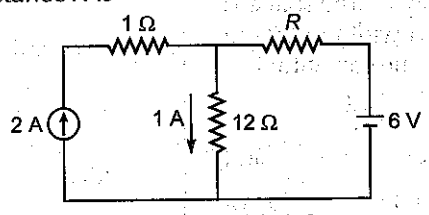
$V_S = 1\angle 0°$ V
$I_S = \sqrt{2}\angle \pi/4$ A
$I_{RL} = \sqrt{2}\ \angle = \pi/4$ A

49. The current I_C in the figure above is
(a) $-j2$ A (b) $-j\frac{1}{\sqrt{2}}$ A (c) $+j\frac{1}{\sqrt{2}}$ A (d) $+j2$ A

50. The power dissipated in the resistor R is
(a) 0.5 W (b) 1 W (c) $\sqrt{2}$ W (d) 2 W

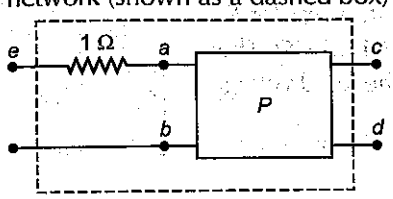
GATE 2010

51. If the 12 Ω resistor draws a current of 1A as shown in the figure given below, the value of resistance R is



(a) 4Ω (b) 6Ω (c) 8 Ω (d) 18 Ω

52. The two-port network P shown in the figure has ports 1 and 2, denoted by terminals (a, b) and (c, d), respectively. It has an impedance matrix Z with parameters denoted by Z_{ij}. A 1 Ω resistor is connected in series with the network at port 1 as shown in the figure. The impedance matrix of the modified two-port network (shown as a dashed box) is



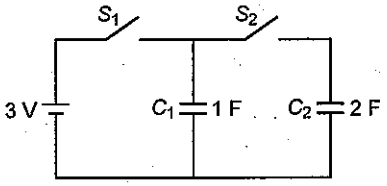
▼ *Solution Points*

(a) $\begin{pmatrix} z_{11}+1 & z_{12}+1 \\ z_{21} & z_{22}+1 \end{pmatrix}$ (b) $\begin{pmatrix} z_{11}+1 & z_{12} \\ z_{21} & z_{22}+1 \end{pmatrix}$

(c) $\begin{pmatrix} z_{11}+1 & z_{12} \\ z_{21} & z_{22} \end{pmatrix}$ (d) $\begin{pmatrix} z_{11}+1 & z_{12} \\ z_{21}+1 & z_{22} \end{pmatrix}$

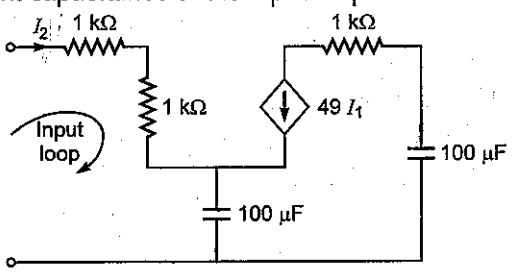
GATE 2009

53. In the figure shown, all elements used are ideal. For time $t<0$, S_1 remains closed and S_2 opened. At $t=0$, S_1 is opened and S_2 is closed. If the voltage V_{C2} across the capacitor C_2 at $t=0$ is zero, the voltage across the capacitor combination at $t=0^+$ will be



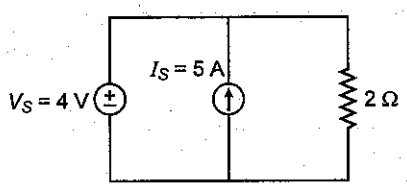
(a) 1 V (b) 2 V (c) 1.5 V (d) 3 V

54. The equivalent capacitance of the input loop of the circuit shown is



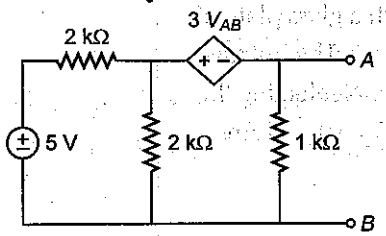
(a) 2 μF (b) 100 μF (c) 200 μF (d) 4μF

55. For the circuit shown, find out the current flowing through the 2Ω resistance. Also identify the changes to be made to double the current through the 2Ω resistance.



(a) (5 A; Put $V_S = 20$ V) (b) (2 A; Put $V_S = 8$ V)
(c) (5 A; Put $I_S = 10$ A) (d) (7A; Put $I_S = 12$ A)

Statements for Linked Answer Questions 56 and 57



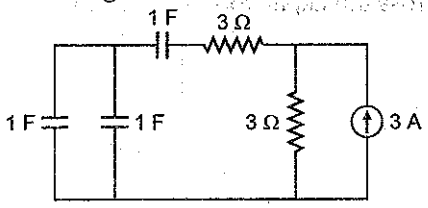
▼ *Solution Points*

56. For the circuit given above, the Thevenin's resistance across the terminals A and B is
(a) 0.5 kΩ (b) 0.2 kΩ (c) 1 kΩ (d) 0.11 kΩ

57. For the circuit given above, the Thevenin's voltage across the terminals A and B is
(a) 1.25 V (b) 0.25 V (c) 1 V (d) 0.5 V

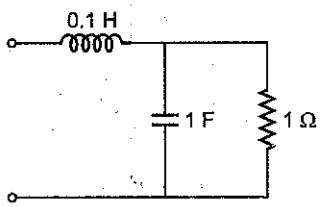
GATE 2008

58. The time constant for the given circuit will be



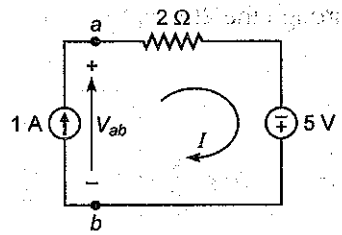
(a) $\frac{1}{9}$ s (b) $\frac{1}{4}$ s (c) 4 s (d) 9 s

59. The resonant frequency for the given circuit will be



(a) 1 rad/s (b) 2 rad/s (c) 3 rad/s (d) 4 rad/s

60. Assuming ideal elements in the circuit shown below, the voltage V_{ab} will be

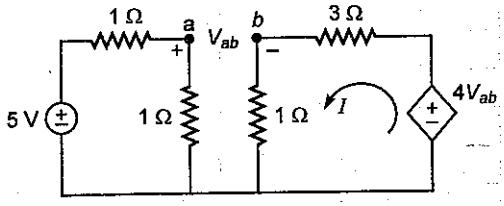


(a) −3 V (b) zero (c) 3 V (d) 5 V

61. A capacitor consists of two metal plates each $500 \times 500\ mm^2$ and spaced 6 mm apart. The space between the metal plates is filled with a glass plate of 4 mm thickness and a layer of paper of 2 mm thickness. The relative permittivities of the glass and paper are 8 and 2 respectively. Neglecting the fringing effect, the capacitance will be (Given that $\varepsilon_0 = 8.85 \times 10^{-12}$ F/m)
(a) 983.33 pF (b) 1475 pF
(c) 6637.5 pF (d) 9956.25 pF

▼ *Solution Points*

62. In the circuit shown in the figure, the value of the current I will be given by

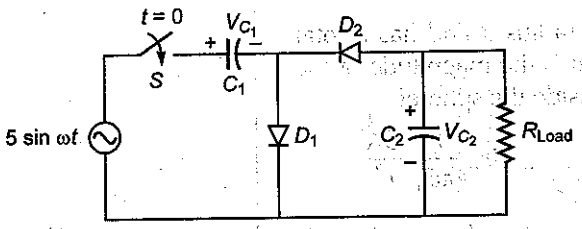


(a) 0.31 A (b) 1.25 A (c) 1.75 A (d) 2.5 A

63. Two point charges $Q_1 = 10\,\mu C$ and $Q_2 = 20\,\mu C$ are placed at coordinates (1, 1, 0) and (−1, −1, 0) respectively. The total electric flux passing through a plane $z = 20$ will be

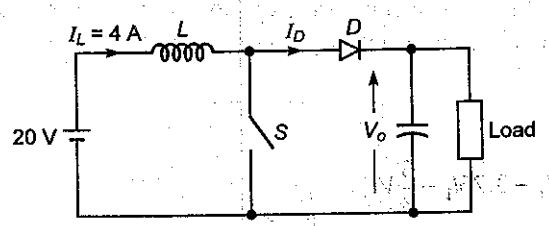
(a) 7.5 μC (b) 13.5 μC
(c) 15.0 μC (d) 22.5 μC

64. In the voltage doubler circuit shown in the figure, the switch S is closed at $t = 0$. Assuming diodes D_1 and D_2 to be ideal, load resistance to be infinite and initial capacitor voltage to be zero, the steady state voltages across capacitors C_1 and C_2 will be



(a) $V_{C1} = 10\text{ V}, V_{C2} = 5\text{V}$ (b) $V_{C1} = 10\text{ V}, V_{C2} = -5\text{V}$
(c) $V_{C1} = 5\text{ V}, V_{C2} = 10\text{ V}$ (d) $V_{C1} = 5\text{ V}, V_{C2} = -10\text{V}$

65. In the circuit shown in the figure, the switch is operated at a duty cycle of 0.5. A large capacitor is connected across the load. The inductor current is assumed to be continuous.
The average voltage across the load and the average current through the diode will respectively be

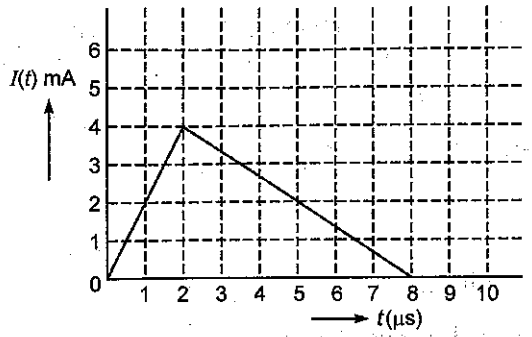


(a) 10 V, 2 A (b) 10 V, 8 A (c) 40 V, 2 A (d) 40 V, 8 A

Statements for Linked Answer Questions 66 and 67

The current $I(t)$ sketched in the figure flows through an initially uncharged 0.3 nF capacitor.

▼ *Solution Points*



66. The charge stored in the capacitor at $t = 5\,\mu s$ will be

(a) 8 nC (b) 10 nC (c) 13 nC (d) 16 nC

67. The capacitor charged upto 5 μs, as per the current profile given in the figure, is connected across an inductor of 0.6 mH. Then the value of voltage across the capacitor after 1 μs will approximately be

(a) 18.8 V (b) 23.5 V (c) −23.5 V (d) −30.6 V

GATE 2007

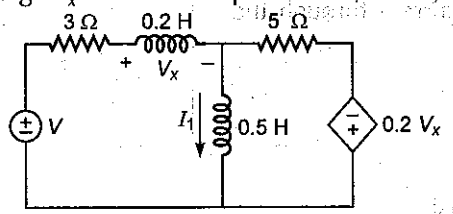
68. A solid sphere made of insulating material has a radius R and has a total charge Q distributed uniformly in its volume. What is the magnitude of the electric field intensity E, at a distance $r\,(0 < r < R)$ inside the sphere?

(a) $\frac{1}{4\pi\varepsilon_0}\frac{Qr}{R^3}$ (b) $\frac{3}{4\pi\varepsilon_0}\frac{Qr}{R^3}$ (c) $\frac{1}{4\pi\varepsilon_0}\frac{Q}{r^2}$ (d) $\frac{1}{4\pi\varepsilon_0}\frac{QR}{r^3}$

69. A single-phase, 230 V, 50 Hz as mains fed step-down transformer (4 : 1) is supplying power to a half-wave uncontrolled AC-DC converter used for charging a battery (12 V DC) with the series current limiting resistor being 19.04 Ω. The charging current is

(a) 2.43 A (b) 1.65 A (c) 1.22 A (d) 1.0 A

70. The state equation for the current I_1 shown in the network shown below in terms of the voltage V_x and the independent source V, is given by

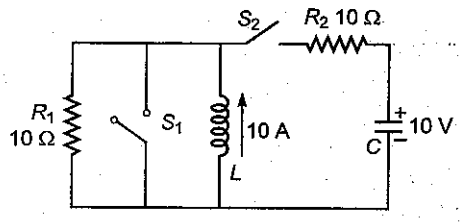


(a) $\frac{dI_1}{dt} = -1.4V_x - 3.75I_1 + \frac{5}{4}V$ (b) $\frac{dI_1}{dt} = -1.4V_x - 3.75I_1 - \frac{5}{4}V$

(c) $\frac{dI_1}{dt} = -1.4V_x + 3.75I_1 + \frac{5}{4}V$ (d) $\frac{dI_1}{dt} = -1.4V_x + 3.75I_1 - \frac{5}{4}V$

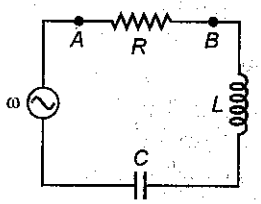
71. In the circuit shown in figure switch S_1 is initially closed and S_2 is opened. The inductor L carries a current of 10 A and the capacitor is charged to 10 V with polarities as indicated. S_2 is initially closed at $t = 0^-$ and S_1 is opened at $t = 0^+$. The current through C and the voltage across L at $t = 0^+$ is

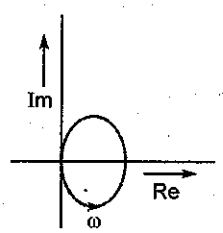
▼ *Solution Points*

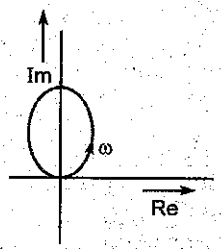


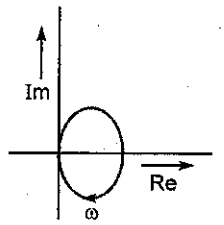
(a) 55 A, 4.5 V
(b) 5.5 A, 45 V
(c) 45 A, 5.5 V
(d) 4.5 A, 55 V

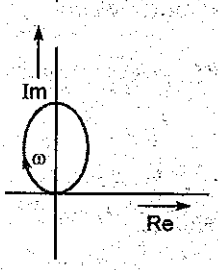
72. The *R-L-C* series circuit shown is supplied from a variable frequency voltage source. The admittance-locus of the *R-L-C* network at terminals *AB* for increasing frequency ω is



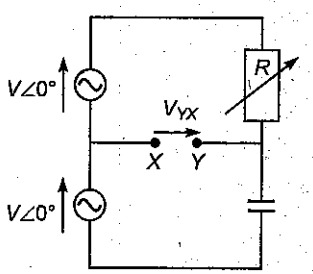
(a)


(b)


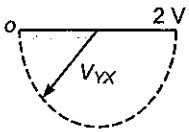
(c)


(d)


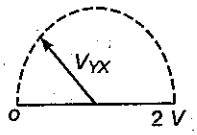
73. In the figure given below all phasors are with reference to the potential at point *O*. The locus of voltage phasor V_{YX} as *R* is varied from zero to infinity is shown by



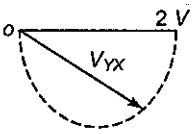
▼ *Solution Points*

(a) 

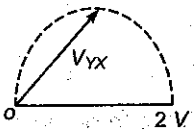
Locus of V_{YX}

(b) 

Locus of V_{YX}

(c) 

Locus of V_{YX}

(d) 

Locus of V_{YX}

74. A 3 V DC supply with an internal resistance of 2 Ω supplies a passive non-linear resistance characterized by the relation $V_{NL} = I_{NL}^2$. The power dissipated in the non-linear resistance is

(a) 1.0 W (b) 1.5 W (c) 2.5 W (d) 3.0 W

75. The matrix A given below is the node incidence matrix of a network. The columns correspond to branches of the network while the rows correspond to nodes. Let $V = [v_1 v_2 \ldots v_6]^T$ denotes the vector of branch voltages while $I = [I_1 I_2 \ldots I_6]^T$ that of branch currents. The vector $E = [E_1 E_2 E_3 E_4]^T$ denotes the vector of node voltages relative to a common ground.

$$A = \begin{bmatrix} 1 & 1 & 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & -1 & 1 & 0 \\ -1 & 0 & 0 & 0 & -1 & -1 \\ 0 & 0 & -1 & 1 & 0 & 1 \end{bmatrix}$$

Which of the following statements is true?

(a) The equations $V_1 - V_2 + V_3 = 0, V_3 + V_4 - V_5 = 0$ and KVL equations for the network for some loops

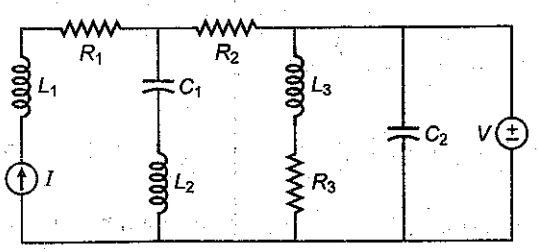
(b) The equations $V_1 - V_3 - V_6 = 0, V_4 + V_5 - V_6 = 0$ are KVL equations for the network for some loops

(c) $E = AV$

(d) $AV = 0$ are KVL equations for the network

GATE 2006

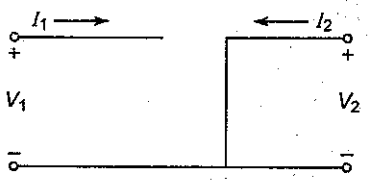
76. In the circuit shown in the figure, the current source $I = 1$A, voltage source $V = 5$ V, $R_1 = R_2 = R_3 = 1\Omega$, $L_1 = L_2 = L_3 = 1$H, $C_1 = C_2 = 1$F. The currents (in ampere) through R_3 and the voltage source V respectively will be



(a) 1, 4 (b) 5, 1 (c) 5, 2 (d) 5, 4

77. The parameter type and the matrix representation of the relevant two-port parameters that describe the circuit shown are

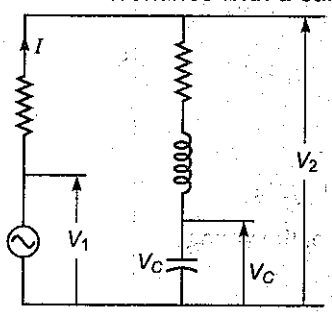
▼ *Solution Points*



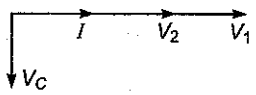
(a) z parameters, $\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$ (b) h parameters, $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

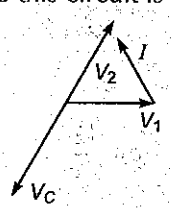
(c) h parameters, $\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$ (d) z parameters, $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

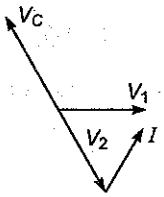
78. The circuit shown in the figure is energized by a sinusoidal voltage source V_1 at a frequency which causes resonance with a current of I.

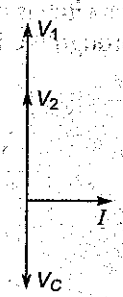


The phasor diagram which is applicable to this circuit is

(a)


(b)


(c)


(d)


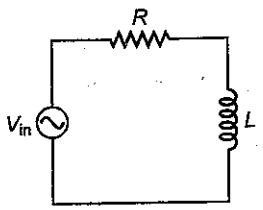
79. An ideal capacitor is charged to a voltage V_0 and connected at $t = 0$ across an ideal inductor L. (The circuit now consists of a capacitor and inductor alone). If we let $\omega_0 = \frac{1}{\sqrt{LC}}$, the voltage across the capacitor at time $t > 0$ is given by

(a) V_0 (b) $V_0 \cos(\omega_0 t)$

(c) $V_0 \sin(\omega_0 t)$ (d) $V_0 e^{-\omega_0 t} \cos(\omega_0 t)$

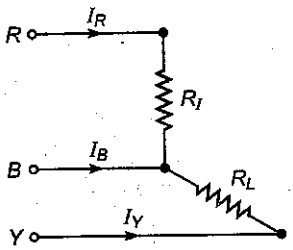
80. The R-L circuit of the figure is fed from a constant magnitude variable frequency sinusoidal voltage source V_{in}. At 100 Hz the R and L elements each have a voltage drop V_{rms}. If the frequency of the source is changed to 50 Hz, the new voltage drop across R is

▼ *Solution Points*



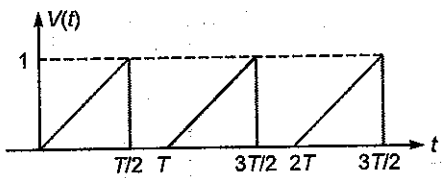
(a) $\sqrt{\frac{5}{8}}\,V_{rms}$ (b) $\sqrt{\frac{2}{3}}\,V_{rms}$ (c) $\sqrt{\frac{8}{5}}\,V_{rms}$ (d) $\sqrt{\frac{3}{2}}\,V_{rms}$

81. For the three-phase circuit shown in the figure, the ratio of the currents $I_R : I_Y : I_B$ is given by



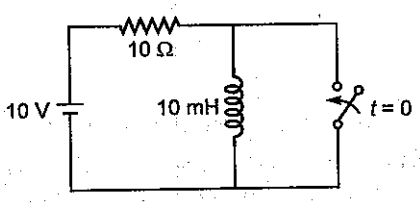
(a) $1:1:\sqrt{3}$ (b) 1:1:2 (c) 1:1:0 (d) $1:1:\sqrt{3/2}$

82. For the triangular waveform shown in the figure, the rms value of the voltage is equal to



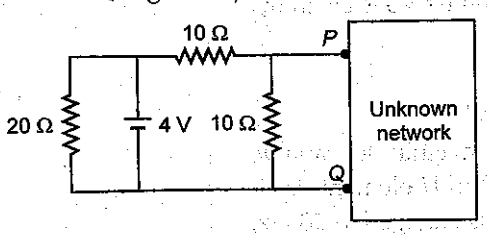
(a) $\sqrt{\frac{1}{6}}$ (b) $\sqrt{\frac{1}{2}}$ (c) $\frac{1}{3}$ (d) $\sqrt{\frac{2}{3}}$

83. The circuit shown in the figure is in steady state, when the switch is closed at $t = 0$. Assuming that the inductance is ideal, the current through the inductor at $t = 0^+$ equals



(a) zero (b) 0.5 A (c) 1 A (d) 2 A

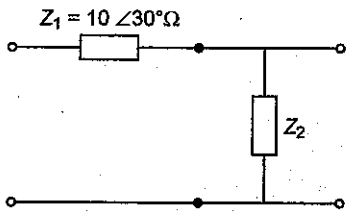
84. In the given figure, the Thevenin equivalent pair (voltage, impedance) as seen at the terminal *P*-*Q* is given by



(a) (2 V, 5 Ω) (b) (2 V, 7.5 Ω) (c) (4 V, 5 Ω) (d) (4 V, 7.5 Ω)

▼ *Solution Points*

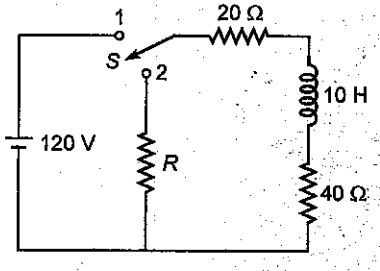
85. Two networks are connected in cascade as shown in figure. With the usual notations the equivalent constants A, B, C and D are obtained. Given that $C = 0.025 \angle 45°$, the value of Z_2 is



(a) $10 \angle 30°\Omega$ (b) $40 \angle -45°\Omega$ (c) $1\ \Omega$ (d) zero

Statements for Linked Answer Questions 86 and 87

A coil of inductance 10 H and resistance 40 Ω is connected as shown in the figure. After the switch S has been in contact with point 1 for a very long time, it is moved to point 2 at $t = 0$.



86. If at $t = 0^+$, the voltage across the coil is 120 V, the value of resistance R is

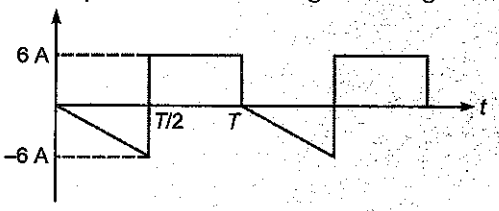
(a) zero (b) 20 Ω (c) 40 Ω (d) 60 Ω

87. For the value of obtained in Q. 86 the time taken for 95% of the stored energy to be dissipated is closed to

(a) 0.10 s (b) 0.15 s (c) 0.50 s (d) 1.0 s

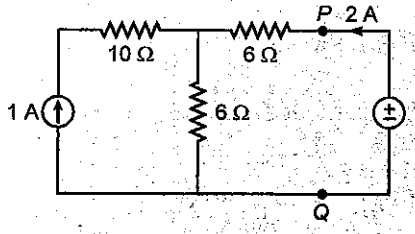
GATE 2004

88. The rms value of the periodic waveform given in figure is



(a) $2\sqrt{6}$ A (b) $6\sqrt{2}$ A (c) $\sqrt{4/3}$ A (d) 1.5 A

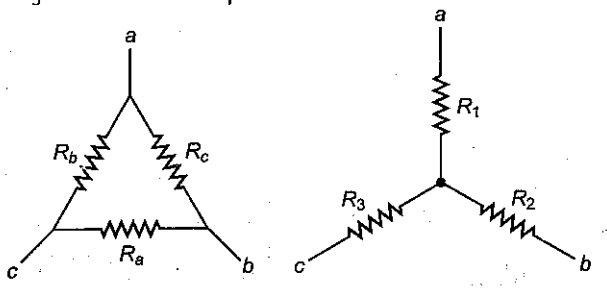
89. In figure, the value of the source voltage is



(a) 12 V (b) 24 V (c) 30 V (d) 44 V

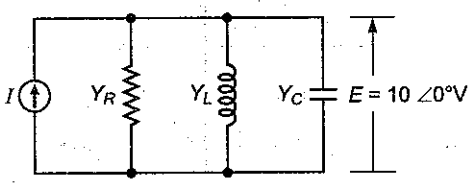
▼ *Solution Points*

90. In figure R_a, R_b and R_c are 20 Ω, 10 Ω and 10 Ω respectively. The resistances R_1, R_2 and R_3 in ohm of an equivalent star-connection are



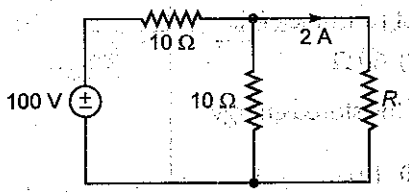
(a) 2.5, 5, 5 (b) 5, 2.5, 5 (c) 5, 5, 2.5 (d) 2.5, 5, 2.5

91. In figure the admittance values of the elements in siemen are $Y_R = 0.5 + j0, Y_L = 0 - j1.5$, $Y_C = 0 + j0.3$ respectively. The value of I as a phasor when the voltage E across the elements is $10 \angle 0°$ V, is



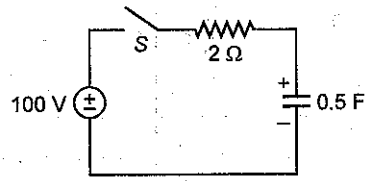
(a) $1.5 + j0.5$ (b) $5 - j18$ (c) $0.5 + j1.8$ (d) $5 - j12$

92. In figure, the value of resistance R in ohm is



(a) 10 (b) 20 (c) 30 (d) 40

93. In figure, the capacitor initially has a charge of 10 C. The current in the circuit 1 s after the switch S is closed will be



(a) 14.7 A (b) 18.5 A (c) 40.0 A (d) 50.0 A

94. The rms value of the resultant current in a wire which carries a DC current of 10 A and a sinusoidal alternating current of peak value 20 A is

(a) 14.1 A (b) 17.3 A (c) 22.4 A (d) 30.0 A

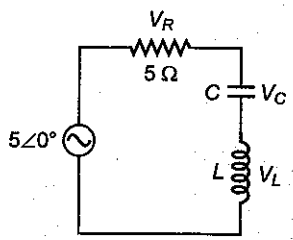
95. The Z matrix of a 2-port network as given by $\begin{bmatrix} 0.9 & 0.2 \\ 0.2 & 0.6 \end{bmatrix}$. The element Y_{22} of the corresponding Y matrix of the same network is given by

(a) 1.2 (b) 0.4 (c) − 0.4 (d) 1.8

GATE 2003

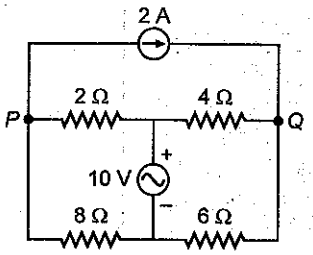
▼ *Solution Points*

96. In the circuit of figure, the magnitudes of V_L and V_C are twice that of V_R. The inductance of the coil is



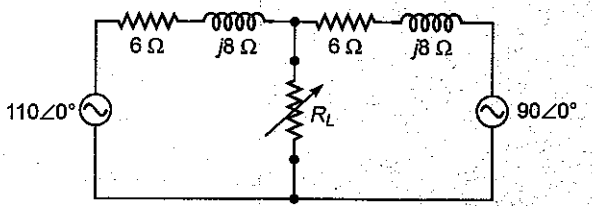
(a) 2.14 mH (b) 5.30 H (c) 3.18 mH (d) 1.32 H

97. In figure, the potential difference between points P and Q is



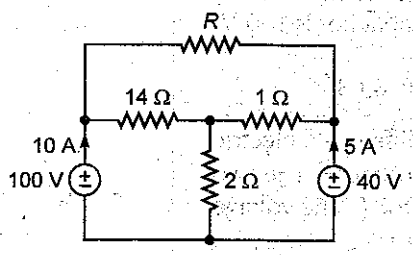
(a) 12 V (b) 10 V (c) –6 V (d) 8 V

98. Two AC sources feed a common variable resistive load as shown in figure Under the maximum power transfer condition, the power absorbed by the load resistance R_L is



(a) 2200 W (b) 1250 W (c) 1000 W (d) 625 W

99. In figure the value of R is



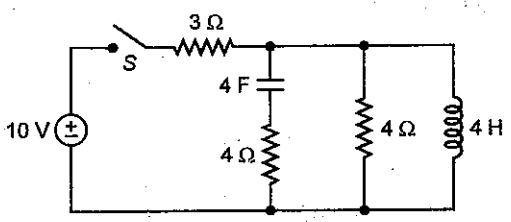
(a) 10 Ω (b) 18 Ω (c) 24 Ω (d) 12 Ω

100. A balanced delta connected load of $(8 + j6)\,\Omega$ per phase is connected to a 400 V, 50 Hz, 3-phase supply lines. If the input power factor is to be improved to 0.9 by connecting a bank of star connected capacitors, the required kVAR of the bank is

(a) 42.7 (b) 10.2 (c) 28.8 (d) 38.4

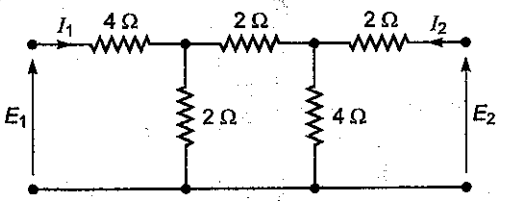
▼ *Solution Points*

101. In the circuit shown in figure, the switch S is closed at time $t = 0$. The voltage across the inductor at $t = 0^+$, is



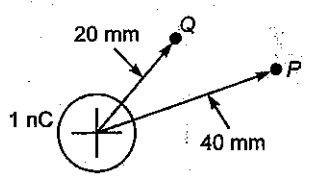
(a) 2 V (b) 4 V (c) – 6 V (d) 8 V

102. The h-parameters for a two-port network are defined by $\begin{bmatrix} E_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{bmatrix} \begin{bmatrix} I_1 \\ E_2 \end{bmatrix}$. For the two-port network shown in figure, the value of h_{12} is given by



(a) 0.125 (b) 0.167 (c) 0.625 (d) 0.25

103. A point charge of +1nC is placed in a space with a permittivity of 8.85×10^{-12} F/m as shown in figure The potential difference V_{PQ} between two points P and Q at distances of 40 mm and 20 mm respectively from the point charge is

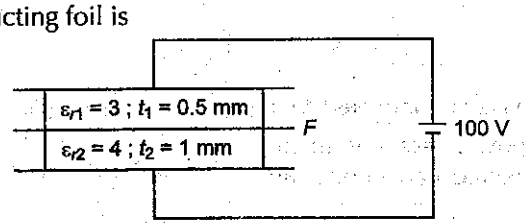


(a) 0.22 kV (b) –225 V (c) –2.24 kV (d) 15 V

104. A parallel plate capacitor has an electrode area of 100 mm^2, with a spacing of 0.1 mm between the electrodes. The dielectric between the plates is air with a permittivity of 8.85×10^{-12} F/m. The charge on the capacitor is 100 V. The stored energy in the capacitor is

(a) 8.85 pJ (b) 440 pJ (c) 22.1 nJ (d) 44.3 nJ

105. A composite parallel plate capacitor is made up of two different dielectric materials with different thicknesses (t_1 and t_2) as shown in figure. The two different dielectric materials are separated by a conducting foil F. The voltage of the conducting foil is



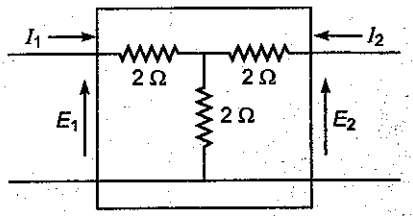
(a) 52 V (b) 60 V (c) 57 V (d) 33 V

GATE 2002

▼ *Solution Points*

106. A two-port network shown in figure is described by the following equations

$$I_1 = Y_{11}E_1 + Y_{12}E_2$$
$$I_1 = Y_{21}E_1 + Y_{22}E_2$$



The admittance parameters Y_{11}, Y_{12}, Y_{21} and Y_{22} for the network shown are

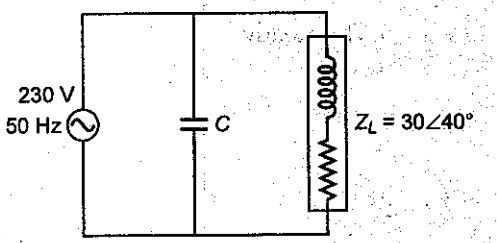
(a) 0.5 mho, 1 mho, 2 mho and 1 mho respectively

(b) $\frac{1}{3}$ mho, $-\frac{1}{6}$ mho, $-\frac{1}{6}$ mho and $\frac{1}{3}$ mho respectively

(c) 0.5 mho, 0.5 mho, 1.5 mho and 2 mho respectively

(d) $-\frac{2}{5}$ mho, $-\frac{3}{7}$ mho, $\frac{3}{7}$ mho and $\frac{2}{5}$ mho respectively

107. In the circuit shown in figure what value of C will cause a unity power factor at the AC source?



(a) 68.1 μF (b) 165 μF (c) 0.681 μF (d) 6.81 μF

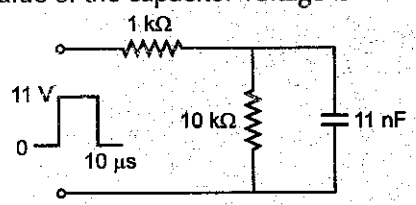
108. A first order, low-pass filter is given with $R = 50\,\Omega$ and $C = 5\,\mu F$. What is the frequency at which the gain of the voltage transfer function of the filter is 0.25?

(a) 4.92 kHz (b) 0.49 kHz (c) 2.46 kHz (d) 24.6 kHz

109. A series R-L-C circuit has $R = 50\,\Omega$; $L = 100\,\mu H$ and $C = 1\,\mu F$. The lower half power frequency of the circuit is

(a) 30.55 kHz (b) 3.055 kHz (c) 51.92 kHz (d) 1.92 kHz

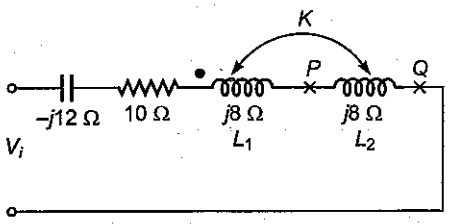
110. An 11 V pulse of 10 μs duration is applied to the circuit shown in figure Assuming that the capacitor is completely discharged prior to applying the pulse, the peak value of the capacitor voltage is



(a) 11 V (b) 5.5 V (c) 6.32 V (d) 0.96 V

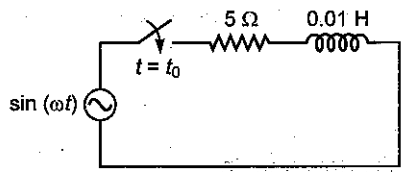
▼ *Solution Points*

111. In the circuit shown in figure, it is found that the input AC voltage (V_i) and current I are in phase. The coupling coefficient is $K = \frac{M}{\sqrt{L_1 L_2}}$, where M is the mutual inductance between the two coils. The value of K and the dot polarity of the coil P-Q are



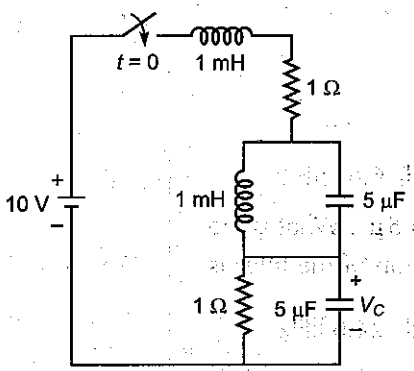
(a) $K = 0.25$ and dot at P (b) $K = 0.5$ and dot at P
(c) $K = 0.25$ and dot at Q (d) $K = 0.5$ and dot at Q

112. Consider the circuit shown in figure. If the frequency of the source is 50 Hz, then a value of t_0 which results in a transient-free response is



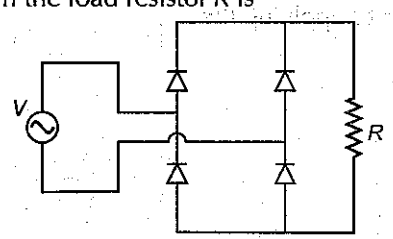
(a) zero (b) 1.78 ms (c) 2.71 ms (d) 2.91 ms

113. In the circuit shown in figure, the switch is closed at time $t = 0$. The steady state value of the voltage V_C is



(a) zero (b) 10 V (c) 5 V (d) 2.5 V

114. In the single-phase diode-bridge rectifier shown in figure, the load resistor is $R = 50\ \Omega$. The source voltage is $V = 200 \sin(\omega t)$, where $\omega = 2\pi \times 50$ rad/ s. The power dissipated in the load resistor R is



(a) $\frac{3200}{\pi}$ W (b) $\frac{400}{\pi}$ W (c) 400 W (d) 800 W

▼ *Solution Points*

GATE 2001

115. The electric field $\vec{E}$ (in V/m) at the point (1, 1, 0) due to a point charge of +1μC located at (−1, 1, 1) (coordinates in metre) is

(a) $\frac{10^{-6}}{20\sqrt{5}\,\pi\varepsilon_0}(2\hat{i}-\hat{k})$ (b) $\frac{10^{-6}}{20\,\pi\varepsilon_0}(2\hat{i}-\hat{k})$

(c) $\frac{-10^{-6}}{20\sqrt{5}\pi\,\varepsilon_0}(2\hat{i}-\hat{k})$ (d) $\frac{-10^{-6}}{20\pi\varepsilon_0}(2\hat{i}-\hat{k})$

116. A connected network of $N>2$ nodes has at most one branch directly connecting any pair of nodes. The graph of the network

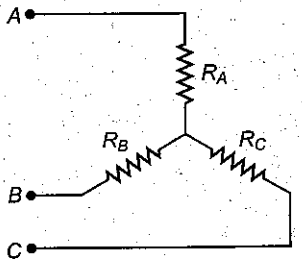
(a) must have at least N branches for one or more closed paths to exist

(b) can have an unlimited number of branches

(c) can only have at most N branches

(d) can have a minimum number of branches not decided by N

117. Consider the star network shown in figure. The resistances between terminals A and B with C open is 6 Ω, between terminals B and C with A open is 11Ω, and between terminals C and A with B open is 9 Ω. Then



(a) $R_A=4\,\Omega, R_B=2\,\Omega, R_C=5\,\Omega$ (b) $R_A=2\,\Omega, R_B=4\,\Omega, R_C=7\,\Omega$

(c) $R_A=3\,\Omega, R_B=3\,\Omega, R_C=4\,\Omega$ (d) $R_A=5\,\Omega, R_B=1\,\Omega, R_C=10\,\Omega$

118. Given the potential function in free space to be $V(x)=(50x^2+50y^2+50z^2)$ V, the magnitude (in V/m) and the direction of the electric field at a point (1, −1, 1), where the dimensions are in metre, are

(a) $100;(\hat{i}+\hat{j}+\hat{k})$ (b) $100/\sqrt{3};(\hat{i}-\hat{j}+\hat{k})$

(c) $100\sqrt{3};[(-\hat{i}+\hat{j}-\hat{k})/\sqrt{3}]$ (d) $100\sqrt{3};[(-\hat{i}-\hat{j}-\hat{k})/\sqrt{3}]$

GATE 2000

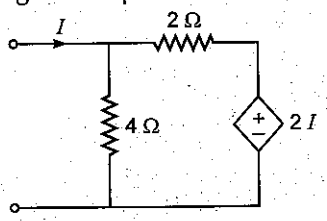
119. A two-port device is defined by the following pair of equations

$$I_1=2V_1+V_2 \quad \text{and} \quad I_2=V_1+V_2$$

Its impedance parameters $(z_{11}, z_{12}, z_{21}, z_{22})$ are given by

(a) (2, 1, 1, 1) (b) (1, −1, −1, 2) (c) (1, 1, 1, 2) (d) (2, −1, −1, 1)

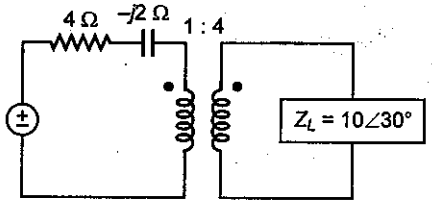
120. The circuit shown in figure is equivalent to a load of



(a) $\frac{4}{3}\Omega$ (b) $\frac{8}{3}\Omega$ (c) 4 Ω (d) 2 Ω

121. The impedance seen by the source in the circuit in figure is given by

▼ *Solution Points*



(a) $(0.54 + j0.313)\ \Omega$
(b) $(4 - j2)\ \Omega$
(c) $(4.54 - j1.69)\ \Omega$
(d) $(4 + j2)\ \Omega$

122. If an AC voltage wave is corrupted with an arbitrary number of harmonics, then the overall voltage waveform differs from its fundamental frequency component in terms of
(a) only the peak values
(b) only the rms values
(c) only the average values
(d) all the three measures (peak, rms and average values)

Answers with Explanations

1. (b) $V(t)=100\sqrt{2}\cos(100\pi t)$

$$I(t)=10\sqrt{2}\sin\left(100\pi t+\frac{\pi}{4}\right)$$

$$=10\sqrt{2}\sin\left(100\pi t+\frac{\pi}{4}+\frac{\pi}{2}-\frac{\pi}{2}\right)$$

$$=10\sqrt{2}\cos\left(100\pi t-\frac{\pi}{4}\right)$$

Hence, the phasor representation

$$I=\frac{10\sqrt{2}}{\sqrt{2}}\angle-\frac{\pi}{4}=10\angle-\frac{\pi}{4}$$

2. (a) Maximum power $P_{max}=\frac{V^2}{R_L}=\frac{10^2}{3}$ (at $R=0$)

3. (b) Here, $\omega=1$ rad/s

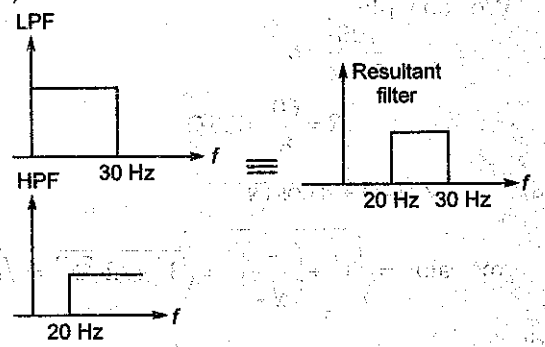
$$\therefore \quad X_L=\omega L=1\ \Omega$$

$$X_C=\frac{1}{\omega C}=1\ \Omega$$

$$I(t)=\frac{1\sin t}{1\Omega}=1\sin t$$

$$\therefore \quad I_{rms}=\frac{1}{\sqrt{2}}\text{ A}$$

4. (d)

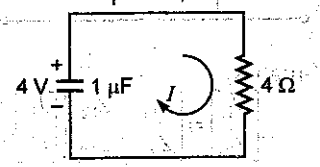


Band-pass filter with pass band $=30-20=10$ Hz

5. (b) As the switch is closed for long time, the capacitor becomes open circuited.

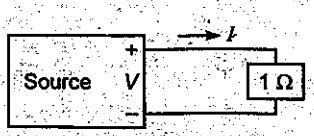
$V_C(0^-)$ = voltage across resistor $=4\times 1=4$ V

When switch is opened,



The current through 1μF capacitor $=\frac{4\text{ V}}{4\ \Omega}=1$ A

6. (b)



Given that load line characteristics

$$V+I=100 \qquad \text{...(i)}$$

Applying Kirchhoff's voltage law in circuit,

$$V=1I$$

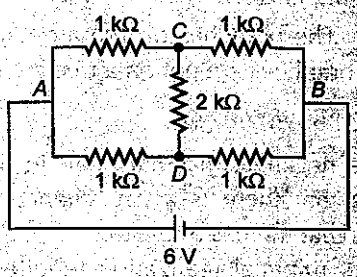
$$\Rightarrow \quad V-I=0 \qquad \text{...(ii)}$$

Subtracting Eq. (ii) from Eq. (i),

$$2I=100$$

$$I=50\text{ A}$$

7. (a)



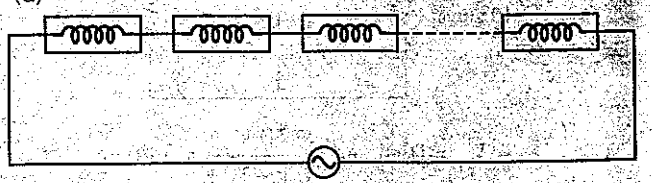
The voltage at C is same as voltage at D.*

$$V_{CD}=0$$

Hence, current through 2kΩ resistance is zero.

*Calculate V_C and V_D.

8. (d)



Let n 200 W/220 V lamps be connected in series.

Voltage across each lamp $=\frac{220}{n}$

Resistance of each lamp $=\frac{(220)^2}{200}$

Power consumed by one lamp $=\left(\frac{220}{n}\right)^2\times\frac{200}{(220)^2}$

Power consumed by n lamps connected in series

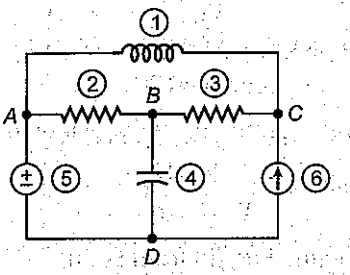
$$=nP_1=\frac{n\times(220)^2\times 200}{n^2\times(220)^2}$$

$$= \frac{200}{n}$$

As per question,

$$\frac{200}{n} = 100 \Rightarrow n = 2$$

9. (a)



The number of branches $= b = 6$

The number of nodes $= n = 4$

The number of chords $= b - n + 1 = 6 - 4 + 1 = 3$

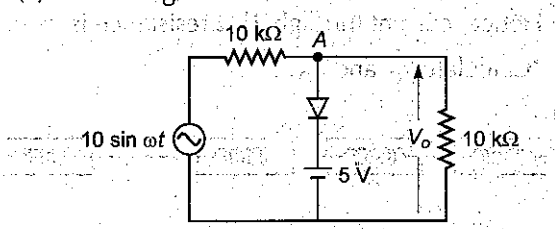
Note The branches of tree are called twigs and those branches are not on a tree are called chords.

10. (b) $Z_0 = 2.38 - j0.667\,\Omega$

As the negative imaginary part in the impedance shows that there is a capacitance and real part shows that there is a resistor.

Hence, minimal realization required resistor and capacitor.

11. (a) The voltage at A is

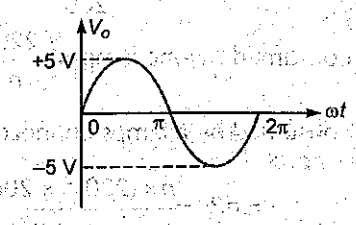


$$V_A = 10 \sin \omega t \frac{10\text{ k}}{10\text{ k} + 10\text{ k}} = 5 \sin \omega t$$

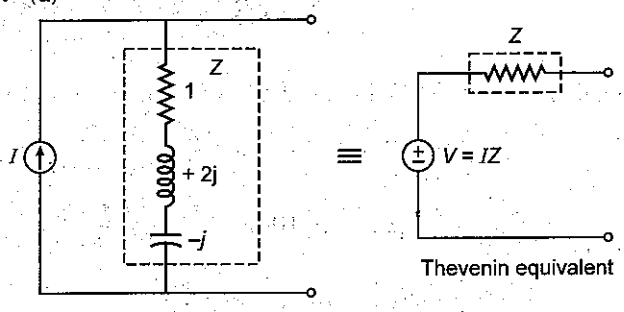
The maximum value of $V_A = 5$ (as $\sin \omega t < 1$)

Hence, the diode will be reverse biased and be open circuited.

$$V_o = 5 \sin \omega t$$



12. (d)



$$Z = 1 + 2j - j = (1 + j)\,\Omega$$

$$V = IZ = 1 \angle 0\,(1 + j) = (1 \angle 0°)(\sqrt{2} < 45°)$$

$$V = \sqrt{2} \angle 45°\ \text{V}$$

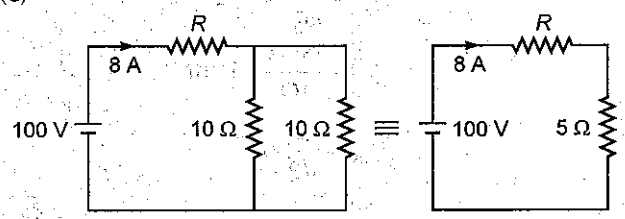
13. (d) $\nabla \cdot E = \nabla \cdot \frac{D}{\varepsilon} = \frac{\rho_v}{\varepsilon}$

i.e., Divergence of electric field is zero for static density.

$$\nabla \cdot B = 0 \qquad \text{(Always)}$$

(Because magnetic field is a closed path).

14. (c)



We can write

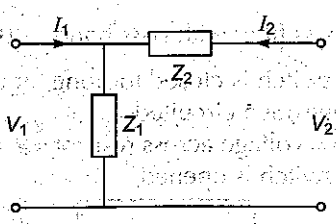
$$\frac{100}{5 + R} = 8$$

$$R = \frac{60}{8} = 7.5\,\Omega$$

15. (a) $V(t) = 3 + 4 \cos(3t)$

$$\text{rms value} = \sqrt{3^2 + \left(\frac{4}{\sqrt{2}}\right)^2} = \sqrt{3^2 + (2\sqrt{2})^2} = \sqrt{17}\ \text{V}$$

16. (d)



From Z-matrix,

$$\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} z_{11} & z_{12} \\ z_{21} & z_{22} \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}$$

i.e., $V_1 = z_{11} I_1 + z_{12} I_2$

$$V_2 = z_{21} I_1 + z_{22} I_2$$

$$z_{11} = \left.\frac{V_1}{I_1}\right|_{I_2 = 0} = z_1$$

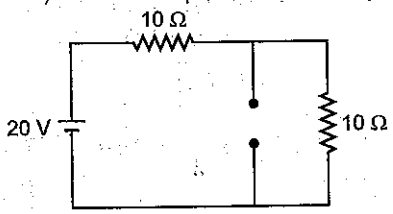
$$z_{12} = \left.\frac{V_1}{I_2}\right|_{I_1 = 0} = z_1$$

$$z_{21} = \left.\frac{V_2}{I_1}\right|_{I_2 = 0} = z_1$$

$$z_{22} = \left.\frac{V_2}{I_2}\right|_{I_1 = 0} = z_1 + z_2$$

$$\begin{bmatrix} z_{11} & z_{12} \\ z_{21} & z_{22} \end{bmatrix} = \begin{bmatrix} z_1 & z_1 \\ z_1 & z_1 + z_2 \end{bmatrix}$$

17. (b) At steady state the capacitor will be open circuited.



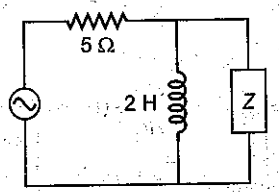
The voltage across capacitor $= 20 \times \frac{10}{10+10} = 10$ V

18. (c) As the electric field is conservative in nature.

Hence, $\nabla \times E = 0$

$\nabla . (\nabla \times E) =$ Null vector

19. (d)



Given $f = 500$ Hz

$\omega = 2\pi f = 1000\ \pi$ rad/s

For parallel resonance, Z must be capacitive. The equivalent admittance

$$Y = Y_L + Y_C = j2\omega + \frac{1}{j\omega C} = j2\omega - j/\omega C$$

$$= j\left(2\omega - \frac{1}{\omega C}\right)$$

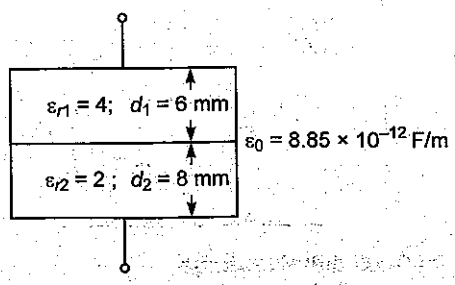
For parallel resonant circuit, the power factor is unity.

$$2\omega - \frac{1}{\omega C} = 0$$

$$2\omega = \frac{1}{\omega C}$$

$$C = \frac{1}{2\omega^2} = 0.05\ \mu\text{F}$$

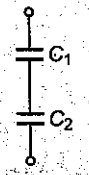
20. (d)



This is equivalent to two capacitors in series with capacitances C_1 and C_2.

where, $C_1 = \frac{A\varepsilon_0\varepsilon_{r_1}}{d_1}$

$C_2 = \frac{A\varepsilon_0\varepsilon_{r_2}}{d_2}$



Hence, $C_{eq} = \frac{C_1 C_2}{C_1 + C_2} = \frac{\frac{A\varepsilon_0\varepsilon_{r_1}}{d_1} \cdot \frac{A\varepsilon_0\varepsilon_{r_2}}{d_2}}{\frac{A\varepsilon_0\varepsilon_{r_1}}{d_1} + \frac{A\varepsilon_0\varepsilon_{r_2}}{d_2}}$

$$= \frac{A\varepsilon_0\varepsilon_{r_1}\varepsilon_{r_2}}{\varepsilon_{r_1} d_2 + \varepsilon_{r_2} d_1}$$

Putting the values,

$$C_{eq} = \frac{(400)^2 \times 10^{-6} \times 8.85 \times 10^{-12} \times 4 \times 2}{4 \times 8 \times 10^{-3} + 6 \times 2 \times 10^{-3}}$$

$$= 257\ \text{pF}$$

21. (c) Inductance of solenoids is given by

$$L = \mu_0 N^2 A x$$

where, $N =$ number of turns

$A =$ cross-sectional area $= \frac{\pi d^2}{4}$ (d is diameter)

$x =$ length

$$L = \frac{4\pi \times 10^{-7} \times (3000)^2 \times \pi \times 60^2 \times 10^{-6}}{4}$$

$$= 31.97\ \text{mH} \approx 32\ \text{mH}$$

22. (b) In balanced load, the phase difference is 120°.

Let $V_a = V_m \sin \omega t$; $I_a = I_m \sin(\omega t - \phi)$

$V_b = V_m \sin(\omega t - 120°)$; $I_b = I_m \sin(\omega t - 120° - \phi)$

$V_c = V_m \sin(\omega t + 120°)$; $I_c = I_m \sin(\omega t + 120° - \phi)$

Instantaneous power is given by

$= V_a I_a + V_b I_b + V_c I_c$

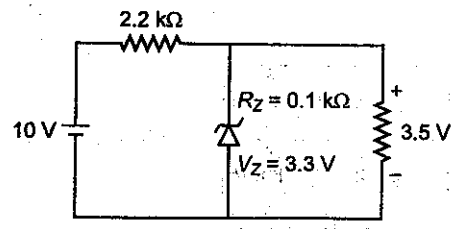
$= (V_m \sin \omega t)(\omega_m \sin(\omega t - \phi)) + \{V_m \sin(\omega t - 120°)$

$\times (I_m \sin(\omega t - 120° - \phi)$

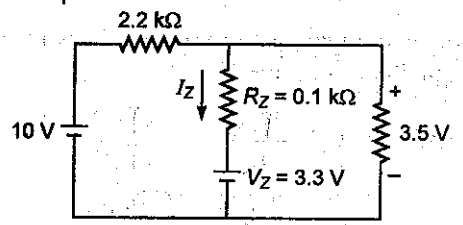
$+ \{V_m \sin(\omega t + 120°) I_m \sin(\omega t + 120° - \phi)$

$= 3VI \cos \phi =$ constant

23. (c)



The equivalent circuit be



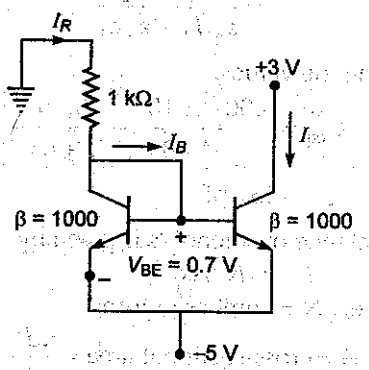
Applying KVL,

$$3.5 - I_Z R_Z - V_Z = 0$$

$$3.5 - 3.3 = I_Z R_Z$$

$$I_Z = \frac{0.2}{0.1\text{k}\Omega} = 2 \text{ mA}$$

24. (c)



Both transistors are perfectly matched.

Here, $V_{BE_1} = V_{BE_2}$

Therefore, $\frac{I_{C_1}}{I_{C_2}} = \exp\left[(V_{BE_1} - V_{BE_2})/V_T\right] = 1$

Also β is same.

$$I_R = \frac{5 - 0.7}{1\text{k}\Omega} = 4.3 \text{ mA}$$

Applying KCL at node B,

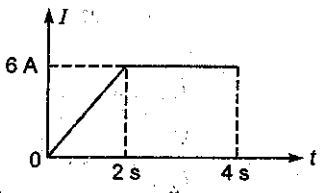
$$I_C + 2I_B - I_R = 0$$

$$\Rightarrow \quad I_C = \beta I_B$$

$$I_C = \left(\frac{\beta}{\beta + 2}\right) I_R \cong I_R \qquad \text{(because } \beta \text{ is very large)}$$

Hence, $I_{C_1} = I_{C_2} = I_R = 4.3$ mA

25. (c)



The voltage across inductor

$$V = L\frac{dI}{dt}$$

Energy absorbed by inductor

$$dW = V\,dq = VI\,dt = LI\frac{dI}{dt}\cdot dt$$

$$dW = LI\,dI$$

For $0 < t < 2\,s$, $I = 3t$

$$dI = 3\,dt$$

$$dW = L \times 3t \times 3\,dt$$

$$= 18\,t\,dt$$

$$W = 18\frac{t^2}{2}\Big|_0^2 = 18 \times \frac{4}{2} = 36 \text{ J}$$

Energy absorbed by resistor $= V\,dq = IR\,I\,dt$

$$dW = I^2 R\,dt$$

$$W_{\text{resistor}} = \int_0^2 (3t)^2 R\,dt = 9 \cdot 1 \cdot \frac{t^3}{3}\Big|_0^2$$

$$= \frac{9 \times 8}{3} = 24 \text{ J}$$

For $2 \le t \le 4$ s, $I = 6$ A

$W_{\text{inductor}} = 0; V = L\left|\frac{dI}{dt}\right| = 0$; as I is constant.

$$W_{\text{resistor}} = \int_2^4 (6)^2 R\,dt$$

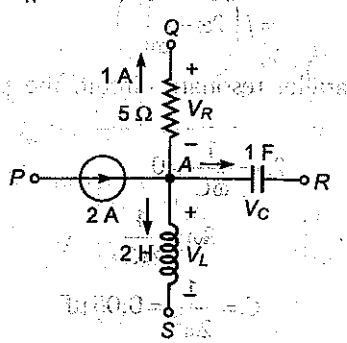
$$= 36 \times 1 \times (4 - 2)$$

$$= 72 \text{ J}$$

Total energy absorbed in the first 4 s

$$= 36 + 24 + 72 = 132 \text{ J}$$

26. (b) Given, $V_R = 5$ V



$$V_C = 4\sin 2t$$

Applying KCL at node A,

$$2 = 1 + C\frac{dV_C}{dt} + \frac{1}{L}\int V\,dt$$

$$2 - 1 - 1\frac{dV_C}{dt} = \frac{1}{2}\int V_L\,dt$$

$$1 - \frac{dV_C}{dt} = \frac{1}{2}\int V_L\,dt$$

$$1 - \frac{d}{dt}(4\sin 2t) = \frac{1}{2}\int V_L\,dt$$

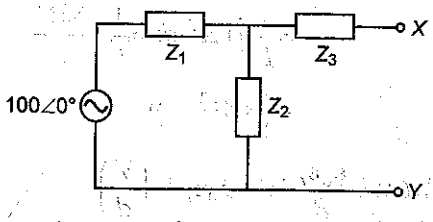
$$\Rightarrow \quad 1 - 4 \times 2\cos 2t = \frac{1}{2}\int V_L\,dt$$

$$\Rightarrow \quad 2(1 - 8\cos 2t) = \int V_L\,dt$$

Differentiating w.r.t. t,

$$\Rightarrow \quad V_L = 32\sin 2t$$

27. (a)

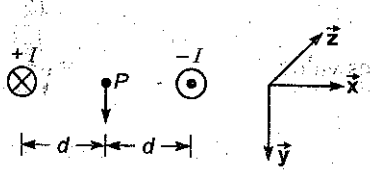


Thevenin equivalent impedance is impedance between X and Y by making voltage source short circuited.

$$\text{Thevenin impedance} = (Z_1 \,||\, Z_2) + Z_3$$

$$= 56.6 \angle 45°$$

28. (a)



The magnetic field intensity at point distance d from the current carrying conductor is

$$H = \frac{I}{2\pi d}\vec{y}$$

$$H = H^+ + H^- = \frac{I}{2\pi d}\vec{y} + \frac{I}{2\pi d}\vec{y}$$

$$H = \frac{I}{\pi d}\vec{y}$$

29. (a) The inductance $\propto \frac{1}{\text{Separation}}$

$$\frac{L_1}{L_2} = \frac{x_2}{x_1} = \frac{x/2}{x} = \frac{1}{2}$$

$$L_2 = 2L_1 = 2L \text{ H/m}$$

30. (d) The voltage $V_C(t)$ across the capacitor $= \frac{1}{C}\int I\,dt$

Given, $\quad I = 5\delta(t)$

$$V_C(t) = \frac{1}{C}\int 5\delta(t)\,dt$$

$$= \frac{1}{C} \times 5\int \delta(t)\,dt$$

$$= \frac{5u(t)}{C}$$

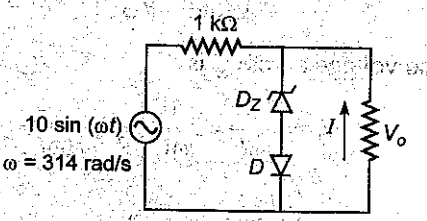
31. (a) Links are branches that do not exist in tree. If B is the number of branches of graph and N is the number of nodes. The branches of tree are $N-1$. Then number of links $= B - N + 1$.

32. (a) As per divergence theorem, the surface integral of vector is same as the volume integral of divergence of vector.

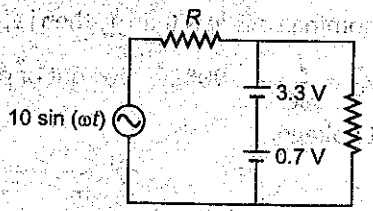
$$\therefore \quad \int_S \vec{F}\cdot d\vec{S} = \int_V \nabla\cdot\vec{F}\,dV$$

33. (a)

34. (b)

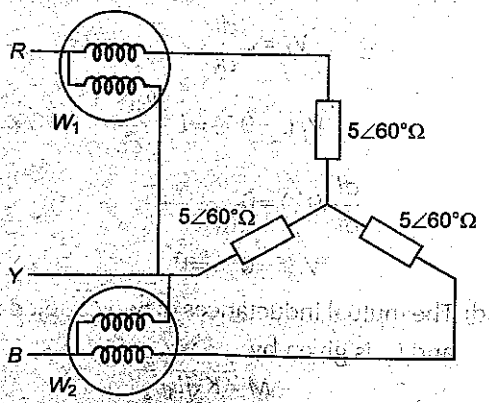


During positive, Zener diode is in breakdown. Hence, diode is in forward bias.



Output voltage $= 3.3 + 0.7 = 4$ V

35. (c)



The wattmeter W_1 reads

$$= \sqrt{3}\, V_P I_P \cos(30° + \theta)$$

$$= \sqrt{3}\, V_P I_P \cos(30° + 60°) \qquad [\because \theta = 60°]$$

$$= \sqrt{3}\, V_P I_P \cos 90° = 0$$

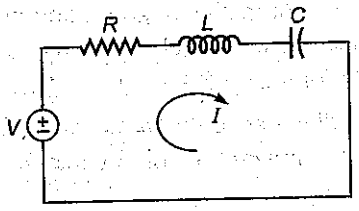
The wattmeter W_2 reads

$$= \sqrt{3}\, V_P I_P \cos(30° - 60°) = \sqrt{3}\, V_P I_P \cos(-30°)$$

$$= \sqrt{3}\, V_P I_P \times \sqrt{3}/2 = \frac{3}{2} V_P I_P = 1000 \text{ W}$$

36. (a)

37. (c)



$$Z = R + j\left(\omega L - \frac{1}{\omega C}\right)$$

$$I = \frac{V}{R + j(\omega L - 1/\omega C)}$$

The voltage across C is

$$V_C = \frac{I}{j\omega C} = \frac{1}{j\omega C}\left[\frac{V}{R + j(\omega L - 1/\omega C)}\right]$$

$$|V_C| = \left(\frac{V}{\omega C\sqrt{R^2 + (\omega L - 1/\omega C)^2}}\right)$$

If denominator is less than 1, then $|V_C| > 1$.

The $V_C = \frac{I}{\omega C} \angle -90°$ i.e., 90° out of phase with the input voltage.

38. (c)

39. (b) The inductor does not allow change in current instantaneously. The voltage across inductor

$$V_L = L\frac{dI}{dt}$$

$$V_L(t = 0^+) = L\frac{dI(0^+)}{dt}$$

$$\frac{dI}{dt}(0^+) = 0$$

$$V_L(t = 0^+) = 0$$

40. (d) The mutual inductances of two coupled inductors L_1 and L_2 is given by

$$M = K\sqrt{L_1 L_2}$$

where K is the coefficient of coupling less than unity.

$$M \le \sqrt{L_1 L_2}$$

41. (c) A passive 2-port network is in steady state. Compared to its input, the steady state output is always less than input power.

42. (d) The force experienced by electron due to electric field is qE.

The force experienced by electron due to magnetic field is $q(\vec{u} \times \vec{B})$.

The total Lorentz force $= q(E + u \times B)$ $(\because q = -e)$

$$= -e(E + u \times B)$$

43. (d) The current $I(t)$ through the inductor for $t \ge 0$

$$I(t) = \frac{1}{L}\int V(t)\, dt$$

$$= \frac{1}{L}\int 12t^2\, dt = \frac{1}{L} \times \frac{12t^3}{3}$$

$$= \frac{1}{1} \times 4t^3 = 4t^3$$

44. (c) $Q = CV = \frac{\varepsilon_0 \varepsilon_r}{d} \cdot V = \varepsilon_0 \varepsilon_r A \cdot \left(\frac{V}{d}\right)$

$$= 2.26 \times 8.85 \times 10^{-12} \times 0.2 \times 0.4 \times 50 \times 10^5 = 8\ \mu\text{C}$$

45. (a) Mean square value = variance $= \frac{[a - (-a)]^2}{12}$

$$= \frac{4a^2}{12} = \frac{a^2}{3}$$

$\therefore$ rms value $= \sqrt{\frac{a^2}{3}} = \frac{a}{\sqrt{3}}$

46. (c)

47. (b) $\because$ Active power $P = V_{1(\text{rms})} \cdot I_{1(\text{rms})} \cos\theta_1$

$$+ V_{3(\text{rms})} \cdot I_{3(\text{rms})} \cos\theta_3 + V_{5(\text{rms})} \cdot I_{5(\text{rms})} \cos\theta_5$$

But $V_{3(\text{rms})}$ and $V_{5(\text{rms})}$ are zero.

$\therefore$ $P = V_{1(\text{rms})} I_{1(\text{rms})} \cos\theta_1$

$$= 100 \times 10 \times \cos\left(\frac{\pi}{3}\right) = 500 \text{ W}$$

48. (c) Since, input power factor depends upon fundamental components.

$\therefore$ $V_i = 100\sqrt{2}\sin(100\,\pi t)$ V

$$I_i = 10\sqrt{2}\sin\left(100\,\pi t - \frac{\pi}{3}\right)$$

$\therefore$ Power factor $= \cos\frac{\pi}{3} = 0.5$

49. (d) Current $I_C = I_S - I_{RL} = \sqrt{2}\angle\frac{\pi}{4} - \sqrt{2}\angle -\frac{\pi}{4}$

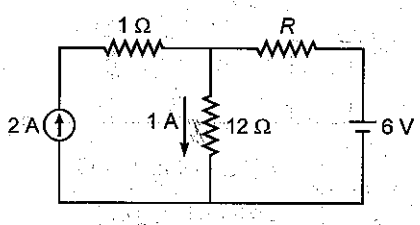
$= j2\,\text{A}$

50. (b) Power dissipated in R = Total power delivered by the source

$\therefore \quad P = VI\cos\phi = 1\times\sqrt{2}\cos\left(\frac{\pi}{4}\right)$

$= 1\,\text{W}$

51. (b)



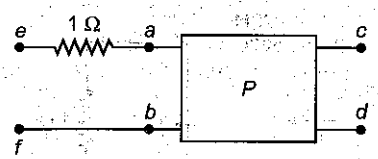
Current through R is 1 A. Voltage across 12Ω resistor is $12\times1 = 12\,\text{V}$

$$\text{Current} = \frac{12-6}{R}$$

$$1 = \frac{12-6}{R}$$

$$R = 6\,\Omega$$

52. (c)



For impedance matrix,

$$V_1 = z_{11}I_1 + z_{12}I_2$$

$$V_2 = z_{21}I_1 + z_{22}I_2$$

After connecting 1Ω,

$$z_{11} = \left.\frac{V_1}{I_1}\right|_{I_2=0} = z_{11P} + 1\Omega,$$

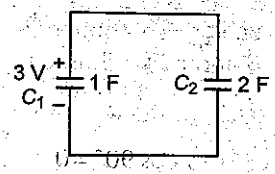
$$z_{12} = \left.\frac{V_1}{I_2}\right|_{I_1=0} = z_{12P}$$

(1Ω has no effect as $I_1 = 0$)

$$z_{21} = \left.\frac{V_2}{I_1}\right|_{I_2=0} = z_{21P}$$

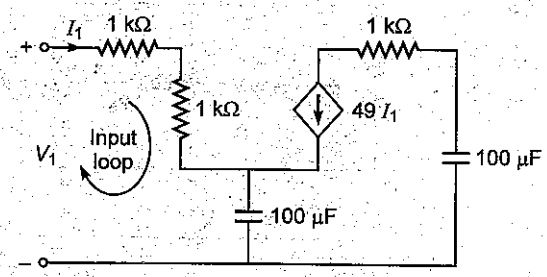
$$z_{22} = \left.\frac{V_2}{I_2}\right|_{I_1=0} = z_{22P}$$

53. (d) For time $t<0$, S_1 closed and S_2 opened. C_1 charged to 3 V. For $t>0$, S_2 closed and S_1 opened.



The C_1 gets discharged and C_2 gets charged till charge both same. The capacitor does not allow to change the voltage instantaneously. Hence, V_{C_1} at $t = 0^+$ is 3 V. Hence, the voltage across combination is 3 V.

54. (a)



Applying KVL in the input loop,

$$V_1 = 1kI_1 + 1kI_1 + (I_1 + 49I_1)X_C$$

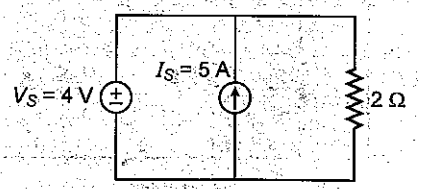
$$V_1 = 2kI_1 + 50I_1\frac{1}{j\omega C} = I_1\left(2k + \frac{1}{j\omega C/50}\right)$$

$$Z_{in} = \frac{V_1}{I_1} = 2k + \frac{1}{j\omega C/50} = R_{in} + \frac{1}{jX_{in}}$$

The input capacitance $C_{in} = C/50$

$$= \frac{100\,\mu\text{F}}{50} = 2\mu\text{F}$$

55. (b)

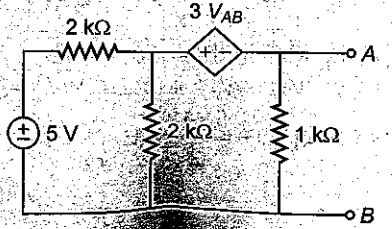


The voltage across the 2 Ω resistor is 4 V.

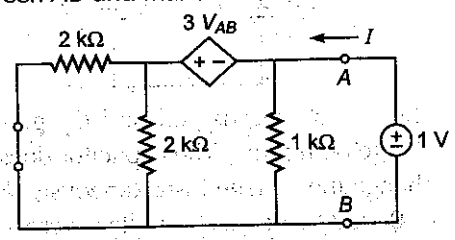
Current through 2 Ω resistor $= \frac{4}{2} = 2$ A

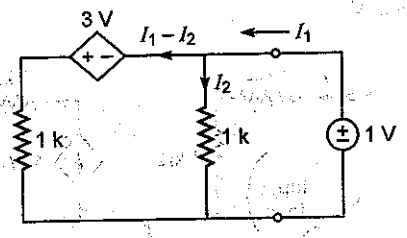
To double the current through 2 Ω resistor we have double the voltage of voltage source.

56. (b)



To calculate R_{Th} we connect 1 V voltage source between AB and make short circuit to 5 V source.





$$I_2 = \frac{1}{1\text{k}} = 1 \text{ mA}$$

Applying KVL,

$$1 + 3 - 1\text{k}(I_1 - 1 \text{ mA}) = 0$$

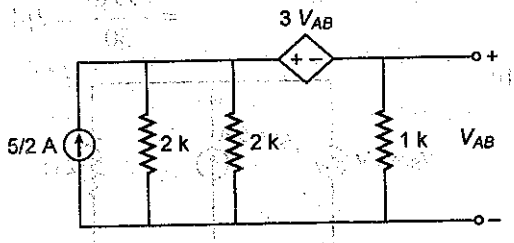
$$4 + 1 = I_1 \times 1\text{k}$$

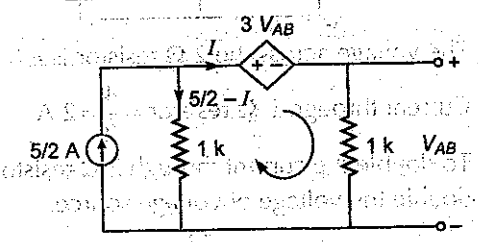
$$I_1 = 5 \text{ mA}$$

$$R_{Th} = \frac{V}{I_1} = \frac{1}{5 \text{ mA}}$$

$$= 0.2\text{k}\Omega$$

57. (d) Converting 5 V to source to current source





$$V_{AB} = I \times 1\text{k}$$

Applying KVL in the loop,

$$-3V_{AB} - V_{AB} + (5/2 - I) \times 1\text{k} = 0$$

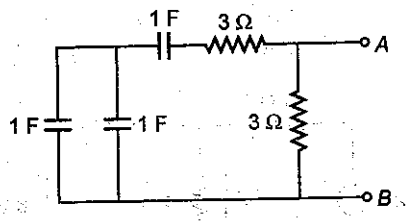
$$4V_{AB} = \frac{5 \times 1\text{k}}{2} = V_{AB}$$

(putting value of I)

$$5V_{AB} = \frac{5}{2} \times 1\text{k}$$

$$V_{AB} = 0.5 \text{ V}$$

58. (c) For finding time constant we have to find the equivalent resistance and equivalent capacitance across the current source.



$$R_{eq} = (3\Omega) + (3\Omega) = 6\Omega$$

$$C_{eq} = (1\text{F} + 1\text{F}) \| (1\text{F})$$

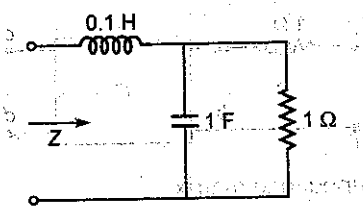
$$= (2\text{F}) \| (1\text{F})$$

$$= \frac{2 \times 1}{2 + 1} = \frac{2}{3}\text{F}$$

Time constant $\tau = R_{eq} \cdot C_{eq}$

$$= 6 \times \frac{2}{3} = 4 \text{ s}$$

59. (c)



$$Z = j\omega(0.1) + \left\{\left(\frac{1}{j\,\omega 1}\right) \| 1\right\}$$

$$= j\omega(0.1) + \left\{\frac{1/j\omega}{1 + 1/j\omega}\right\}$$

$$= j\omega(0.1) + \left\{\frac{1}{1 + j\omega}\right\}$$

Multiplying denominator and numerator of second term by $(1 - j\omega)$,

$$Z = j\omega(0.1) + \left\{\frac{1 - j\omega}{1 + \omega^2}\right\}$$

$$= \frac{1}{1 + \omega^2} + j\left\{0.1\omega - \frac{\omega}{1 + \omega^2}\right\}$$

At resonance frequency, imaginary term becomes zero.

$$0.1\omega_r - \frac{\omega_r}{1+\omega_r^2} = 0$$

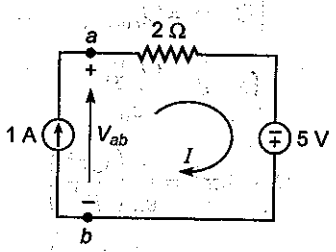
$$\Rightarrow \quad 0.1\omega_r = \frac{\omega_r}{1+\omega_r^2}$$

$$\Rightarrow \quad \frac{1}{1+\omega_r^2} = 0.1 \Rightarrow 1+\omega_r^2 = 10$$

$$\Rightarrow \quad \omega_r^2 = \sqrt{9}$$

$$\Rightarrow \quad \omega_r = 3 \text{ rad/s}$$

60. (a)

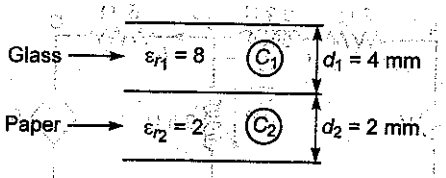


The current in the loop will be 1 A.
Applying KVL in the loop,

$$V_{ab} - 2\times 1 + 5 = 0$$

$$V_{ab} = -3 \text{ V}$$

61. (b)



The capacitance of parallel plate capacitor

$$C = \frac{\varepsilon A}{d} = \frac{\varepsilon_0 \varepsilon_r A}{d}$$

The capacitors C_1 and C_2 are in series.

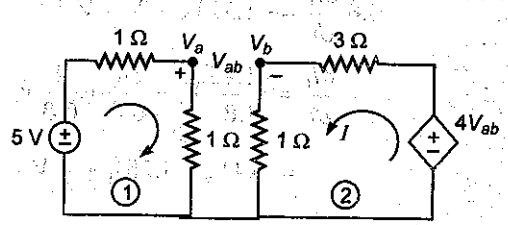
Hence, $$C_{eq} = \frac{C_1 C_2}{C_1 + C_2} = \frac{\frac{\varepsilon_0 \varepsilon_{r1} A}{d_1} \times \frac{\varepsilon_0 \varepsilon_{r_2} A}{d_2}}{\frac{\varepsilon_0 \varepsilon_{r1} A}{d_1} + \frac{\varepsilon_0 \varepsilon_{r_2} A}{d_2}}$$

$$= \frac{A \varepsilon_0 \varepsilon_{r_1} \varepsilon_{r_2}}{\varepsilon_{r_1} d_2 + \varepsilon_{r_2} d_1}$$

$$= \frac{500 \times 500 \times 10^{-6} \times 8.854 \times 10^{-12} \times 8 \times 2}{(8\times 2 + 2\times 4)\times 10^{-3}}$$

$$= 1475 \text{ pF}$$

62. (b)



The current flows in loop (1) is $\frac{5}{2}$ A.

Hence, $$V_a = \frac{5}{2}\times 1 = \frac{5}{2} \text{ V}$$

The current flows in loop (2) is I.

$$I = \frac{4V_{ab}}{4} = V_{ab}$$

Hence, $$V_b = 1I$$
$$= V_{ab}$$

$$V_{ab} = V_a - V_b \Rightarrow V_{ab} = 5/2 - V_{ab} \Rightarrow V_{ab} = 5/4 \text{ V}$$

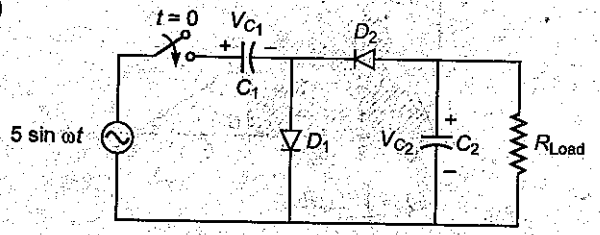
Hence, $I = V_{ab} = 1.25$ A

63. (c) The net flux passing through the plane $z = 20$ is equal to the charge enclosed by the plane. [Gauss Law].

Total charge enclosed by plane $= \frac{Q_1 + Q_2}{2}$

Net flux $= \frac{10 + 20}{2} = 15 \mu\text{C}$

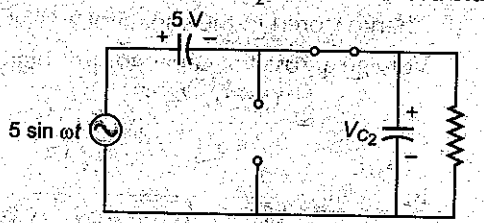
64. (c)



During the positive half cycle, the capacitor will be charged upto 5 V.

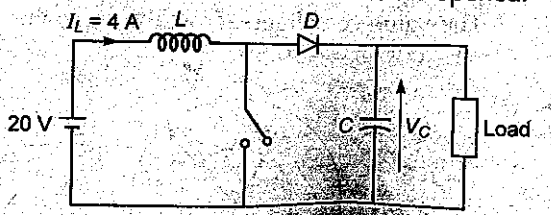
$$V_{C_1} = 5 \text{ V}$$

During the negative half cycle, the diode D_1 is reverse biased and diode D_2 will be forward biased.



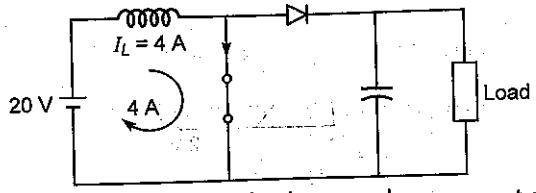
The capacitor will be charged upto 5 V + 5 = 10 V

65. (a) Consider the case when the switch is opened.



Diode D is in forward bias. Then the current through diode $= I_L = 4$ A.
The voltage across the capacitor = 20 V

Consider the case when the switch S is closed.

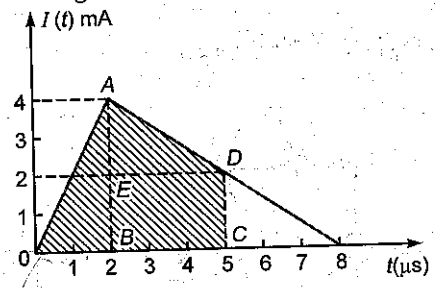


The current flows in the loop and no current will flow in diode. The current through diode = 0.
The voltage across capacitor = 0
As the duty cycle of switch is 0.5.

$$\text{Average output voltage} = \frac{20+0}{2} = 10 \text{ V}$$

$$\text{Average diode current} = \frac{4+0}{2} = 2 \text{ A}$$

66. (c) The charge stored in capacitor



$$Q = \int I(t)\,dt$$

$$= \text{shaded area}$$

$$= \Delta OAB + \square EBCD + \Delta AED$$

$$= \frac{1}{2} \times 2 \times 4 + 3 \times 2 + \frac{1}{2} \times 3 \times 2$$

$$= 4 + 6 + 3 = 13 \text{ nC}$$

67. (d) Charge stored in the capacitor = 13 nC

Voltage across the capacitor just before

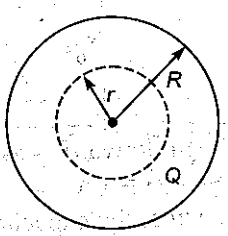
$$= \frac{13 \times 10^{-9}}{0.3 \times 10^{-9}} = 43.33 \text{ V}$$

But $$\omega_0 = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{0.6 \times 10^{-3} \times 0.3 \times 10^{-9}}}$$

$$= 2.36 \times 10^6$$

The value of voltage across the capacitor after 1μs $= V_C \cos \omega t = -30.6$ V.

68. (a)



Charge enclosed in volume of radius r

$$Q_{encl} = \frac{Q}{\frac{4}{3}\pi R^3} \times \frac{4}{3}\pi r^3$$

$$= \frac{Qr^3}{R^3}$$

From Gauss theorem, the flux emerging in area is equal to charge enclosed.

$$\int D \cdot dS = Q_{encl}$$

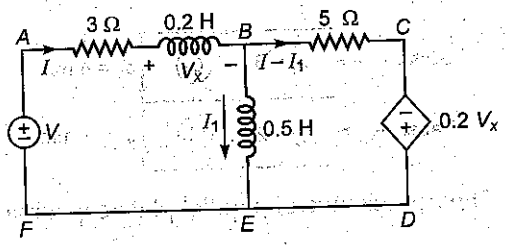
$$\int E \cdot dS = \frac{Q_{encl}}{\varepsilon_0}$$

$$E \times 4\pi r^2 = \frac{1}{\varepsilon_0}\left(\frac{Qr^3}{R^3}\right)$$

$$E = \frac{Qr}{4\pi\varepsilon_0 R^3}$$

69. (d)

70. (a)



Applying KVL in loop $ABEFA$,

$$V - 3I - V_x - 0.5\frac{dI_1}{dt} = 0 \quad \ldots(i)$$

Applying KVL in loop $BCDEB$,

$$-5(I - I_1) + 0.2V_x + 0.5\frac{dI_1}{dt} = 0 \quad \ldots(ii)$$

$$5I = 0.2V_x + 0.5\frac{dI_1}{dt} + 5I_1$$

Putting the value of I in Eq. (i),

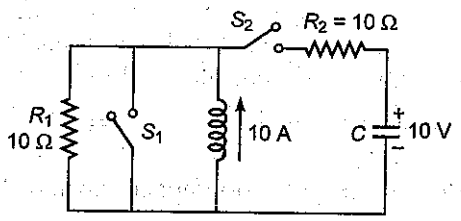
$$V - 0.12V_x - 0.3\frac{dI_1}{dt} - 3I_1 - V_x - 0.5\frac{dI_1}{dt} = 0$$

$$0.8\frac{dI_1}{dt} = -1.12V_x - 3I_1 + V$$

$$\frac{dI_1}{dt} = \frac{-1.12}{0.8}V_x - \frac{3I_1}{0.8} + \frac{1}{0.8}V$$

$$= -14V_x - 3.75I_1 + \frac{5}{4}V$$

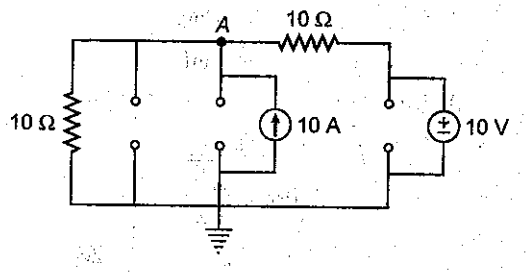
71. (d)



When S_1 is closed and S_2 is opened at $t<0$, the current through inductor is 10 A and voltage across capacitor is 10 V.

At $t=0^+$,

The S_1 is opened and S_2 is closed, then the circuit will be



Applying KCL at node A,

$$\frac{V_A}{10}+\frac{V_A-10}{10}=10$$

$$2V_A-10=100$$

$$V_A=55\ \text{V}$$

The current through capacitor $=\dfrac{55-10}{10}=4.5\ \text{A}$

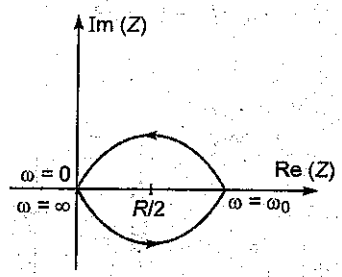
72. (a) The impedance of series resonant circuit

$$Z(j\omega)=R+j\left(\omega L-\frac{1}{\omega C}\right)$$

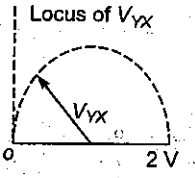
$$|Z(j\omega)|=\sqrt{R^2+\left(\omega L-\frac{1}{\omega C}\right)^2}$$

The admittance $|Y(j\omega)|=\dfrac{1}{\sqrt{R^2+\left(\omega L-\dfrac{1}{\omega C}\right)^2}}$

The admittance locus will be



73. (b)

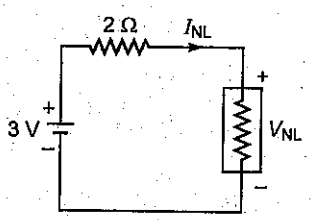


$$V_{YX}=V_X-V_Y$$

$=V-0$ for $R=\infty$

$=V-2V$ for $R=0$

74. (a)



Applying KVL in loop,

$$3-2I_{NL}-V_{NL}=0$$

$$3-2I_{NL}-I_{NL}^2=0$$

$$I_{NL}^2+2I_{NL}-3=0$$

$$I_{NL}^2+3I_{NL}-I_{NL}-3=0$$

$$I_{NL}(I_{NL}+3)-1(I_{NL}+3)=0$$

$$I_{NL}=1,-3$$

Hence, $I_{NL}=1\ \text{A}$

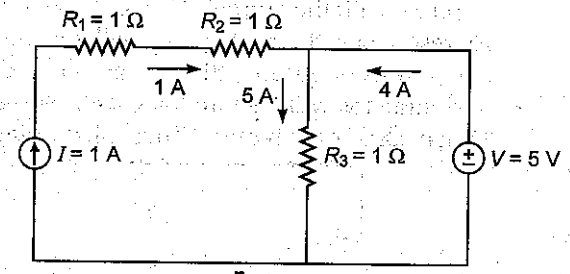
Power dissipated $=V_{NL}I_{NL}=I_{NL}^2I_{NL}=I_{NL}^3=1\ \text{W}$

75. (c)

$$\begin{array}{c} \\ E_1 \\ E_2 \\ E_3 \\ E_4 \end{array}\begin{array}{c} \begin{array}{cccccc} V_1 & V_2 & V_3 & V_4 & V_5 & V_6 \end{array} \\ \begin{bmatrix} 1 & 1 & 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & -1 & 1 & 0 \\ -1 & 0 & 0 & 0 & -1 & -1 \\ 0 & 0 & -1 & 1 & 0 & 1 \end{bmatrix} \end{array}$$

$$E=[E_1\ E_2\ E_3\ E_4]^T=AV$$

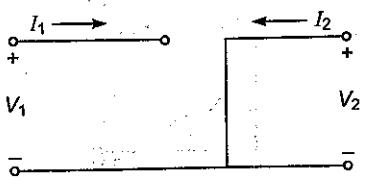
76. (d) At steady state, the capacitor will be open circuited and inductor will be short circuited. The circuit becomes



The current in $R_3=\dfrac{5}{1}=5\ \text{A}$

The current in voltage source $=5-1=4\ \text{A}$

77. (b)



The hybrid parameters of the relevant two-port network are used.

$$V_1 = h_{11}I_1 + h_{12}V_2$$

$$I_2 = h_{21}I_1 + h_{22}V_2$$

$$h_{11} = \left.\frac{V_1}{I_1}\right|_{V_2 = 0} = 1$$

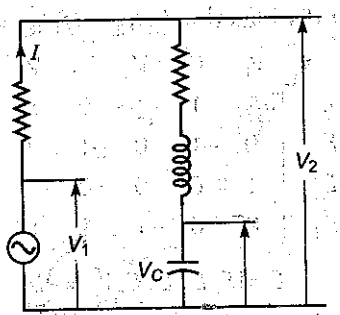
$$h_{12} = \left.\frac{V_1}{V_2}\right|_{I_1 = 0} = 0$$

$$h_{21} = \left.\frac{I_2}{I_1}\right|_{V_2 = 0} = 0$$

$$h_{22} = \left.\frac{I_2}{V_2}\right|_{I_1 = 0} = 1$$

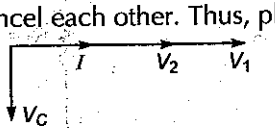
$$h\text{-parameters} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

78. (a)



The voltage applied and the current through R is in phase. The voltage across capacitor will be 90° out of phase with the current.

At resonance the voltage V_2 will be the voltage across resistor because the voltage across capacitor and inductor will be same and 180° out of phase hence, cancel each other. Thus, phase diagram



79. (b) When the capacitor is connected to inductor, capacitor will discharge through inductor and again capacitor will charge to V_0 by the current in inductor.

Thus, a sustained oscillation will take place, no decaying component will be available, as per assumption the capacitor and inductor ideal. The frequency of oscillation $\omega_0 = \frac{1}{\sqrt{LC}}$;

Hence, voltage across capacitor will be $V_0 \cos \omega_0 t$.

80. (c) Given, input variable voltage $= V_{in}$

$$V_{rms} = \frac{V_{in}}{\sqrt{2}}$$

Given at 100 MHz,

$$V_R = V_L$$

$$IR = I\omega L$$

$$R = \omega L$$

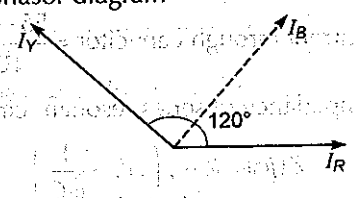
At 50 MHz,

$$I_{in} = \frac{V_{in}}{\sqrt{R^2 + \frac{\omega^2 L^2}{4}}}$$

$$= \frac{V_{in}}{\sqrt{\omega^2 L^2 + \frac{\omega^2 L^2}{4}}} = \frac{2V_{in}}{\sqrt{5}\,\omega L}$$

$$V_{R\,new} = \frac{2V_{in}}{\sqrt{5}\,\omega L} \times R = \frac{2V_{in}}{\sqrt{5}} = \frac{2\sqrt{2}V_{rms}}{\sqrt{5}} = \sqrt{\frac{8}{5}}\,V_{rms}$$

81. (a) The phasor diagram



$$\vec{I}_B = \vec{I}_R + \vec{I}_Y$$

$$|\vec{I}_B|^2 = |\vec{I}_R|^2 + |\vec{I}_Y|^2 + 2|\vec{I}_R||\vec{I}_Y| \cos \frac{120°}{2}$$

Given, $|\vec{I}_R| = |\vec{I}_Y|$

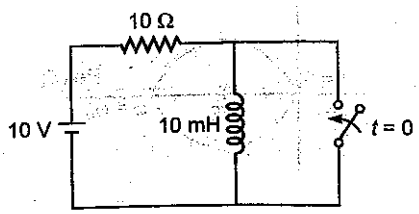
$$|\vec{I}_B|^2 = 3|\vec{I}_R|^2$$

$$|\vec{I}_B| = \sqrt{3}|\vec{I}_R|$$

$$I_R : I_Y : I_B = 1:1:\sqrt{3}$$

82. (a)

83. (c)

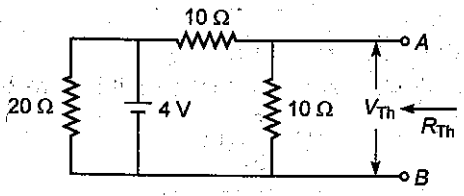


In the steady state, the inductor is short circuited and current flowing through it $= \frac{10}{10} = 1$ A.

The switch is closed at $t = 0$. The current through inductor doesn't change instantaneously.

$\therefore \quad I_L(t=0^+) = I_L(t=0^-) = 1$ A

84. (b)



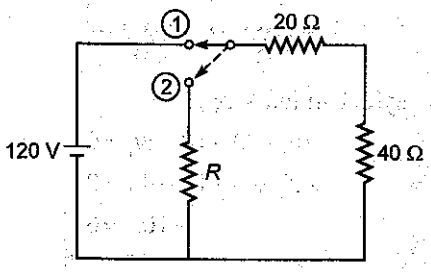
$$V_{Th} = 4 \times \frac{10}{10+10} = 2 \text{ V}$$

To calculate R_{Th}, the voltage source 4 V is short circuited,

$$R_{Th} = (20+10) \| 10$$
$$= \frac{30 \times 10}{40} = 7.5\,\Omega$$

85. (a)

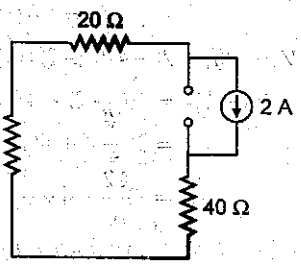
86. (a) As the point 1 is connected for long time, the inductor will be in steady state.



The current flowing through 40 Ω and inductor will be

$$I_L = \frac{120}{60} = 2\,\text{A}$$

When the switch is connected to point (2), at $t = 0^+$; inductor current does not change instantaneously.



Given, $\quad 120 = 2(R + 40 + 20)$

$$R = 0$$

87. (c) Time taken to dissipate 95% of the stored energy = Time taken to increase the current to 95%

$$I(t) = I(t=0^+)\, e^{-Rt/L}$$

$$R = 40 + 20 = 60\,\Omega$$

$$I(t) = 2e^{-\frac{60}{10}t}$$

$$0.05 = 2e^{-6t}$$

$$t = 0.61 \text{ s} \simeq 0.5 \text{ s}$$

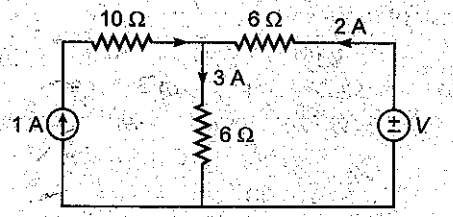
88. (a) $\quad I_{rms} = \left[\frac{1}{T}\int_0^T I^2(T)\right]$

$$= \left[\frac{1}{T}\int_0^{T/2}\left(-\frac{12T}{T}\right)^2 dT + \int_{T/2}^{T} 6^2\, dT/2\right]$$

$$I_{rms}^2 = 24$$

$$I_{rms} = 2\sqrt{6} \text{ A}$$

89. (c)



Current through 6 Ω is 3 A.

Applying KVL,

$$V - 12 - 18 = 0$$

$$V = 30 \text{ V}$$

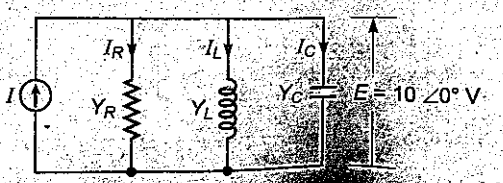
90. (a) From delta-star conversion,

$$R_1 = \frac{R_b R_c}{R_a + R_b + R_c} = \frac{10 \times 10}{20+10+10} = \frac{100}{40} = 2.5$$

$$R_2 = \frac{R_a R_c}{R_a + R_b + R_c} = \frac{20 \times 10}{40} = 5$$

$$R_3 = \frac{R_a R_b}{R_a + R_b + R_c} = \frac{20 \times 10}{40} = 5$$

91. (d)

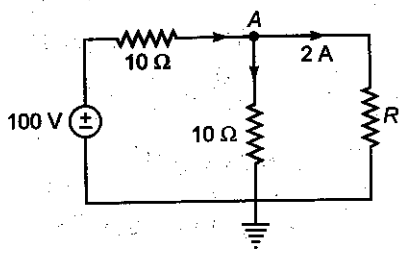


From Kirchhoff's current law,

$$I = I_R + I_L + I_C$$
$$= EY_R + EY_L + EY_C$$

$$=10(0.5+j0)+10(0-j1.5)+10(0+j0.3)$$
$$=5-j15+j3$$
$$=5-j12$$

92. (b)



Let potential of point A be V_A.

Applying KCL at point A,

$$\frac{100-V_A}{10}=\frac{V_A}{10}+2 \Rightarrow V_A=40\text{ V}$$

Hence, $2\times R=40\text{ V}$

$$R=20\,\Omega$$

93. (a) The initial voltage on capacitor $=\frac{Q}{C}=\frac{10}{0.5\text{ F}}=20\text{ V}$

As the capacitor does not allow change in voltage instantaneously.

$$V_C(0^-)=V_C(0^+)=20\text{ V}$$

At $t=\infty$; steady state $V_C(\infty)=100$ V

$$V_C(t)=V_C(\infty)+[V_C(0^+)-V_C(\infty)]\,e^{-t/RC}$$
$$=100+(20-100)\,e^{-t/2\times 0.5}$$
$$=100-80e^{-t}$$

The current $I_C(t)=C\frac{dV_C}{dt}=0.5\times 80\times e^{-t}$

$$I_C(t)=40\,e^{-t}$$
$$I_C(t=1\text{s})=40\,e^{-1}=14.71\text{ A}$$

94. (b) rms value of DC current $=10\text{ A}=I_{\text{rms1}}$

rms value of sinusoidal current $=\left(\frac{20}{\sqrt{2}}\right)=I_{\text{rms2}}$

rms value of resultant current

$$=\sqrt{I_{\text{rms1}}^2+I_{\text{rms2}}^2}$$
$$=\sqrt{10^2+\left(\frac{20}{\sqrt{2}}\right)^2}=17.32\text{ A}$$

95. (d) $[Y]=[Z]^{-1}$

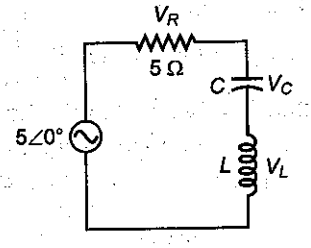
$$\begin{bmatrix}Y_{11} & Y_{12}\\ Y_{21} & Y_{22}\end{bmatrix}=\begin{bmatrix}Z_{11} & Z_{12}\\ Z_{21} & Z_{22}\end{bmatrix}^{-1}$$

Hence, $Y_{11}=\frac{Z_{22}}{\Delta Z}$; $Y_{12}=\frac{-Z_{12}}{\Delta Z}$; $Y_{21}=\frac{-Z_{21}}{\Delta Z}$;

$$Y_{22}=\frac{Z_{11}}{\Delta Z}$$

$$Y_{22}=\frac{0.9}{(0.9\times 0.6-0.2\times 0.2)}=\frac{0.9}{0.5}=1.8$$

96. (c)

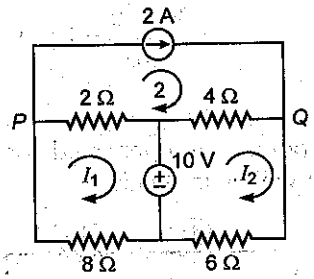


Given, $V_L=V_C=2V_R$

As $V_L=V_C \Rightarrow$ means that circuit is in resonance.

$$I=\frac{5}{5}=1\text{A}$$
$$V_R=5\times 1=5\text{ V}$$
$$V_L=V_C=2\times 5=10\text{ V}$$
$$\omega L=10$$
$$L=\frac{10}{2\pi f}=3.18\text{ mH}$$

97. (c)



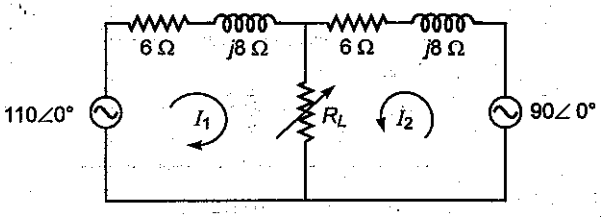
Applying KVL in left loop,

$$-2(I_1-2)-10-8I_1=0$$
$$-2I_1+4-10-8I_1=0$$
$$-10I_1=6$$
$$I_1=-\frac{6}{10}$$

Applying KVL in right loop,

$$-4(I_2-2)-6I_2+10=0$$
$$-4I_2+8-6I_2+10=0$$
$$-10I_2=-18$$
$$I_2=+8/10$$
$$V_P-2(I_1-2)-4(I_2-2)=V_Q$$
$$V_P-V_Q=2(I_1-2)+4(I_2-2)$$
$$=2I_1-4+4I_2-8$$
$$=-\frac{12}{10}-4+\frac{72}{10}-8$$
$$=\frac{-12-40+72-80}{10}$$
$$V_{PQ}=-6\text{ V}$$

98. (d)



For maximum power transfer to load

$$R_L = (6\,\Omega + j8\Omega) \,||\, (6\Omega + j8\Omega)$$
$$= 5\,\Omega$$

Applying KVL in left loop,

$$110 = (6 + j8)\, I_1 + 5(I_1 + I_2)$$
$$110 = (6 + j8 + 5)\, I_1 + 5I_2 \quad \ldots(i)$$

Applying KVL in right loop,

$$90 = (6 + j8 + 5)\, I_2 + 5I_1 \quad \ldots(ii)$$

Solving Eqs. (i) and (ii), we get

$$I_1 = 5.5 - j2.75$$
$$I_2 = 4.5 - j2.2$$

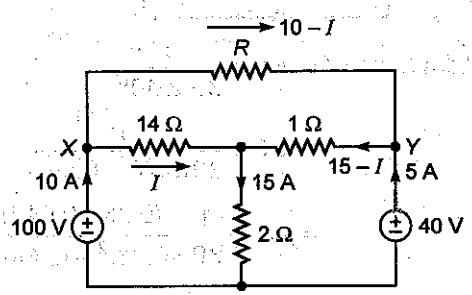
Total current through $R_L = (10 - j4.55)$ A

$$= 11.15 \text{ A}$$

Maximum power absorbed by R_L

$$= I^2R = (11.15)^2 \times 5 = 625 \text{ W}$$

99. (d)



The current distribution is shown in figure.

Applying KVL in the left loop,

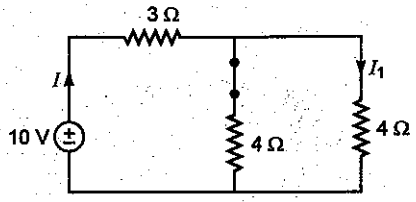
$$100 - 14I - 30 = 0$$
$$I = 5 \text{ A}$$
$$V_{XY} = 14I - (15 - I)1$$
$$\Rightarrow \quad R(10 - I) = 70 - 10 = 60$$
$$\Rightarrow \quad R = \frac{60}{5} = 12\ \Omega$$

100. (b)

101. (b) As at $t = 0$, the switch is closed. Hence, at $t = 0^+$, the voltage across capacitor is zero and current through inductor is zero.

$$V_C(0^+) = 0$$
$$I_L(0^+) = 0$$

And capacitor acts as a short circuit and inductor (at $t = 0^+$) acts as an open circuit.



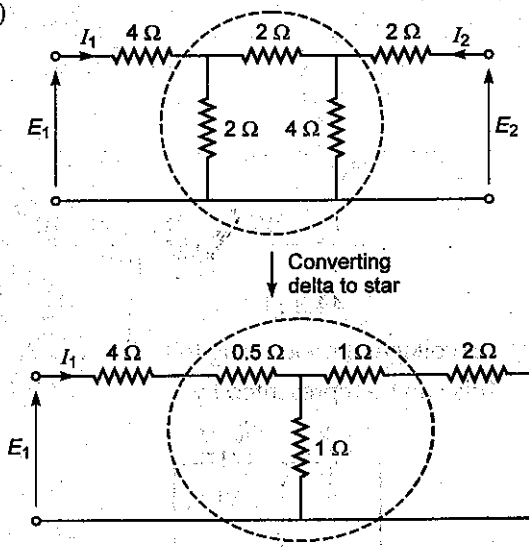
$$I = \frac{10}{5} = 2\text{A}$$

$$I_1 = 2 \times \frac{4}{4+4} = 1 \text{ A}$$

Voltage across the inductor at $t = 0^+$

$$= 4 \times 1 = 4 \text{ V}$$

102. (d)



For h-parameter,

$$\begin{bmatrix} E_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{bmatrix} \begin{bmatrix} I_1 \\ E_2 \end{bmatrix}$$

or

$$E_1 = h_{11}I_1 + h_{12}E_2$$
$$I_2 = h_{21}I_1 + h_{22}E_2$$
$$h_{12} = \left.\frac{E_1}{E_2}\right|_{I_1 = 0}$$

$$\Rightarrow \quad E_2 - 4I_2 = 0$$

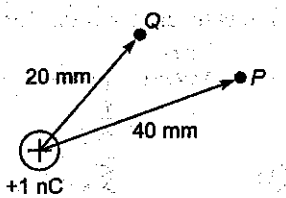
and $E_1 = I_2$

$\Rightarrow E_2 - 4E_1 = 0$

$\Rightarrow h_{12} = \left.\frac{E_1}{E_2}\right|_{I_1 = 0}$

$= \frac{1}{4}$

103. (b)



$$V_{PQ} = V_P - V_Q = \frac{q}{4\pi\varepsilon_0 r_1} - \frac{q}{4\pi\varepsilon_0 r_2}$$

$$= \frac{q}{4\pi\varepsilon_0}\left[\frac{1}{r_1} - \frac{1}{r_2}\right]$$

$$= 9\times10^9 \times 10^{-9}\left[\frac{1}{40\times10^{-3}} - \frac{1}{20\times10^{-3}}\right]$$

$$= -225\ \text{V}$$

104. (d) $E = \frac{1}{2}CV^2$

$$C = \frac{\varepsilon_0 A}{d}$$

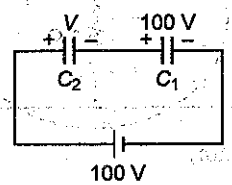
$$= \frac{1}{2}\frac{\varepsilon_0 A}{d}V^2$$

$$= \frac{1}{2}\times\frac{8.85\times10^{-12}\times100\times10^{-6}\times10^4}{10^{-4}}$$

$$= 44.3\ \text{nJ}$$

105. (b) Let voltage of conducting foil $= V$

This can be represented by



As charge on both capacitors is same.

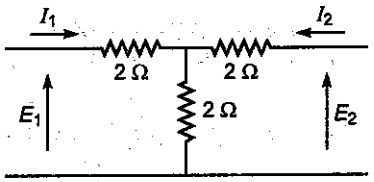
$$C_1(100 - V) = C_2 V$$

$$\frac{\varepsilon r_1}{t_1}(100 - V) = \frac{\varepsilon r_2}{t_2}V$$

$$6(100 - V) = 4V$$

$$V = 60\ \text{V}$$

106. (b)



For admittance parameters,

$$I_1 = Y_{11}E_1 + Y_{12}E_2$$

$$I_2 = Y_{21}E_1 + Y_{22}E_2$$

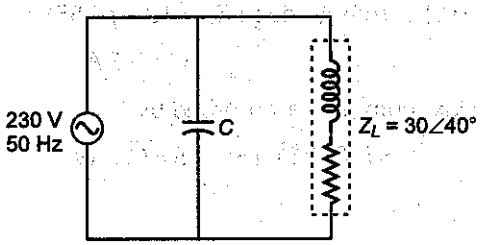
$$Y_{11} = \left.\frac{I_1}{E_1}\right|_{E_2 = 0} = \frac{1}{3}$$

$$Y_{12} = \left.\frac{I_1}{E_2}\right|_{E_1 = 0} = -\frac{1}{6}$$

$$Y_{21} = \left.\frac{I_2}{E_1}\right|_{E_2 = 0} = -\frac{1}{6}$$

$$Y_{22} = \left.\frac{I_2}{E_2}\right|_{E_1 = 0} = \frac{1}{3}$$

107. (a)



$$\text{Admittance } Y = j\omega C + \frac{1}{30\angle 40°}$$

$$= j\omega C + \frac{1}{30(0.76 + 0.64j)}$$

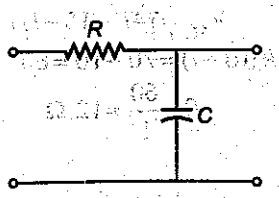
$$= j\omega C + \frac{1}{30}\frac{(0.76 - 0.64j)}{[(0.76)^2 + (0.64)^2]}$$

$$j\left(\omega C - \frac{0.64}{30[(0.76)^2 + (0.64)^2]}\right) + \text{Real part}$$

Equating imaginary part to zero (at unity power factor)

$$C = 68.1\,\mu\text{F}$$

108. (c)



$$TF = \frac{1/j\omega C}{R + \frac{1}{j\omega C}} = \frac{1}{1 + j\omega RC}$$

$$|TF| = \frac{1}{\sqrt{1 + \omega^2 R^2 C^2}} = 0.25 \text{ (Given)}$$

$$\frac{1}{0.25} = 1 + \omega^2 R^2 C^2$$

$$4 - 1 = \omega^2 R^2 C^2$$

$$\omega^2 = \frac{3}{R^2 C^2} = \frac{3}{(50)^2 \times 25 \times 10^{-12}}$$

$$f = 2.46 \text{ kHz}$$

109. (b) At lower half power frequency,

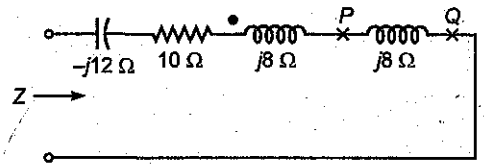
$$\omega_1 L - \frac{1}{\omega_1 C} = -R$$

$$2\pi f_1 \times 100 \times 10^{-6} - \frac{1}{2\pi f \times 1 \times 10^{-6}} = -50$$

$$f_1 = 3.055 \text{ kHz}$$

110. (a)

111. (c)



$$Z = 10 - j12 + j8 + j8 \pm j2K\sqrt{L_1 L_2}$$

$$= 10 + 4j \pm j2K\sqrt{L_1 L_2}$$

As voltage and current are in phase, the imaginary part of impedance must be zero.

$$4 \pm 2K\sqrt{L_1 L_2} = 0$$

For resultant to be zero,

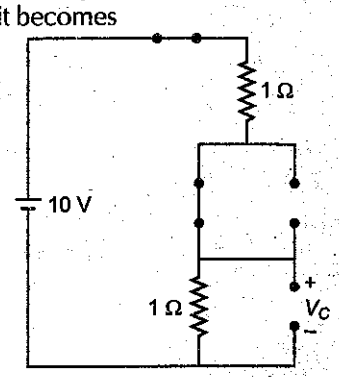
Mutual inductance $= -2K\sqrt{L_1 L_2}$

Hence, dot connection to be at Q.

$$K = \frac{4}{2\sqrt{8 \times 8}} = 0.25$$

112. (b)

113. (c) At steady state, the inductor becomes short circuited and capacitor becomes open circuited. The circuit becomes



From voltage divider rule,

$$V_C = 10 \times \frac{1}{1+1} = 5 \text{ V}$$

114. (a)

115. (a) Electric field due to point charge

$$\vec{E} = \frac{q}{4\pi\varepsilon_0} \frac{\vec{r}}{|r|^3}$$

$$\vec{r} = 2\hat{i} - \hat{k}$$

$$\vec{E} = \frac{10^{-6}(2\hat{i} - \hat{k})}{4\pi\varepsilon_0 (\sqrt{5})^3}$$

$$= \frac{10^{-6}}{20\sqrt{5}\pi\varepsilon_0}(2\hat{i} - \hat{k}) \text{ V/m}$$

116. (a) Must have at least N branches for one or more closed paths to exist.

117. (b) Given,

$$R_A + R_B = 6 \quad \ldots(i)$$

$$R_B + R_C = 11 \quad \ldots(ii)$$

$$R_C + R_A = 9 \quad \ldots(iii)$$

Adding Eqs. (i), (ii) and (iii), we get

$$R_A + R_B + R_C = \frac{26}{2} = 13$$

Hence, $R_A = 2\ \Omega, R_B = 4\ \Omega, R_C = 7\ \Omega$

118. (c) $\vec{E} = -\left[\frac{\partial V}{\partial x}\hat{i} + \frac{\partial V}{\partial y}\hat{j} + \frac{\partial V}{\partial t}\hat{k}\right]$

$$= -[100x\,\hat{i} + 100y\,\hat{j} + 100z\,\hat{k}]$$

$$\vec{E}\,|\,\text{at } (1,-1,1) = -100[\hat{i} - \hat{j} + \hat{k}] \text{ V/m}$$

$$\text{Magnitude} = 100\sqrt{3}$$

$$\text{Direction} = \frac{-\hat{i} + \hat{j} - \hat{k}}{\sqrt{3}}$$

119. (b) Given,

$$I_1 = 2V_1 + V_2 \quad \ldots(i)$$

$$I_2 = V_1 + V_2 \quad \ldots(ii)$$

Subtracting Eq. (ii) from Eq. (i),

$$V_1 = I_1 - I_2$$

Putting the value of V_1 in Eq. (ii),

$$I_2 = I_1 - I_2 + V_2$$

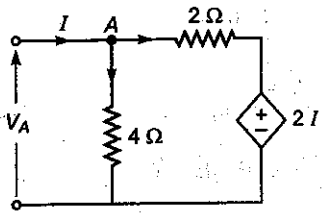
$$V_2 = -I_1 + 2I_2$$

Comparing with 2-parameter general expression,

$$V_1 = z_{11}I_1 + z_{12}I_2$$
$$V_2 = z_{21}I_1 + z_{22}I_2$$

Hence, $(z_{11}, z_{12}, z_{21}, z_{22})$ is $(1, -1, -1, 2)$.

120. (b)



Applying KCL at A,

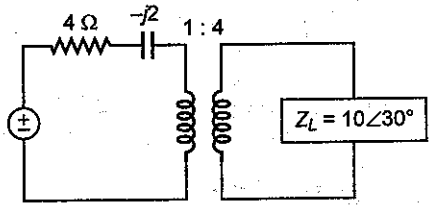
$$\frac{V_A}{4} + \frac{V_A - 2I}{2} = I$$

$$\frac{V_A}{4} + \frac{V_A}{2} = 2I$$

$$\frac{3V_A}{4} = 2I$$

$$\text{Equivalent load} = \frac{V_A}{I} = \frac{8}{3}\Omega$$

121. (c)



The load impedance seen by source

$$= 10\angle 30° \times \left(\frac{1}{4}\right)^2$$

$$= 0.54 + j0.31$$

The source impedance $= 4 - j2$

The total impedance seen by source

$$= (4.54 - j1.69)\,\Omega$$

122. (d) Harmonic distortion effects are all peak, rms and average values.

3

Signals and Systems

Syllabus

Signals and Systems *Representation of continuous and discrete-time signals, Shifting and scaling operations, Linear, time-invariant and causal systems, Fourier series representation of Continuous periodic signals, sampling theorem, Fourier, Laplace and Z transforms.*

GATE 2011 *(1 Mark Questions)*

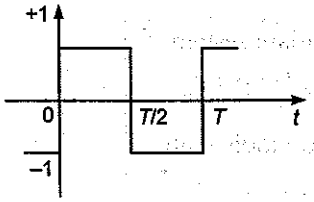
▼ *Solution Points*

1. Given two continuous time signals $x(t) = e^{-t}$ and $y(t) = e^{-2t}$ which exist for $t > 0$, the convolution $z(t) = x(t) * y(t)$ is

(a) $e^{-t} - e^{-2t}$ (b) e^{-3t} (c) e^{+t} (d) $e^{-t} + e^{-2t}$

GATE 2010

2. The second harmonic component of the periodic waveform given in the figure has an amplitude of



(a) zero (b) 1 (c) 2/ (d) $\sqrt{5}$

3. The system represented by the input-output relation $y(t) = \int_{-\infty}^{\infty} x(\tau)\, d\tau, t > 0$ is

(a) linear and causal (b) linear but not causal
(c) causal but not linear (d) neither linear nor causal

GATE 2009

4. A linear time-invariant system with an impulse response $h(t)$ produces output $y(t)$ when input $x(t)$ is applied. When the input $x(t - \tau)$ is applied to a system with impulse response $h(t - \tau)$, the output will be

(a) $y(t)$ (b) $y\{2(t - \tau)\}$ (c) $y(t - \tau)$ (d) $y(t - 2\tau)$

▼ *Solution Points*

GATE 2008

5. A signal $e^{-\alpha t}\sin(\omega t)$ is the input to a linear time-invariant system. Given, K and ϕ are constants, the output of the system will be of the form $Ke^{-\beta t}\sin(vt+\phi)$, where

(a) β needs not be equal to α but v equals to ω

(b) v needs not be equal to ω but β equals to α

(c) β equals to α and v equals to ω

(d) β needs not be equal to α and v needs not be equal to ω

6. X is a uniformly distributed random variable that takes values between 0 and 1. The value of $E\{X^3\}$ will be

(a) zero (b) $\frac{1}{8}$ (c) $\frac{1}{4}$ (d) $\frac{1}{2}$

7. A function $y(t)$ satisfies the following differential equation

$$\frac{dy(t)}{dt} + y(t) = \delta(t)$$

where, $\delta(t)$ is the delta function. Assuming zero initial condition, and denoting the unit step function by $u(t)$, $y(t)$ can be of the form

(a) e^t (b) e^{-t} (c) $e^t u(t)$ (d) $e^{-t}u(t)$

8. The impulse response of a causal linear time-invariant system is given as $h(t)$. Now consider the following two statements :

Statement (I) : Principle of superposition holds.

Statement (II) : $h(t) = 0$ for $t < 0$

Which one of the following statements is correct?

(a) Statement (I) is correct and Statement (II) is wrong

(b) Statement (II) is correct and Statement (I) is wrong

(c) Both Statement (I) and Statement (II) are wrong

(d) Both Statement (I) and Statement (II) are correct

GATE 2007

9. Let a signal $a_1\sin(\omega_1 t + \phi_1)$ be applied to a stable linear time-invariant system. Let the corresponding steady state output be represented as $a_2 F(\omega_2 t + \phi_2)$. Then which of the following statements is true?

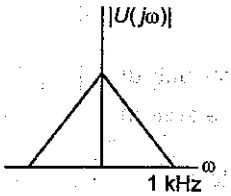
(a) F is not necessarily a 'sine' or 'cosine' function but must be periodic with $\omega_1 = \omega_2$

(b) F must be a 'sine' or 'cosine' function with $a_1 = a_2$

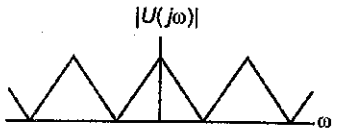
(c) F must be a 'sine' function with $\omega_1 = \omega_2$ and $\phi_1 = \phi_2$

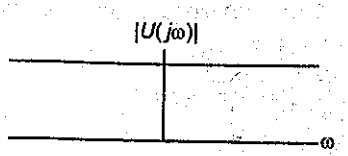
(d) F must be a 'sine' or 'cosine' function with $\omega_1 = \omega_2$

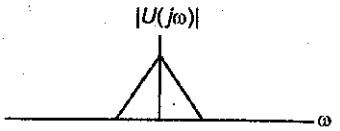
10. The frequency spectrum of a signal is shown in the figure. If this signal is ideally sampled at intervals of 1 ms, then the frequency spectrum of the sampled signal will be

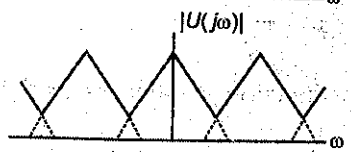


▼ *Solution Points*

(a) 

(b) 

(c) 

(d) 

GATE 2006

11. Which of the following is true?
(a) A finite signal is always bounded
(b) A bounded signal always possesses finite energy
(c) A bounded signal is always zero outside the interval $[-t_0, t_0]$ for some t_0
(d) A bounded signal is always finite

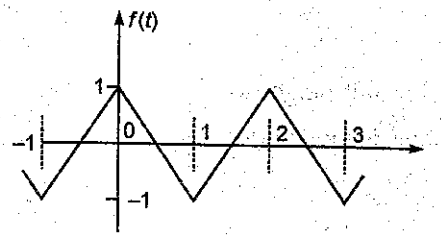
12. $x(t)$ is a real valued function of real variable with period T. Its trigonometric Fourier series expansion contains no terms of frequency $\omega = 2\pi(2k)/T$; $k = 1, 2, \ldots$. Also, no sine terms are present. Then, $x(t)$ satisfies the equation

(a) $x(t) = -x(t-T)$
(b) $x(t) = x(T-t) = -x(-t)$
(c) $x(t) = x(T-t) = -x(t-T/2)$
(d) $x(t) = x(t-T) = x(t-T/2)$

13. A discrete real all-pass system has a pole at $z = 2\angle 30°$. It, therefore
(a) also has a pole at $1/2\angle 30°$
(b) has a constant phase response over the z-plane; $\arg|H(\tau)| = \text{constant}$
(c) is stable only if it is anticausal
(d) has a constant phase response over the unit circle; $\arg|H(e^{j\Omega})| = \text{constant}$

GATE 2002

14. Fourier series for the waveform $f(t)$ shown in figure is



(a) $\frac{8}{\pi^2}\left[\sin(\pi t) + \frac{1}{9}\sin(3\pi t) + \frac{1}{25}\sin(5\pi t) + \ldots\right]$

(b) $\frac{8}{\pi^2}\left[\sin(\pi t) - \frac{1}{9}\sin(3\pi t) + \frac{1}{25}\sin(5\pi t) + \ldots\right]$

(c) $\frac{8}{\pi^2}\left[\cos(\pi t) + \frac{1}{9}\cos(3\pi t) + \frac{1}{25}\cos(5\pi t) + \ldots\right]$

(d) $\frac{8}{\pi^2}\left[\cos(\pi t) - \frac{1}{9}\cos(3\pi t) + \frac{1}{25}\cos(5\pi t) + \ldots\right]$

▼ *Solution Points*

15. Let $s(t)$ be the step response of a linear system with zero initial conditions; then the response of this system to an input $u(t)$ is

(a) $\int_0^t s(t-\tau)\,u(\tau)\,d\tau$ (b) $\frac{d}{dt}\left[\int_0^t s(t-\tau)\,u(\tau)\,d\tau\right]$

(c) $\int_0^t s(t-\tau)\left[\int_0^t u(\tau_1)\,d\tau_1\right]d\tau$ (d) $\int_0^t s(t-\tau)^2\,u(\tau)\,d\tau$

16. Let $Y(s)$ be the Laplace transformation of the function $y(t)$, then the final value of the function is

(a) $\lim_{s\to 0} Y(s)$ (b) $\lim_{s\to\infty} Y(s)$

(c) $\lim_{s\to 0} sY(s)$ (d) $\lim_{s\to\infty} sY(s)$

GATE 2011

(2 Marks Questions)

17. The response $h(t)$ of a linear time-invariant system to an impulse $\delta(t)$, under initially relaxed condition is $h(t) = e^{-t} + e^{-2t}$. The response of this system for a unit step input $u(t)$ is

(a) $u(t) + e^{-t} + e^{-2t}$ (b) $(e^{-t} + e^{-2t})\,u(t)$

(c) $(1.5 - e^{-t} - 0.5\,e^{-2t})\,u(t)$ (d) $e^{-t}\,\delta(t) + e^{-2t}\,u(t)$

18. Let the Laplace transform of a function $f(t)$ which exists for $t > 0$ be $F_1(s)$ and the Laplace transform of its delayed version $f(t-\tau)$ be $F_2(s)$. Let $F_1^*(s)$ be the complex conjugate of $F_1(s)$ with the Laplace variable set as $s = \sigma + j\omega$. If $G(s) = \frac{F_2(s).F_1^*(s)}{|F_1(s)|^2}$, then the inverse Laplace transform of $G(s)$ is

(a) an ideal impulse $\delta(t)$

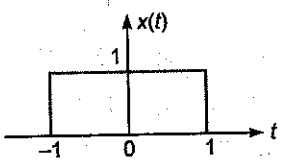
(b) an ideal delayed impulse $\delta(t-\tau)$

(c) an ideal step function $u(t)$

(d) an ideal delayed step function $u(t-\tau)$

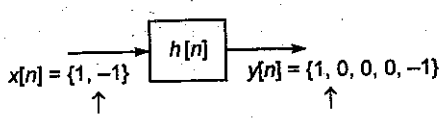
GATE 2010

19. $x(t)$ is a positive rectangular pulse from $t = -1$ to $t = +1$ with unit height as shown in the figure. The value of $\int_{-\infty}^{\infty} |X(\omega)|^2\,d\omega$ {where, $X(\omega)$ is the Fourier transform of $x(t)$} is



(a) 2 (b) 2π (c) 4 (d) 4π

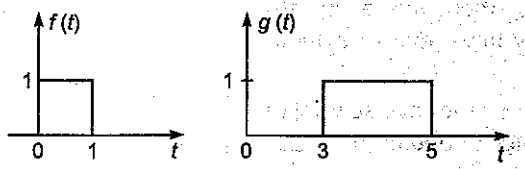
20. Given the finite length input $x[n]$ and the corresponding finite length output $y[n]$ of an LTI system as shown below, the impulse response $h[n]$ of the system is



(a) $h[n] = \{1, 0, 0, 1\}$ (b) $h[n] = \{1, 0, 1\}$

(c) $h[n] = \{1, 1, 1, 1\}$ (d) $h[n] = \{1, 1, 1\}$

Common Data for Questions 21 and 22
Given $f(t)$ and $g(t)$ as shown below.



21. $g(t)$ can be expressed as

(a) $g(t) = f(2t-3)$ (b) $g(t) = f\left(\frac{t}{2}-3\right)$

(c) $g(t) = f\left(2t-\frac{3}{2}\right)$ (d) $g(t) = f\left(\frac{t}{2}-\frac{3}{2}\right)$

22. The Laplace transform of $g(t)$ is

(a) $\frac{1}{s}(e^{3s} - e^{5s})$ (b) $\frac{1}{s}(e^{-5s} - e^{-3s})$

(c) $\frac{e^{-3s}}{s}(1 - e^{-2s})$ (d) $\frac{1}{s}(e^{5s} - e^{3s})$

GATE 2009

23. A cascade of 3 linear time-invariant systems is causal and unstable. From this, we conclude that

(a) each system in the cascade is individually causal and unstable
(b) at least one system is unstable and at least one sysem is causal
(c) at least one system is causal and all systems are unstable
(d) the majorities are unstable and the minorities are causal

24. The Fourier series coefficient of a periodic signal $x(t)$ expressed as $x(t) = \Sigma_{k=-\infty}^{\infty} a_k e^{j2\pi kt/T}$ are given by $a_{-2} = 2 - j1; a_{-1} = 0.5 + j0.2;$ $a_0 = j2; a_1 = 0.5 - j0.2; a_2 = 2 + j1;$ and $a_k = 0;$ for $|k| > 2$. Which of the following is true?

(a) $x(t)$ has finite energy because only finitely many coefficients are non-zero
(b) $x(t)$ has zero average value because it is periodic
(c) The imaginary part of $x(t)$ is constant
(d) The real part of $x(t)$ is even

25. The z-transform of a signal $x[n]$ is given by $4z^{-3} + 3z^{-1} + 2 - 6z^2 + 2z^3$. It is applied to a system with a transfer function $H(z) = 3z^{-1} - 2$. Let the output be $y(n)$. Which of the following is true?

(a) $y(n)$ is non-causal with finite support
(b) $y(n)$ is causal with infinite support
(c) $y(n) = 0; |n| > 3$
(d) $\text{Re}[Y(z)]_{z=e^{j\theta}} = -\text{Re}[Y(z)]_{z=e^{-j\theta}};\ \text{Im}[Y(z)]_{z=e^{j\theta}}$
$= \text{Im}[Y(z)]_{z=e^{-j\theta}}; -\pi \le \theta < \pi$

▼ Solution Points

▼ *Solution Points*

GATE 2008

26. Given a sequence $x[n]$ to generate the sequence $y[n] = x[3 - 4n]$, which one of the following procedures would be correct?

(a) First delay $x[n]$ by 3 samples to generate $z_1[n]$, then pick every 4th sample of $z_1[n]$ to generate $x_2[n]$, and then finally time reverse $z_2[n]$ to obtain $y[n]$

(b) First advance $x[n]$ by 3 samples to generate $z_1[n]$, then pick every 4th sample of $z_1[n]$ to generate $z_2[n]$ and then finally time reverse $z_2[n]$ to obtain $y[n]$

(c) First pick every 4th sample of $x[n]$ to generate $v_1[n]$, time reverse $v_1[n]$ to obtain $v_2[n]$, and finally advance $v_2[n]$ by 3 samples to obtain $y[n]$

(d) First pick every 4th sample of $x[n]$ to generate $v_1[n]$, time reverse $v_1[n]$ to obtain $v_2[n]$, and finally delay $v_2[n]$ by 3 samples to obtain $y[n]$

27. A system with input $x(t)$ and output $y(t)$ is defined by the input-output relation

$$y(t) = \int_{-\infty}^{2t} x(\tau)\, d\tau$$

The system will be

(a) causal, time-invariant and unstable

(b) causal, time-invariant and stable

(c) non-causal, time-invariant and unstable

(d) non-causal, time-variant and unstable

28. A signal $x(t) = \text{sinc}(\alpha t)$, where α is a real constant $\left(\text{sinc}(x) = \dfrac{\sin(\pi x)}{\pi x}\right)$ is the input to a linear time-invariant system whose impulse response $h(t) = \text{sinc}(\beta t)$ where, β is a real constant. If $\min(\alpha, \beta)$ denotes the minimum of α and β, and similarly $\max(\alpha, \beta)$ denotes the maximum of α and β, and K is a constant, which one of the following statements is true about the output of the system?

(a) It will be of the form K sinc (γt), where $\gamma = \min(\alpha, \beta)$

(b) It will be of the form K sinc (γt), where $\gamma = \max(\alpha, \beta)$

(c) It will be of the form K sinc (αt)

(d) It cannot be a sin c type of signal

29. Let $x(t)$ be a periodic signal with time period T. Let $y(t) = x(t - t_0) + x(t + t_0)$ for some t_0. The Fourier series coefficients of $y(t)$ are denoted by b. If $b_k = 0$ for all odd k, then t_0 can be equal to

(a) $\dfrac{T}{8}$ (b) $\dfrac{T}{4}$ (c) $\dfrac{T}{2}$ (d) $2T$

30. $H(z)$ is a transfer function of a real system. When a signal $x(n) = (t + j)^n$ is the input to such a system, the output is zero. Further, the Region of Convergence (ROC) of $\left(1 - \dfrac{1}{2} z^{-1}\right) H(z)$ is the entire Z-plane (except $z = 0$). It can then be inferred that $H(z)$ can have a minimum of

(a) one pole and one zero (b) one pole and two zeros

(c) two poles and one zero (d) two poles and two zeros

31. Given $X(z) = \dfrac{z}{(z-a)^2}$ with $|z| > a$, the residue of $X(z)\, z^{n-1}$ at $z = a$ for $n \geq 0$ will be

(a) a^{n-1} (b) a^n (c) na^n (d) na^{n-1}

▼ *Solution Points*

32. Let $x(t) = \text{rect}\left(t - \frac{1}{2}\right)$ {where, rect $(x) = 1$ for $-\frac{1}{2} \le x \le \frac{1}{2}$ and zero otherwise}.

Then, if $\text{sinc}(x) = \frac{\sin(\pi x)}{\pi x}$, the Fourier transform of $x(t) + x(-t)$ will be given by

(a) $\text{sinc}\left(\frac{\omega}{2\pi}\right)$ (b) $2\,\text{sinc}\left(\frac{\omega}{2\pi}\right)$

(c) $2\,\text{sinc}\left(\frac{\omega}{2\pi}\right)\cos\left(\frac{\omega}{2}\right)$ (d) $\text{sinc}\left(\frac{\omega}{2\pi}\right)\sin\left(\frac{\omega}{2}\right)$

33. The transfer function of a linear time-invariant system is given as

$$G(s) = \frac{1}{s^2 + 3s + 2}$$

The steady state value of the output of this system for a unit impulse input applied at time instant $t = 1$ will be

(a) zero (b) 0.5 (c) 1 (d) 2

GATE 2007

34. A signal $x(t)$ is given by

$$x(t) = \begin{cases} 1, -T/4 < t \le 3T/4 \\ -1, 3T/4 < t \le 7T/4 \\ -x(t+T) \end{cases}$$

Which among the following gives the fundamental Fourier term of $x(t)$?

(a) $\frac{4}{\pi}\cos\left(\frac{\pi t}{T} - \frac{\pi}{4}\right)$ (b) $\frac{4}{\pi}\cos\left(\frac{\pi t}{2T} + \frac{\pi}{4}\right)$

(c) $\frac{4}{\pi}\sin\left(\frac{\pi t}{T} - \frac{\pi}{4}\right)$ (d) $\frac{4}{\pi}\sin\left(\frac{\pi t}{2T} + \frac{\pi}{4}\right)$

35. If $u(t), r(t)$ denote the unit step and unit ramp functions respectively and $u(t) * r(t)$ their convolution, then the function $u(t+1) * r(t-2)$ is given by

(a) $(1/2)(t+1)(t+2)$ (b) $(1/2)(t-1)(t-2)$

(c) $(1/2)(t-1)^2 u(t-1)$ (d) None of these

36. $X(z) = 1 - 3z^{-1}, Y(z) = 1 + 2z^{-1}$ are Z-transforms of two signals $x[n], y[n]$ respectively. A linear time-invariant system has the impulse response $h[n]$ defined by these two signals as $h[n] = x[n-1] * y[n]$, where $*$ denotes discrete time convolution. Then the output of the system for the input $\delta[n-1]$

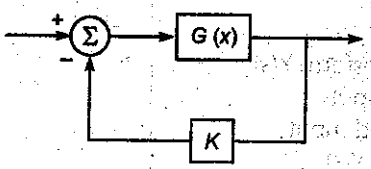
(a) has Z-transform $z^{-1}X(z)\,Y(z)$

(b) equals $\delta[n-2] - 3\delta[n-3] + 2\delta[n-4] - 6\delta[n-5]$

(c) has Z-transform $1 - 3z^{-1} + 2z^{-2} - 6z^{-3}$

(d) does not satisfy any of the above three

37. Consider the discrete time system shown in the figure where the impulse response of $G(z)$ is $g(0) = 0, g(1) = g(2) = 1, g(3) = g(4) = \ldots = 0$.



▼ *Solution Points*

This system is stable for range of values of K

(a) $(-1, 1/2)$ (b) $(-1, 1)$

(c) $(-1/2, 1)$ (d) $(-1/2, 2)$

Statements for Linked Answer Questions 38 and 39

38. A signal is processed by a causal filter with transfer function $G(s)$. For a distortion-free output signal waveform, $G(s)$ must

(a) provide zero phase shift for all frequency

(b) provide constant phase shift for all frequency

(c) provide linear phase shift that is proportional to frequency

(d) provide a phase shift that is inversely proportional to frequency

39. $G(z) = \alpha z^{-1} + \beta z^{-3}$ is a low-pass digital filter with a phase characteristic same as that of the above question if

(a) $\alpha = \beta$ (b) $\alpha = -\beta$ (c) $\alpha = \beta^{(1/3)}$ (d) $\alpha = \bar{\beta}^{(1/3)}$

GATE 2006

40. $x[n] = 0; n < -1, n > 0, x[-1] = -1, x[0] = 2$ is the input and $y[n] = 0; n < -1, n > 2, y[-1] = -1 = y[1], y[0] = 3, y[2] = -2$ is the output of a discrete-time LTI system. The system impulse response $h[n]$ will be

(a) $h[n] = 0; n < 0, n > 2, h[0] = 1, h[1] = h[2] = -1$

(b) $h[n] = 0; h < -1, n > 1, h[1] = 1, n[0] = h[1] = 2$

(c) $h[n] = 0; n < 0, n > 3, h[0] = -1, h[1] = 2, h[2] = 1$

(d) $h[n] = 0; n < -2, n > 1, h[-2] = h[1] = h[-1] = -h(0) = 3$

41. The discrete-time signal

$$x[n] \leftrightarrow X(z) = \Sigma_{n=0}^{\infty} \frac{3^n}{2+n} z^{2n},$$

where $\leftrightarrow$ denotes a transform pair relationship, is orthogonal to the signal

(a) $y_1[n] \leftrightarrow Y_1(z) = \Sigma_{n=0}^{\infty} \left(\frac{2}{3}\right)^n z^{-n}$

(b) $y_2[n] \leftrightarrow Y_2(z) = \Sigma_{n=0}^{\infty} (5^n - n) z^{-(2n+1)}$

(c) $y_3[n] \leftrightarrow Y_3(z) = \Sigma_{n=-\infty}^{\infty} 2^{-|n|} z^{-n}$

(d) $y_4[n] \leftrightarrow Y_4(z) = 2z^{-4} + 3z^{-2} + 1$

42. A continuous-time system is described by $y(t) = e^{-|x(t)|}$, where $y(t)$ is the output and $x(t)$ is the input. $y(t)$ is bounded

(a) only when $x(t)$ is bounded

(b) only when $x(t)$ is non-negative

(c) only for $t \geq 0$ if $x(t)$ is bounded for $t \geq 0$

(d) even when $x(t)$ is not bounded

43. The running integrator, given by

$$y(t) = \int_0^t \pi R^2$$

(a) has no finite singularities in its double sided Laplace transform $Y(s)$

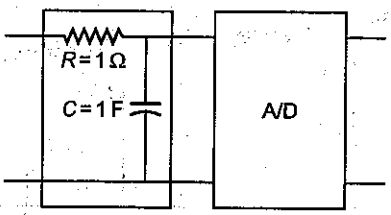
(b) produces a bounded output for every causal bounded input

(c) produces a bounded output for every anticausal bounded input

(d) has no finite zeros in its double sided Laplace transform $Y(s)$

▼ *Solution Points*

Statements for Linked Answer Questions 44 and 45
It is required to design an anti-aliasing filter for an 8 bit *ADC*. The filter is a first order *RC* filter with $R = 1\Omega$ and $C = 1F$. The *ADC* is designed to span a sinusoidal signal with peak to peak amplitude equal to the full scale range of the *ADC*.



44. The transfer function of the filter and its roll-off respectively are
(a) $1/(1+RCs)$, – 20 dB/decade
(b) $(1+RCs)$, – 40 dB/decade
(c) $1/(1+RCs)$, – 40 dB/decade
(d) $[RCs/(1+RCs)]$, – 20 dB/decade

45. What is the SNR (in dB) of the ADC? Also find the frequency (in decade) at the filter output at which the filter attenuation just exceeds the SNR of the ADC.
(a) 50 dB, 2 decade
(b) 50 dB, 2.5 decade
(c) 60 dB, 2 decade
(d) 60 dB, 2.5 decade

46. The Laplace transform of a function $f(t)$ is $F(s) = \dfrac{5s^2 + 23s + 6}{s(s^2 + 2s + 2)}$. As $t \to \infty$, $f(t)$ approaches
(a) 3
(b) 5
(c) $\dfrac{17}{2}$
(d) ∞

47. The Fourier series for the function $f(x) = \sin^2 x$ is
(a) $\sin x + \cos x$
(b) $1 - \cos 2x$
(c) $\sin 2x + \cos 2x$
(d) $0.5 - 0.5\cos 2x$

48. If $u(t)$ is the unit step and $\delta(t)$ is the unit impulse function, the inverse z-transform of $f(z) = \dfrac{1}{z+1}$ for $k > 0$ is
(a) $(-1)^k \delta(k)$
(b) $\delta(k) - (-1)^k$
(c) $(-1)^k u(k)$
(d) $u(k) - (-1)^k$

49. Given the relationship between the input $u(t)$ and the output $y(t)$ to be

$$y(t) = \int_0^t (2 + t - \tau)\, e^{-3(t-\tau)}\, u(\tau)\, d\tau,$$

The transfer function $Y(s)/U(s)$ is
(a) $\dfrac{2e^{-2s}}{s+3}$
(b) $\dfrac{s+2}{(s+3)^2}$
(c) $\dfrac{2s+5}{s-3}$
(d) $\dfrac{2s+7}{(s+3)^2}$

Answers with Explanations

1. (a) $x(t) = e^{-t}$

$y(t) = e^{-2t}$

$z(t) = x(t) * y(t)$

or $z(s) = x(s) \cdot y(s) = \frac{1}{s+1} \cdot \frac{1}{s+2}$

$$z(t) = L^{-1}\{z(s)\} = L^{-1}\left\{\frac{1}{s+1} - \frac{1}{s+2}\right\}$$

$$z(t) = e^{-t} - e^{-2t}$$

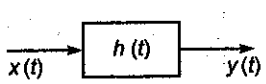
2. (a) For the periodic waveform all harmonics have the same amplitude and it is equal to the amplitude of waveform.

3. (b) Given, $Y(t) = \int_{-\infty}^{\infty} x(\tau)\, d\tau$

The system is linear because the principles of homogeneity and superposition hold for integration operator.

The system is not causal because the output depends upon the future value.

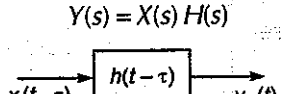
4. (d)



$$y(t) = x(t) * h(t)$$

Taking Fourier transform,

$$Y(s) = X(s)\, H(s)$$



$$y_1(t) = x(t-\tau) * h(t-\tau)$$

Taking Fourier transform,

$$Y_1(s) = X(s)\, e^{-s\tau} \cdot H(s)\, e^{-s\tau}$$

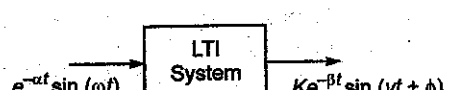
$$Y_1(s) = X(s) \cdot H(s)\, e^{-2\tau s}$$

$$= Y(s)\, e^{-2\tau s}$$

By inverse transform,

$$y_1(t) = y(t - 2\tau)$$

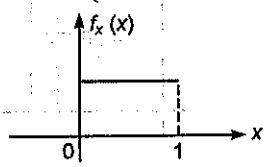
5. (a)



As the signal passes through a LTI system. The output has the same frequency components. Hence, ω is equal to ν. α and β are attenuation constants and it need not to be equal as system may change its value.

6. (c) Given, X is uniformly distributed between 0 and 1.

Hence, $f_x(x) = \begin{cases} 1, & 0 < x < 1 \\ 0, & \text{otherwise} \end{cases}$



$$E(x) = \int_{-\infty}^{\infty} x f_x(x)\, dx$$

$$E(x^3) = \int_{-\infty}^{\infty} x^3 f_x(x)\, dx = \int_0^1 x^3 dx = \left[\frac{x^4}{4}\right]_0^1 = \frac{1}{4}$$

7. (d) Given, differential equation

$$\frac{dy(t)}{dt} + y(t) = \delta(t)$$

Taking Laplace transform,

$$SY(s) + Y(s) = 1$$

$$Y(s) = 1/S + 1$$

Taking inverse Laplace,

$$y(t) = e^{-t} u(t)$$

8. (d) For linear system the principles of superposition and homogeneity must follow.

$$x_1(t) \rightarrow y_1(t); \quad x_2(t) \rightarrow y_2(t)$$

Then $a_1 x_1(t) + a_2 x_2(t) \rightarrow a_1 y_1(t) + a_2 y_2(t)$

A system is causal if and only if its impulse response is zero for t less than zero.

$$h(t) = 0 \text{ for } t < 0.$$

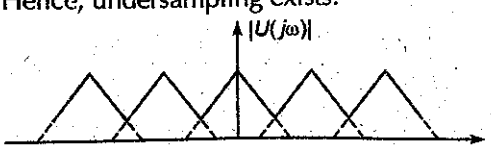
9. (d) If a sinusoidal signal is applied to a linear time invariant (LTI) system. The output is also sinusoidal with same frequency, may be with attenuation and different phase shift.

10. (d) The bandwidth $B = 1$ kHz

Nyquist sampling period

$$= \frac{1}{2B} = \frac{1}{2 \times 1 \times 10^{-3}} = 0.5 \text{ ms}$$

Sampling rate = 1ms > Nyquist sample rate;
Hence, undersampling exists.



11. (b) Energy of signal $= \int_{-\infty}^{\infty} |f(t)|^2\, dt$

If the signal is bounded, then the integration has finite value hence energy of signal is finite.

12. (d) The trigonometric Fourier series expansion does not contain term of frequency $\omega = 2\pi(2K)/T$ where $K = 1, 2, 3, \ldots$ mean that the signal is half-wave symmetrical *i.e.*,

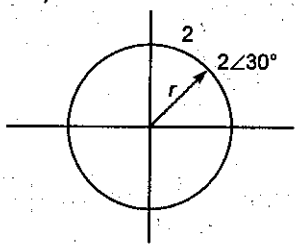
$$x(t) = x(t - T/2)$$

The series contains no sine terms mean that the signal exhibits even symmetry.

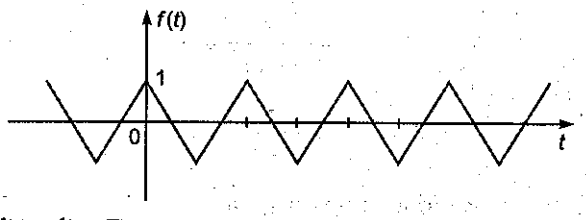
$$x(t) = x(t - T)$$

$$x(t) = x(t - T) = x(t - T/2)$$

13. (c) For causal system, if all the poles r inside the unit circle the system is stable and inverse is true for anticausal system.



14.(c)



$f(t) = f(t - T) \Rightarrow$ even symmetry.

The signal does not contain series expansion sine term.

15. (b) The time derivative of step response is impulse response of the system.

$$h(t) = \frac{d}{dt} s(t)$$

The output $y(t) = h(t) * u(t)$

$$= \int h(t - \tau)\, u(\tau)\, d\tau$$

$$= \int \frac{d}{dt} s(t - \tau)\, u(\tau)\, d\tau$$

$$y(t) = \frac{d}{dt} \int s(t - \tau)\, u(\tau)\, d\tau$$

16. (c) According to final value theorem,

$$\lim_{t \to \infty} y(t) = \lim_{s \to 0} sY(s)$$

17. (c) As we know,

Transfer function = L (impulse function)

$$\therefore \quad \text{TF} = \frac{1}{s+1} + \frac{1}{s+2}$$

$$\text{Step response} = L^{-1}\left[\frac{1}{s(s+1)} + \frac{1}{s(s+2)}\right]$$

$$= L^{-1}\left[\left(\frac{1}{s} - \frac{1}{s+1}\right) + \left(\frac{0.5}{s} - \frac{0.5}{s+2}\right)\right]$$

$$= (1 - e^{-t} + 0.5 - 0.5\, e^{-2t})\, U(t)$$

$$= (1.5 - e^{-t} - 0.5\, e^{-2t})\, U(t))$$

18. (b) $F_2(s) = F(t - \tau)$

$$F_2(t) = L^{-1}\,[F(t - \tau)] = e^{-st} F_1(s)$$

$$\because \quad G(s) = \frac{F_2(s) \cdot F_1^*(s)}{|F_1(s)|^2} = e^{-st}$$

$$G(t) = L^{-1}\,[G(s)] = \delta(t - \tau)$$

19. (d) Using Parseval's theorem,

$$\int_{-\infty}^{\infty} x^2(t)\, dt = \frac{1}{2\pi} \int_{-\infty}^{\infty} |X(\omega)|^2\, d\omega$$

$$\int_{-\infty}^{\infty} |X(\omega)|^2\, d\omega = 2\pi \int_{-\infty}^{\infty} x^2(t)\, dt$$

$$= 2\pi \int_{-1}^{1} 1 \cdot dt = 2\pi \times 2 = 4\pi$$

20. (c) $Y(z) = 1 - z^{-4}$

$$X(z) = 1 - z^{-1}$$

$$H(z) = \frac{Y(z)}{X(z)} = \frac{(z^4 - 1)\, z}{z^4 (z - 1)} = \frac{(z^4 - 1)}{z^3 (z - 1)}$$

$$\frac{(z+1)(z^2+1)}{z^3} = \frac{z^3 + z + z^2 + 1}{z^3}$$

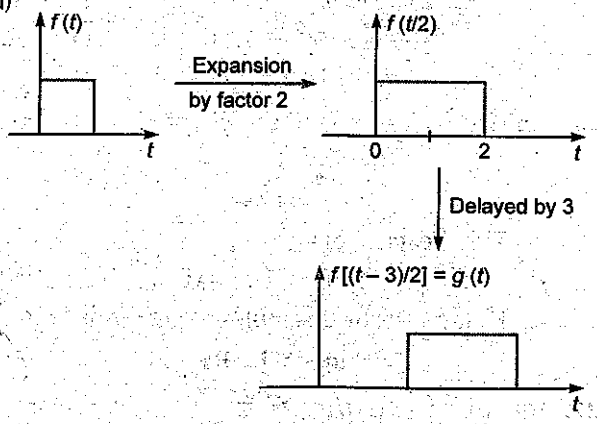
$$\Rightarrow \quad 1 + z^{-2} + z^{-1} + z^{-3}$$

$$= 1 + z^{-1} + z^{-2} + z^{-3}$$

$$= [1, 1, 1, 1]$$

$$\uparrow$$

21. (d)



22. (c) $g(t) = u(t - 3) - u(t - 5)$

$$L\{g(t)\} = L\{u(t - 3) - u(t - 5)\}$$

$$= \frac{1}{s} e^{-3s} - \frac{1}{s} e^{-5s}$$
$$= \frac{1}{s}(e^{-3s} - e^{-5s})$$
$$= \frac{e^{-3s}}{s}(1 - e^{-2s})$$

23. (a) A cascade of 3 linear systems is causal and unstable. If each system in the cascade is individually causal and unstable.

24. (a) $$x(t) = \sum_{k=-\infty}^{\infty} a_k e^{j2\pi kt/T}$$
$$= \sum_{k=-\infty}^{\infty} a_k e^{j2\pi kt/T}$$

Putting the value,

$$x(t) = (2 - j1)\, e^{-j\frac{4\pi t}{T}} + (0.5 + j0.2)\, e^{-j2\pi t/T}$$
$$+ (j2)\, e^{0} + (0.5 - j0.2)\, e^{j2\pi t/T} + (2 + j1)\, e^{j\frac{4\pi t}{T}}$$
$$= (2 - j1)\left[\cos\frac{4\pi t}{T} - j\sin\frac{4\pi t}{T}\right]$$
$$+ (0.5 + j0.2)\left[\cos\frac{2\pi t}{T} - j\sin\frac{2\pi t}{T}\right]$$
$$+ j2 + (2 + j1)\left[\cos\frac{4\pi t}{T} + j\sin\frac{4\pi}{t}\right]$$

Simplifying and separating real and imaginary parts,

$$x(t) = \left[4\cos\frac{4\pi t}{T} - 2\sin\frac{4\pi t}{T} + \cos\frac{2\pi t}{T} + 0.4\sin\frac{2\pi t}{T} + j2\right]$$

Imaginary part of $X(t)$ is constant.

It has finite energy since all coefficients are non-zero.

25. (a) $y(n) = x(n) * h(n)$

Taking z-transform,

$$Y(z) = X(z)\, H(z)$$
$$= (4z^{-3} + 3z^{-1} + 2 - 6z^2 + 2z^3)(3z^{-1} - 2)$$
$$= -4z^3 + 18z^2 - 18z + 12z^{-4} + 9z^{-2} - 4$$

26. (d) First pick every 4th sample of $x[n]$ to generate $v_1[n]$ *i.e.*,

$$v_1[n] = x[4n]$$

Time reversal of $v_1[n]$

$$v_2[n] = v_1[-n] = x[-4n]$$

Delay $v_2[n]$ by 3 samples to get $y[n]$

$$y[n] = x[3 - 4n]$$

27. (d) $y(t) = \int_{-\infty}^{2t} x(\tau)\, d\tau$

- is non-causal system because output value depends upon future value.
- is unstable system because for bound input $x(\tau)$. the integration is not bounded and hence, output is not bounded.
- is time-variant due to term t.

28. (a)

29. (b) Given, $x(t)$ is the periodic function with period T.

$$\omega_0 = 2\pi/T$$
$$y(t) = x(t - t_0) + x(t + t_0)$$

Fourier series coefficient $b_k = 2\left(\frac{e^{-jk\omega_0 t_0} + e^{jk\omega_0 t_0}}{2}\right)$

$$b_k = 2\cos k\,\omega_0 t_0 = 0 \text{ for all odd } k$$
$$\omega_0 t_0 = \frac{\pi}{2}$$
$$\frac{2\pi}{T} t_0 = \frac{\pi}{2}$$
$$t_0 = \frac{T}{4}$$

30. (a)

31. (d) Given, $X(z) = \frac{z}{(z-a)^2}$ with $R \in |z| > |a|$

Let $f(z) = X(z)\, z^{n-1}$

$$= \frac{z}{(z-a)^2} z^{n-1}$$
$$= \frac{z^n}{(z-a)^2}$$

Residue of $f(z)$ at $z = a$

$$= \frac{1}{1!}\frac{d}{dz}(z-a)^2 f(z)$$
$$= \frac{d}{dz} z^n$$
$$= nz^{n-1}$$

For $z = a$, $F(a) = na^{n-1}$

32. (c) Given, $x(t) = \text{rect}\left(t - \frac{1}{2}\right)$

i.e., $x(t) = 1,\ 0 \le t \le 1$
$= 0,$ otherwise

Taking Fourier transform

$$X(\omega) = \int_{-\infty}^{\infty} X(t)\, e^{-j\omega t}\, dt$$
$$= \int_0^1 1 \cdot e^{-j\omega t}\, dt$$
$$= \frac{1}{j\omega}(1 - e^{-j\omega})$$

$$= \frac{1}{j\omega}\left(\frac{e^{j\omega/2} - e^{-j\omega/2}}{e^{j\omega/2}}\right)$$

$$= \frac{2}{\omega}\left[\frac{e^{j\omega/2} - e^{-j\omega/2}}{2j}\right] e^{-j\omega/2}$$

$$X(\omega) = \frac{\sin\frac{\omega}{2}}{\frac{\omega}{2}} e^{-j\omega/2}$$

From FT property, we know that if

$$x(t) \xleftrightarrow{FT} X(\omega)$$

$$x(-t) \xleftrightarrow{FT} X(-\omega)$$

$$X(-\omega) = \frac{\sin\left(\frac{-\omega}{2}\right)}{\left(-\frac{\omega}{2}\right)} e^{-j(-\omega/2)}$$

$$= \frac{\sin \omega/2}{\omega/2} e^{j\omega/2}$$

$$x(t) + x(-t) \xleftrightarrow{FT} X(\omega) + X(-\omega)$$

$$X(\omega) + X(-\omega) = \frac{\sin \omega/2}{\omega/2} e^{-j\omega/2} \cdot e + \frac{\sin \omega/2}{\omega/2} e^{j\omega/2}$$

$$= \frac{\sin \omega/2}{\omega/2}[e^{-j\omega/2} + e^{j\omega/2}]$$

$$= \frac{\sin \omega/2}{\omega/2}(2\cos \omega/2)$$

$$= 2\sin c(\omega/2\pi)\cos \omega/2$$

33. (a) The output

$$Y(s) = G(s)\,R(s)$$

$$= \frac{1}{s^2 + 3s + 2} R(s)$$

Given, $r(t) = s(t-1)$

$$R(s) = e^{-s}$$

$$Y(s) = \frac{e^{-s}}{s^2 + 3s + 2}$$

Steady state value

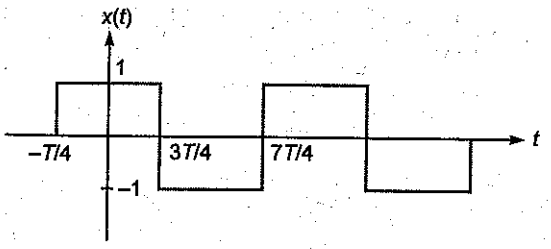
$$\lim_{t \to \infty} y(t) = \lim_{s \to 0} sY(s)$$

$$= \lim_{s \to 0} \frac{se^{-s}}{s^2 + 3s + 2} = 0$$

34. (c) By property of symmetry of Fourier transform of $x(t)$,

$$x(t) = \frac{4}{\pi}\sin\left(\frac{\pi t}{T} - \frac{\pi}{4}\right)$$

Alternate Method



35. (c) LT of $u(t) = \frac{1}{s}$ and $r(t) = \frac{1}{s^2}$

Hence, $L[u(t+1)] = \frac{e^s}{s}$

$$L[r(t-2)] = \frac{1}{s^2} e^{-2s}$$

$$L[u(t+1) * r(t-2)] = L[u(t+1)] \times L[r(t-2)]$$

$$= \frac{1}{s} e^s \times \frac{1}{s^2} e^{-2s}$$

$$= \frac{1}{s^3} e^{-s}$$

$$u(t-1) * r(t-2) = L^{-1}\left[\frac{1}{s^3} e^{-s}\right]$$

$$= \frac{1}{2}(t-1)^2\, u(t-1)$$

36. (d)

37. (b) $G(z) = z^{-1} + z^{-2}$

The characteristics equation is

$$1 + KG(z) = 0$$

$$1 + K(z^{-1} + z^{-2}) = 0$$

$$1 + \frac{K}{z} + \frac{K}{z^2} = 0$$

$$z^2 + zK + K = 0$$

For system to be stable,

$$|K| < 1$$

$$-1 < K < 1$$

38. (c) For distortion-free output filter should provide linear phase shift that is proportional to frequency.

39. (a)

40. (a) For finite duration convolution,

$x[n]$ have M terms,

$h[n]$ have N terms,

Then, $y[n]$ should have terms $(M + N - 1)$.

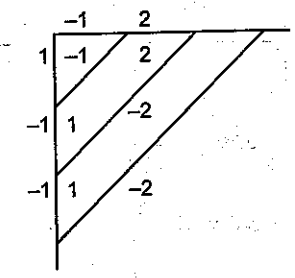
Here, $x[n] = \{-1, \underset{\uparrow}{2}\}$

$y[n] = \{-1, 3, -1, -2\}$

So, $h[n]$ should have only 3 terms and $h[n]$ have value starting from origin or $[n=0]$ only because $y[n]$ have value starting from $[n=-1]$.

$\therefore$ Consider option (a),

$$y[n]=\{-1,3,-1,-2\}$$



41. (a)

42. (d) Given that, $y(t)=e^{-|x(t)|}$

i.e., $y(t)=e^{-x(t)}$ for $x(t)>0$

$=e^{x(t)}$ for $x(t)<0$

$y(t)$ is convergent ($y(t)\to 0$ as $x(t)\to\infty$) even if $x(t)$ is not bounded.

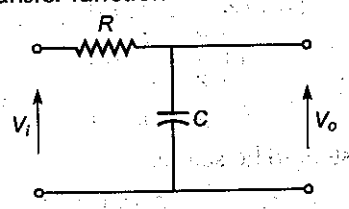
For example, $x(t)=t$ is not bounded but $e^{-|t|}$ is bounded signal.

43. (b) The running integrator

$$y(t)=\int_{-\infty}^{t} x(t')\,dt'$$

produces bounded output for every causal bounded input.

44. (a) The transfer function of filter



$$T(s)=\frac{1/sC}{R+1/sC}=\frac{1}{1+sRC}$$

i.e., There exists simple pole at $s=-1/RC$

Hence, roll-off factor is –20 dB/decade.

45. (a)

46. (a) Given, $F(s)=\dfrac{5s^2+23s+6}{s(s^2+2s+2)}$

From final value theorem,

$$\lim_{t\to\infty} f(t)=\lim_{s\to 0} sF(s)$$

$$=\lim_{s\to 0} s\times\frac{5s^2+23s+6}{s(s^2+2s+2)}$$

$$=\frac{6}{2}=3$$

47. (d) $f(x)=\sin^2 x$

Expanding in frequency component,

$$f(x)=\frac{1-\cos 2x}{2}$$

$$=0.5-0.5\cos 2x$$

48. (b) $F(z)=\dfrac{1}{z+1}$

$$=\frac{z+1-z}{z+1}$$

$$F(z)=1-\frac{z}{z+1}$$

Taking inverse z-transform,

$$f(k)=\delta(k)-(-1)^k$$

49. (d) We can write

$$y(t)=\left[\frac{2}{3}(1-e^{-3t})-\frac{t}{3}e^{-3t}+\frac{1}{9}(1-e^{-3t})\right]u(t)$$

Taking LT,

$$Y(s)=\left[\frac{2}{3}\left(\frac{1}{s}-\frac{1}{s+3}\right)\right]-\frac{1}{3(s+3)^2}+\frac{1}{9}\left(\frac{1}{s}-\frac{1}{s+3}\right)u(s)$$

$$\frac{Y(s)}{U(s)}=\frac{2s+7}{(s+3)^2}$$

❐

4

Electrical Machines

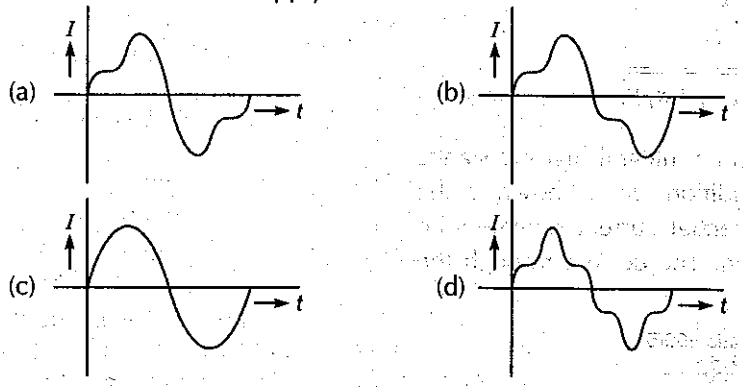
Syllabus

Electrical Machines *Single-phase transformer—equivalent circuit, phasor diagram, tests, regulation and efficiency. Three-phase transformer—connections, parallel operation. Auto-transformer—energy conversion principles. DC machines—types, windings, generator characteristics, armature reaction and commutation, starting and speed control of motors. Three phase induction motors—principles, types, performance characteristics, starting and speed control. Single-phase induction motors. Synchronous machines—performance, regulation and parallel operation of generators, motor starting, characteristics and applications. Servo and stepper motors.*

GATE 2011 *(1 Mark Questions)*

▼ *Solution Points*

1. A single-phase air core transformer, fed from a rated sinusoidal supply is operating at no load. The steady state magnetizing current drawn by the transformer from the supply will have the wavèform



2. A negative sequence relay is commonly used to protect
(a) an alternator (b) a transformer
(c) a transmission line (d) a bus bar

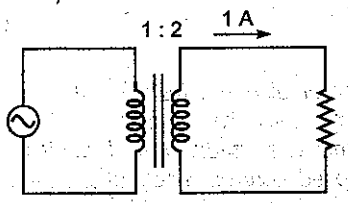
3. A 4-point starter is used to start and control the speed of a
(a) DC shunt motor with armature resistance control
(b) DC shunt motor with field weakening control
(c) DC series motor
(d) DC compound motor

▼ *Solution Points*

4. A three-phase, salient pole synchronous motor is connected to an infinite bus. It is operated at no load at normal excitation. The field excitation of the motor is first reduced to zero and then increased in the reverse direction gradually. Then the armature current
   (a) increases continuously
   (b) first increases and then decreases steeply
   (c) first decreases and then increases steeply
   (d) remains constant

GATE 2010

5. A single-phase transformer has a turns ratio of 1 : 2 and is connected to a purely resistive load as shown in the figure. The magnetizing current drawn is 1 A and the secondary current is 1 A. If core losses and leakage reactances are neglected, the primary current is



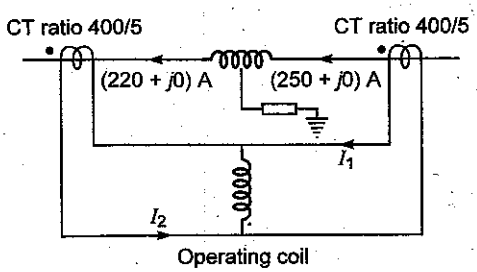
(a) 1.41 A (b) 2 A (c) 2.24 A (d) 3 A

6. A balanced three-phase voltage is applied to a star-connected induction motor, the phase to neutral voltage being V. The stator resistance, rotor resistance referred to the stator, stator leakage reactance, rotor leakage reactance referred to the stator, and the magnetizing reactance are denoted by r_s, r_r, x_s, x_r and x_m respectively. The magnitude of the starting current of the motor is given by

(a) $\dfrac{V}{\sqrt{(r_s + r_r)^2 + (x_s + x_r)^2}}$ (b) $\dfrac{V}{\sqrt{r_s^2 + (x_s + x_m)^2}}$

(c) $\dfrac{V}{\sqrt{(r_s + r_r)^2 + (x_m + x_r)^2}}$ (d) $\dfrac{V}{\sqrt{r_s^2 + (x_m + x_r)^2}}$

7. Consider a stator winding of an alternator with an integral high-resistance ground fault. The currents under the fault condition are as shown in the figure. The winding is protected using a differential current scheme with current transformers of ratio 400/5 A as shown. The current through the operating coil is



(a) 0.17875 A (b) 0.2 A (c) 0.375 A (d) 60 kA

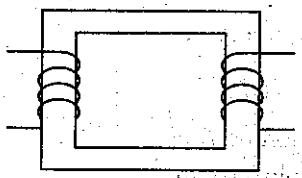
GATE 2009

▼ *Solution Points*

8. A field excitation of 20 A in a certain alternator results in an armature current of 400 A in short circuit and a terminal voltage of 2000 V on open circuit. The magnitude of the internal voltage drop within the machine at a load current of 200 A is

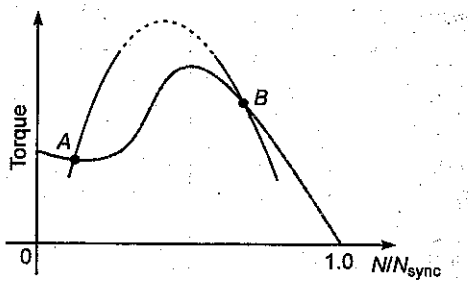
(a) 1 V (b) 10 V (c) 100 V (d) 1000 V

9. The single-phase, 50 Hz, iron core transformer in the circuit has both the vertical arms of cross-sectional area 20 cm^2 and both the horizontal arms of cross-sectional area 10 cm^2. If the two windings shown were wound instead on opposite horizontal arms, the mutual inductance will



(a) double (b) remain same
(c) be halved (d) become one quarter

10. A 3-phase squirrel cage induction motor supplied from a balanced 3-phase source drives a mechanical load. The torque-speed characteristics of the motor (solid curve) and of the load (dotted curve) are shown. Of the two equilibrium points *A* and *B*, which of the following options correctly describes the stabilities of *A* and *B*?

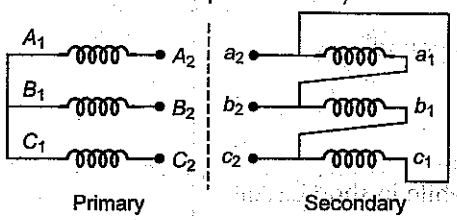


(a) *A* is stable, *B* is unstable (b) *A* is unstable, *B* is stable
(c) Both are stable (d) Both are unstable

11. Distributed winding and short chording winding employed in AC machines will result in

(a) increase in emf and reduction in harmonics
(b) reduction in emf and increase in harmonics
(c) increase in both emf and harmonics
(d) reduction in both emf and harmonics

12. Three single-phase transformers are connected to form a 3-phase transformer bank. The transformers are connected in the following manner. The transformer connection will be represented by



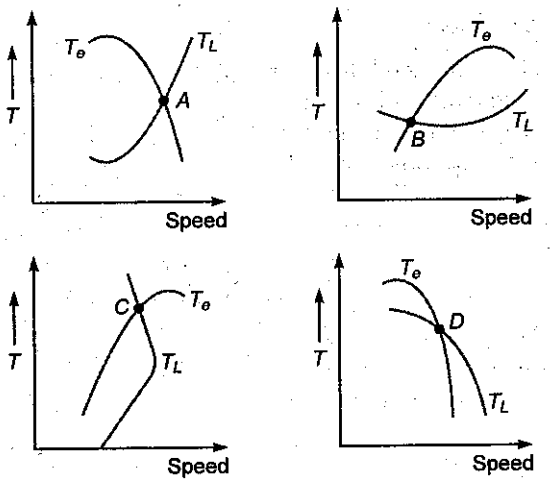
(a) Yd_0 (b) Yd_1 (c) Yd_6 (d) Yd_{11}

▼ *Solution Points*

13. In a stepper motor, the detent torque means
(a) minimum of the static torque with the phase winding excited
(b) maximum of the static torque with the phase winding excited
(c) minimum of the static torque with the phase winding unexcited
(d) maximum of the static torque with the phase winding unexcited

14. It is desired to measure parameters of 230 V/115 V, 2 kVA, single-phase transformer. The following wattmeters are available in a laboratory
W_1 : 250 V, 10 A, low power factor
W_2 : 250 V, 5 A, low power factor
W_3 : 150 V, 10 A, high power factor
W_4 : 150 V, 5 A, high power factor
The wattmeters used in open circuit test and short circuit test of the transformer will respectively be
(a) W_1 and W_2 (b) W_2 and W_4 (c) W_1 and W_4 (d) W_2 and W_3

15. The electromagnetic torque T_e of a drive and its connected load torque T_L are as shown below. Out of the operating points A, B, C and D, the stable ones are



(a) A, C, D (b) B, C (c) A, D (d) B, C, D

GATE 2007

16. In a transformer, zero voltage regulation at full load is
(a) not possible
(b) possible at unity power factor load
(c) possible at leading power factor load
(d) possible at lagging power factor load

17. The DC motor, which can provide zero speed regulation at full load without any controller, is
(a) series (b) shunt
(c) cumulative compound (d) differential compound

GATE 2006

18. In transformers, which of the following statements is valid?
(a) In an open circuit test, copper losses are obtained while in short circuit test, core losses are obtained
(b) In an open circuit test, current is drawn at high power factor

▼ Solution Points

(c) In a short circuit test, current is drawn at zero power factor
(d) In an open circuit test, current is drawn at low power factor

19. For a single-phase capacitor start induction motor, which, of the following statements is valid?
(a) The capacitor is used for power factor improvement
(b) The direction of rotation can be changed by reversing the main winding terminals
(c) The direction of rotation cannot be changed
(d) The direction of rotation can be changed by interchanging the supply terminals

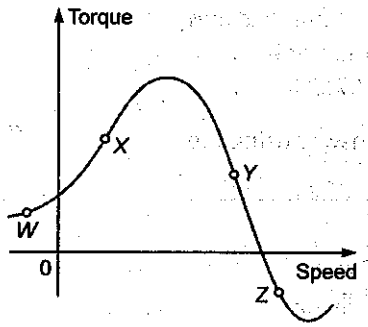
20. In a DC machine, which of the following statements is true?
(a) Compensating winding is used for neutralizing armature reaction while interpole winding is used for producing residual flux
(b) Compensating winding is used for neutralizing armature reaction while interpole winding is used for improving commutation
(c) Compensating winding is used for improving commutation while interpole winding is used for neutralizing armature reaction
(d) Compensating winding is used for improving commutation while interpole winding is used for producing residual flux

GATE 2005

21. The equivalent circuit of a transformer has leakage reactances X_1, X_2' and magnetizing reactance X_M. Their magnitudes satisfy
(a) $X_1 >> X_2' >> X_M$ (b) $X_1 << X_2' << X_M$
(c) $X_1 \approx X_2' >> X_M$ (d) $X_1 \approx X_2' << X_M$

22. Which three-phase connection can be used in a transformer to introduce a phase difference of 30° between its output and corresponding input time voltages?
(a) Star-Star (b) Star-Delta
(c) Delta-Delta (d) Delta-Zigzag

23. On the torque/speed curve of induction motor shown in the figure four points of operation are marked as W, X, Y and Z. Which one of them represents the operation at a slip greater than 1?



(a) W (b) X (c) Y (d) Z

24. For an induction motor, operating at a slip s, the ratio of gross power output to air gap power is equal to
(a) $(1-s)^2$ (b) $(1-s)$ (c) $\sqrt{(1-s)}$ (d) $(1-\sqrt{s})$

▼ *Solution Points*

25. For a linear electromagnetic circuit, which of the following statements is true?
(a) Field energy is equal to the co-energy
(b) Field energy is greater than co-energy
(c) Field energy is lesser than co-energy
(d) Co-energy is zero

GATE 2004

26. A 500 kVA, 3-phase transformer has iron losses of 300 W and full load copper losses of 600 W. The percentage load at which the transformer is expected to have maximum efficiency is
(a) 50.0% (b) 70.7% (c) 141.4% (d) 200.0%

27. For a given stepper motor, which of the following torque has the highest numerical value?
(a) Detent torque (b) Pull-in torque
(c) Pull-out torque (d) Holding torque

28. Which of the following motors definitely has a permanent magnet rotor?
(a) DC commutator motor (b) Brushless DC motor
(c) Stepper motor (d) Reluctance motor

29. The type of single-phase induction motor having the highest power factor at full load is
(a) shaded pole type (b) split-phase type
(c) capacitor-start type (d) capacitor-run type

30. The direction of rotation of a 3-phase induction motor is clockwise when it is supplied with 3-phase sinusoidal voltage having phase sequence *A-B-C*. For counterclockwise rotation of the motor, the phase sequence of the power supply should be
(a) *B-C-A* (b) *C-A-B*
(c) *A-C-B* (d) *B-C-A* or *C-A-B*

GATE 2003

31. A single-phase transformer has a maximum efficiency of 90% at full load and unity power factor. Efficiency at half load at the same power factor is
(a) 86.7% (b) 88.26% (c) 88.9% (d) 87.8%

32. Match the following List I with List II and select the correct answer using the codes given below

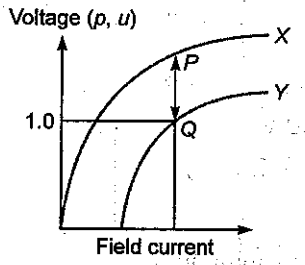
List I	List II
P. Food mixer	1. Permanent magnet DC motor
Q. Cassette tape recorder	2. Single-phase induction motor
R. Domestic water pump	3. Universal motor
S. Escalator	4. Three-phase induction motor
	5. DC series motor
	6. Stepper motor

▼ *Solution Points*

	P	Q	R	S
(a)	3	6	4	5
(b)	1	3	2	4
(c)	3	1	2	4
(d)	3	2	1	4

33. A stand alone engine driven synchronous generator is feeding a partly inductive load. A capacitor is now connected across the load to completely nullify the inductive current. For this operating condition
(a) the field current and fuel input have to be reduced
(b) the field current and fuel input have to be increased
(c) the field current has to be increased and fuel input left unaltered
(d) the field current has to be reduced and fuel input left unaltered

34. Curves *X* and *Y* in figure denote open circuit and full-load zero power factor (ZPF) characteristics of a synchronous generator, *Q* is a point on the ZPF characteristics at 1.0 pu voltage. The vertical distance *PQ* in figure gives the voltage drop across



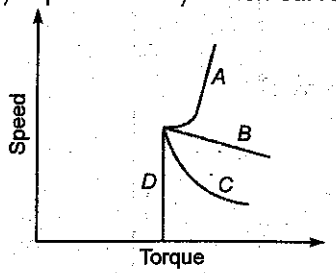
(a) synchronous reactance (b) magnetizing reactance
(c) Potier reactance (d) leakage reactance

35. No-load test on a 3-phase induction motor was conducted at different supply voltages and a plot of input power *versus* voltage was drawn. This curve was extrapolated to intersect the *y*-axis. This intersection point yields
(a) core loss (b) stator copper loss
(c) stray load loss (d) friction and windage loss

GATE 2002

36. If a 400 V, 50 Hz, star connected, 3-phase squirrel cage induction motor is operated from a 400 V, 75 Hz supply, the torque that the motor can now provide while drawing rated current from the supply
(a) reduces
(b) increases
(c) remains the same
(d) increases or reduces depending upon the rotor resistance

37. A DC series motor fed from rated supply voltage is overloaded and its magnetic circuit is saturated. The torque-speed characteristic of this motor will be approximately represented by which curve of figure?

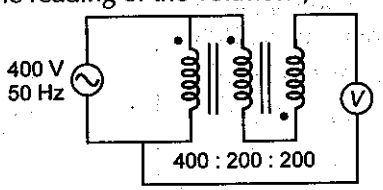


(a) Curve *A* (b) Curve *B* (c) Curve *C* (d) Curve *D*

▼ *Solution Points*

38. A 1 kVA, 230 V/100 V, single-phase, 50 Hz transformer having negligible winding resistance and leakage inductance is operating under saturation, while 250 V, 50 Hz sinusoidal supply is connected to the high voltage winding. A resistive load is connected to the low voltage winding which draws rated current. Which one of the following quantities will not be sinusoidal?
(a) Voltage induced across the low voltage winding
(b) Core flux
(c) Load current
(d) Current drawn from the source

39. A 400 V/200 V/200 V, 50 Hz three winding transformer is connected as shown in figure. The reading of the voltmeter, V will be



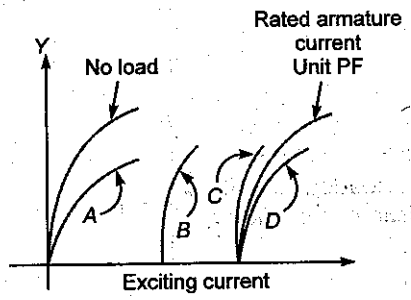
(a) zero (b) 400 V (c) 600 V (d) 800 V

GATE 2001

40. A single-phase transformer is to be switched to the supply to have minimum inrush current. The switch should be closed at
(a) maximum supply voltage (b) zero supply voltage
(c) $\frac{1}{\sqrt{2}}$ maximum supply voltage (d) $\frac{1}{2}$ maximum supply voltage

41. It is desirable to eliminate 5th harmonic voltage from the phase voltage of an alternator. The coils should be short-pitched by an electrical angle of
(a) 30° (b) 36° (c) 72° (d) 18°

42. Figure shows the magnetization curves of an alternator at rated armature current, unity power factor and also at no load. The magnetization curve for rated armature current, 0.8 power factor leading is given by



(a) curve A (b) curve B (c) curve C (d) curve D

43. The core flux of a practical transformer with a resistive load
(a) is strictly constant with load changes
(b) increases linearly with load
(c) increases as the square root of the load
(d) decreases with increased load

▼ *Solution Points*

44. X_d, X'_d and X''_d are steady state d-axis synchronous reactance, transient d-axis reactance and sub-transient d-axis reactance of a synchronous machine respectively. Which of the following statements is true?

(a) $X_d > X'_d > X''_d$ (b) $X''_d > X'_d > X_d$ (c) $X'_d > X''_d > X_d$ (d) $X_d > X''_d > X'_d$

GATE 2000

45. A 3-phase, 4-pole squirrel cage induction motor has 36 stator and 28 rotor slots. The number of phases in the rotor is

(a) 3 (b) 9 (c) 7 (d) 8

46. The compensating winding in a DC machine

(a) is located in armature slots for compensation of the armature reaction
(b) is located on commutating poles for improving the commutation
(c) is located on pole shoes for avoiding the flashover at the commutator surface
(d) is located on pole shoes to avoid the sparking at the brushes

47. In a Constant Voltage Transformer (CVT), the output voltage remains constant due to

(a) capacitor (b) input inductor
(c) saturation (d) tapped windings

48. The phase sequence of a three-phase alternator will reverse if

(a) the field current is reversed keeping the direction of rotation same
(b) the field current remains the same but the direction of rotation is reversed
(c) the field current is reversed and the number of poles is doubled
(d) the number of poles is doubled without reversing the field current

49. A 1.8° step, 4-phase stepper motor has a total of 40 teeth on 8 poles of stator. The number of rotor teeth for this motor will be

(a) 40 (b) 50 (c) 100 (d) 80

GATE 2011 *(2 Marks Questions)*

50. A three-phase 440 V, 6 pole, 50 Hz, squirrel cage induction motor is running at a slip of 5%. The speed of stator magnetic field with respect to rotor magnetic field and speed of rotor with respect to stator magnetic field are

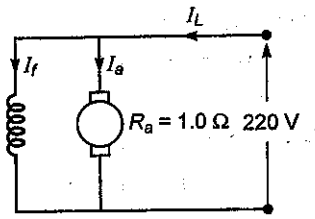
(a) zero, – 5 rpm (b) zero, 955 rpm
(c) 1000 rpm, – 5 rpm (d) 1000 rpm, 955 rpm

51. The direct axis and quadrature axis reactances of a salient pole alternator are 1.2 pu and 1.0 pu respectively. The armature resistance is negligible. If this alternator is delivering rated kVA at UPF and at rated voltage then its power angle is

(a) 30° (b) 45° (c) 60° (d) 90°

52. A 220 V, DC shunt motor is operating at a speed of 1440 rpm. The armature resistance is 1.0 Ω and armature current is 10 A. If the excitation of the machine is reduced by 10%, the extra resistance to be put in the armature circuit to maintain the same speed and torque will be

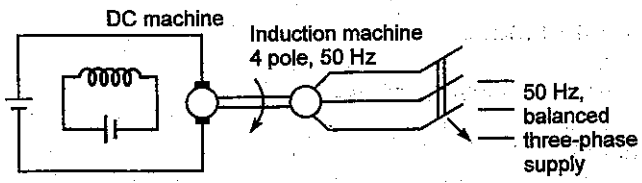
▼ *Solution Points*



(a) 1.79 Ω (b) 2.1 Ω (c) 3.1 Ω (d) 18.9 Ω

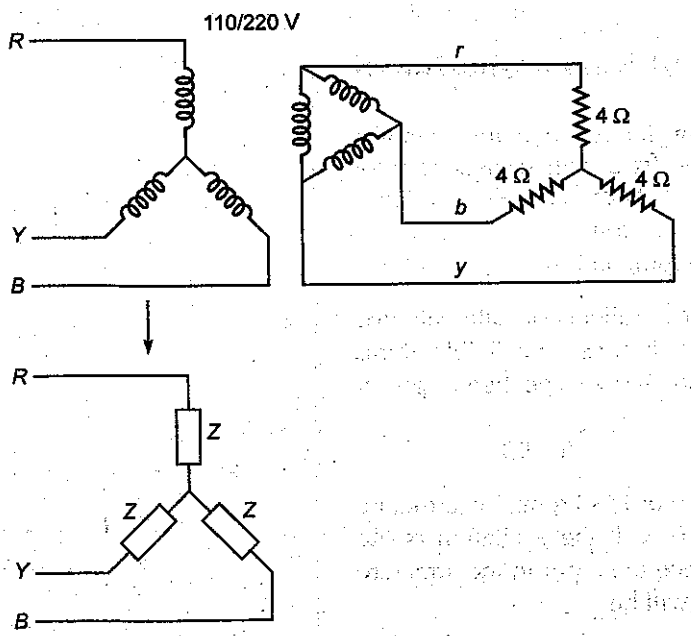
GATE 2010

53. A separately excited DC machine is coupled to a 50 Hz, three-phase, 4-pole induction machine as shown in the figure. The DC machine is energized first and the machines rotate at 1600 rpm. Subsequently the induction machine is also connected to a 50 Hz, three-phase source, the phase sequence being consistent with the direction of rotation. In steady state,



(a) both machines act as generators
(b) the DC machine acts as a generator and the induction machine acts as a motor
(c) the DC machine acts as a motor and the induction machine acts as a generator
(d) both machines act as motors

54. A balanced star-connected and purely resistive load is connected at the secondary of a star-delta transformer as shown in the figure. The line-to-line voltage rating of the transformer is 110V/220V. Neglecting the non-idealities of the transformer, the impedance Z of the equivalent star-connected load, referred to the primary side of the transformer, is

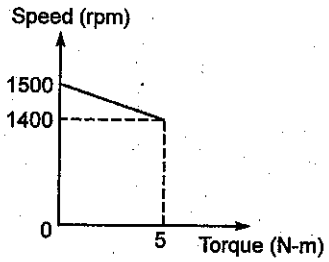


▼ *Solution Points*

(a) $(3 + j0)\,\Omega$ (b) $(0.866 - j0.5)\,\Omega$
(c) $(0.866 + j0.5)\,\Omega$ (d) $(1 + j0)\,\Omega$

Common Data for Questions 55 and 56

A separately excited DC motor runs at 1500 rpm under no-load with 200 V applied to the armature. The field voltage is maintained at its rated value. The speed of the motor, when it delivers a torque of 5 N-m, is 1400 rpm as shown in the figure. The rotational losses and armature reaction are neglected.

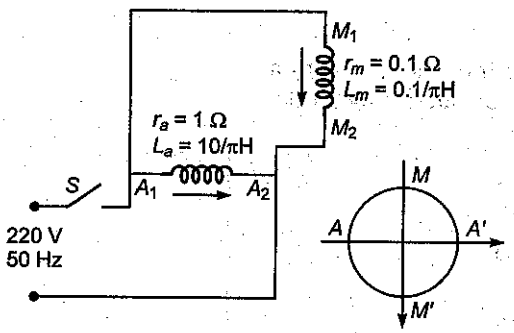


55. The armature resistance of the motor is
(a) 2 Ω (b) 3.4 Ω (c) 4.4 Ω (d) 7.7 Ω

56. For the motor to deliver a torque of 2.5 N-m at 1400 rpm, the armature voltage to be applied is
(a) 125.5 V (b) 193.3 V (c) 200 V (d) 241.7 V

GATE 2009

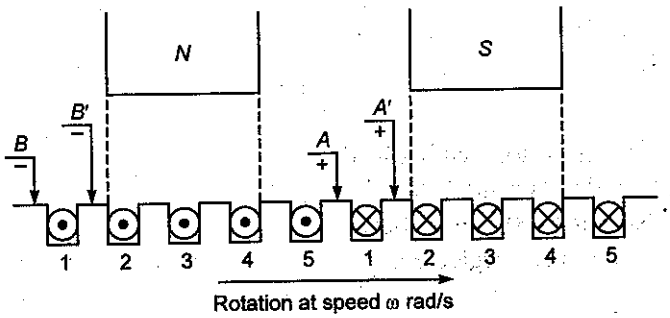
57. A 220 V, 50 Hz, single-phase induction motor has the following connection diagram and winding orientations shown. MM' is the axis of the main stator winding (M_1M_2) and AA' is that of the auxiliary winding (A_1A_2). Directions of the winding axes indicate direction of flux when currents in the windings are in the directions shown. Parameters of each winding are indicated. When switch S is closed, the motor

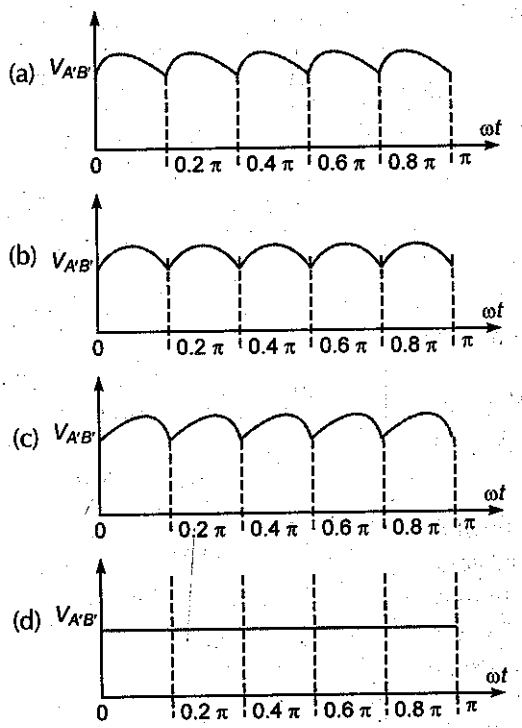


(a) rotates clockwise
(b) rotates anticlockwise
(c) does not rotate
(d) rotates momentarily and comes to a halt

58. Figure shows the extended view of a 2 pole DC machine with 10 armature conductors. Normal brush positions are shown by A and B, placed at the interpolar axis. If the brushes are now shifted, in the direction of rotation, to A' and B' as shown, the voltage waveform $V_{A'B'}$ will resemble

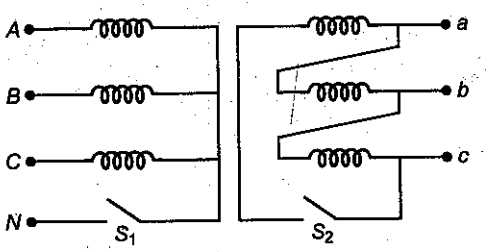
▼ *Solution Points*





Common Data for Questions 59 and 60

The star-delta transformer shown below is excited on the star side with a balanced, 4-wire, 3-phase, sinusoidal voltage supply of rated magnitude. The transformer is under no-load condition.



59. With both S_1 and S_2 opened, the core flux waveform will be

(a) a sinusoid at fundamental frequency
(b) flat topped with third harmonic
(c) peaky with third harmonic
(d) None of the above

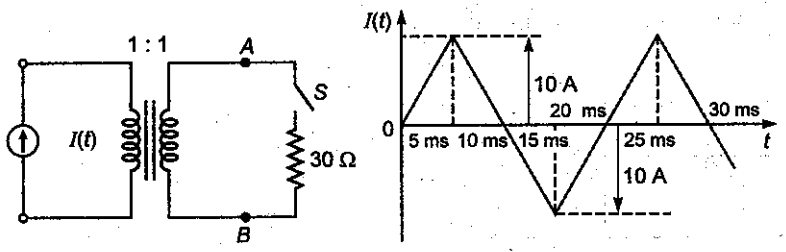
▼ *Solution Points*

60. With S_2 closed and S_1 opened, the current waveform in the delta winding will be

(a) a sinusoid at fundamental frequency
(b) flat topped with third harmonic
(c) only third harmonic
(d) None of the above

Common Data for Questions 61 and 62

The circuit diagram shows a two winding, lossless transformer with no leakage flux, excited from a current source $I(t)$, whose waveform is also shown. The transformer has a magnetizing inductance of $400/\pi$ mH.



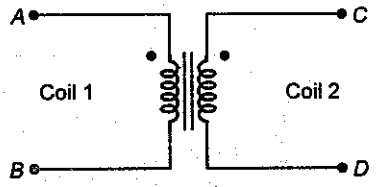
61. The peak voltage across A and B, with S opened is

(a) $400/\pi$ V
(b) 800 V
(c) $4000/\pi$ V
(d) $800/\pi$ V

62. If the waveform of $I(t)$ is changed to $I(t) = 10 \sin(100\ \pi t)$ A, the peak voltage across A and B with S closed is

(a) 400 V
(b) 240 V
(c) 320 V
(d) 160 V

Statements for Linked Answer Questions 63 and 64



The figure above shows coils 1 and 2, with dot markings as shown, having 4000 and 6000 turns respectively. Both coils have a rated current of 25 A. Coil 1 is excited with single-phase, 400 V, 50 Hz supply.

63. The coils are to be connected to obtain a single-phase, 400/1000 V, auto transformer to drive a load of 10 kVA. Which of the options given should be exercised to realize the required auto transformer?

(a) Connect A and D; Common B
(b) Connect B and D; Common C
(c) Connect A and C; Common B
(d) Connect A and C; Common D.

64. In the auto transformer obtained in Q. 63, the currents in coil 1 and coil 2 respectively, are

(a) 25 A and 10 A
(b) 10 A and 25 A
(c) 10 A and 15 A
(d) A and 10 A

GATE 2008

▼ *Solution Points*

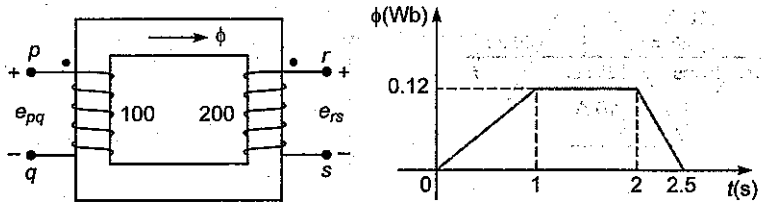
65. A 230 V, 50 Hz, 4-pole, single-phase induction motor is rotating in the clockwise (forward) direction at a speed of 1425 rpm. If the rotor resistance at standstill is 7.8 Ω, then the effective rotor resistance in the backward branch of the equivalent circuit will be

(a) 2Ω (b) 4Ω (c) 78Ω (d) 156Ω

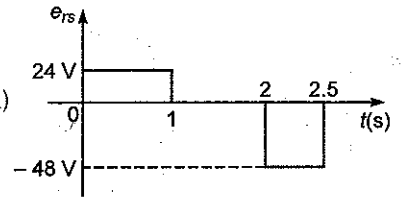
66. A 400 V, 50 Hz, 30 HP, three-phase induction motor is drawing 50 A current at 0.8 power factor lagging. The stator and rotor copper losses are 1.5 kW and 900 W respectively. The friction and windage losses are 1050 W and the core losses are 1200 W. The air-gap power of the motor will be

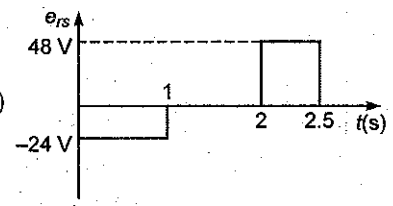
(a) 23.06 kW (b) 24.11 kW (c) 25.01 kW (d) 26.21 kW

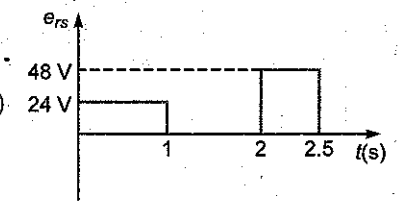
67. The core of a two-winding transformer is subjected to a magnetic flux variation as indicated in the figure.

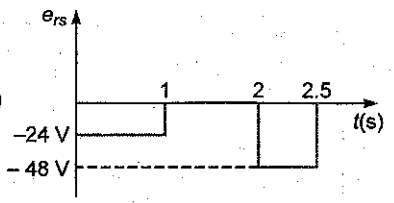


The induced emf (e_{rs}) in the secondary winding as a function of time will be of the form

(a)


(b)


(c)


(d)


68. A 400 V, 50 Hz, 4-pole, 1400 rpm, star connected squirrel cage induction motor has the following parameters referred to the stator

$$R_r' = 1.0\,\Omega, X_r = X_r' = 1.5\,\Omega$$

Neglect stator resistance and core and rotational losses of the motor.

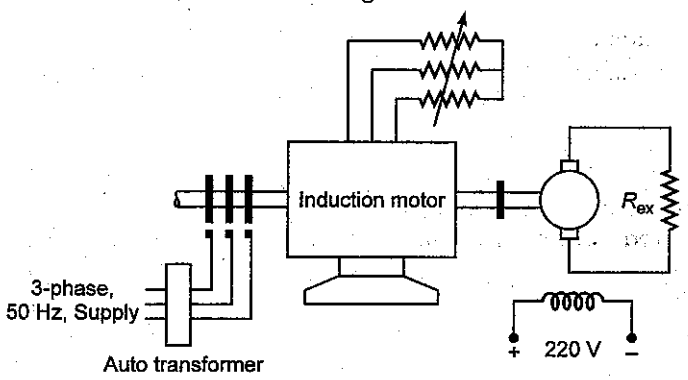
▼ *Solution Points*

The motor is controlled from a 3-phase voltage source inverter with constant V/f control. The stator line-to-line voltage (rms) and frequency to obtain the maximum torque at starting will be

(a) 20.6 V, 2.7 Hz (b) 133.3 V, 16.7 Hz
(c) 266.6 V, 33.3 Hz (d) 323.3 V, 40.3 Hz

Common Data for Questions 69 and 70

A 3-phase, 440 V, 50 Hz, 4-pole, slip ring induction motor is fed from the rotor side through an auto transformer and the stator is connected to a variable resistance as shown in the figure.



The motor is coupled to a 220 V, separately excited DC generator feeding power to fixed resistance of 10 Ω. Two-wattmeter method is used to measure the input power to induction motor. The variable resistance is adjusted such that the motor runs at 1410 rpm and the following readings were recorded $W_1 = 1800$ W, $W_2 = -200$ W.

69. The speed of rotation of stator magnetic field with respect to rotor structure will be

(a) 90 rpm in the direction of rotation
(b) 90 rpm in the opposite direction of rotation
(c) 1500 rpm in the direction of rotation
(d) 1500 rpm in the opposite direction of rotation

70. Neglecting all losses of both the machines, the DC generator power output and the current through resistance (R_{ex}) will respectively be

(a) 96 W, 3.10 A (b) 120 W, 3.46 A
(c) 1504 W, 12.26 A (d) 1880 W, 13.71 A

Statements for Linked Answer Questions 71 and 72

A 240 V, DC shunt motor draws 15 A while supplying the rated load at a speed of 80 rad/s. The armature resistance is 0.5 Ω and the field winding resistance is 80 Ω.

71. The net voltage across the armature resistance at the time of plugging will be

(a) 6 V (b) 234 V (c) 240 V (d) 474 V

72. The external resistance to be added in the armature circuit to limit the armature current to 125% of its rated value is

(a) 31.1 Ω (b) 31.9 Ω (c) 15.1 Ω (d) 15.9 Ω

▼ *Solution Points*

Statements for Linked Answer Questions 73 and 74

A synchronous motor is connected to an infinite bus at 1.0 pu voltage and draws 0.6 pu current at unity power factor. Its synchronous reactance is 1.0 pu and resistance is negligible.

73. The excitation voltage (E) and load angle (δ) will respectively be

(a) 0.8 pu and 36.86° lag (b) 0.8 pu and 36.86° lead
(c) 1.17 pu and 30.96° lead (d) 1.17 pu and 30.96° lag

74. Keeping the excitation voltage same, the load on the motor is increased such that the motor current increases by 20%. The operating power factor will become

(a) 0.995 lagging (b) 0.995 leading
(c) 0.791 lagging (d) 0.848 leading

GATE 2007

75. A three-phase synchronous motor connected to AC mains is running at full load and unity power factor. If its shaft load is reduced by half, with field current held constant, its new power factor will be

(a) unity
(b) leading
(c) lagging
(d) dependent on machine parameters

76. A 100 kVA, 415 V (line), star connected synchronous machine generates rated open circuit voltage of 415 V at a field current of 15 A. The short circuit armature current at a field current of 10 A is equal to the rated armature current. The per unit saturated synchronous reactance is

(a) 1.731 (b) 1.5 (c) 0.666 (d) 0.577

77. A three-phase, three-stack, variable reluctance step motor has 20 poles on each rotor and stator stack. The step angle of this step motor is

(a) 3° (b) 6° (c) 9° (d) 18°

78. A single-phase 50 kVA, 250 V/500 V two winding transformer has an efficiency of 95% at full load, unity power factor. If it is reconfigured as a 500 V/750 V auto transformer, its efficiency at its new rated load at unity power factor will be

(a) 95.752% (b) 97.851% (c) 98.276% (d) 99.241%

79. A three-phase squirrel cage induction motor has a starting torque of 150% and a maximum torque of 300% with respect to rated torque at rated voltage and rated frequency. Neglect the stator resistance and rotational losses. The value of slip for maximum torque is

(a) 13.48% (b) 16.24% (c) 18.92% (d) 26.79%

Common Data for Questions 80, 81 and 82

A three-phase squirrel cage induction motor has a starting current of seven times the full load current and full load slip of 5%.

80. If an auto transformer is used for reduced voltage starting to provide 1.5 pu starting torque, the auto transformer ratio (%) should be

(a) 57.77% (b) 72.56% (c) 78.25% (d) 81.33%

81. If a star-delta starter is used to start this induction motor, the per unit starting torque will be

(a) 0.607 (b) 0.816 (c) 1.225 (d) 1.616

▼ *Solution Points*

82. If a starting torque of 0.5 pu is required then, the per unit starting current should be
(a) 4.65 (b) 3.75 (c) 3.16 (d) 2.13

Statements for Linked Answer Questions 83 and 84

An inductor designed with 400 turns coil wound on an iron core of 16 cm^2 cross-sectional area and with a cut of an air gap length of 1 mm. The coil is connected to a 230 V, 50 Hz AC supply. Neglect coil resistance, core loss, iron reluctance and leakage inductance. ($\mu_0 = 4\pi \times 10^{-7}$ H/m)

83. The current in the inductor is
(a) 18.08 A (b) 9.04 A (c) 4.56 A (d) 2.28 A

84. The average force on the core to reduce the air gap will be
(a) 832.29 N (b) 1666.22 N (c) 3332.47 N (d) 6664.84 N

GATE 2006

85. A 220 V DC machine supplies 20 A at 200 V as a generator. The armature resistance is 0.2 Ω. If the machine is now operated as a motor at same terminal voltage and current but with the flux increased by 10%, then ratio of motor speed to generator speed is
(a) 0.87 (b) 0.95 (c) 0.96 (d) 1.06

86. A synchronous generator is feeding a zero power factor (lagging) load at rated current. The armature reaction is
(a) magnetizing (b) demagnetizing
(c) cross-magnetizing (d) ineffective

87. Two transformers are to be operated in parallel such that they share load in proportion to their kVA ratings. The rating of the first transformer is 500 kVA and its pu leakage impedance is 0.05 pu. If the rating of second transformer is 250 kVA, then its pu leakage impedance is
(a) 0.20 (b) 0.10 (c) 0.05 (d) 0.025

88. The speed of a 4-pole induction motor is controlled by varying the supply frequency while maintaining the ratio of supply voltage to supply frequency (V/f) constant. At rated frequency of 50 Hz and rated voltage of 400 V its speed is 1440 rpm. Find the speed at 30 Hz, if the load torque is constant.
(a) 882 rpm (b) 864 rpm (c) 840 rpm (d) 828 rpm

89. A 3-phase, 4-pole, 400 V, 50 Hz, star connected induction motor has following circuit parameters $r_1 = 1.0\,\Omega$, $r_2' = 0.5\,\Omega$, $x_1 = x_2' = 1.2\,\Omega$, $x_m = 35\,\Omega$. The starting torque when the motor is started direct-on-line is (use approximate equivalent circuit model)
(a) 63.6 N-m (b) 74.3 N-m (c) 190.8 N-m (d) 222.9 N-m

90. A 3-phase, 10 kW, 400 V, 4-pole, 50 Hz, star connected induction motor draws 20 A on full load. Its no load and blocked rotor test data are given below.

No load test	400 V	6 A	1002 W
Blocked rotor test	90 V	15 A	762 W

Neglecting copper loss in no load test and core loss in blocked rotor test, estimate motor's full load efficiency.
(a) 76% (b) 81% (c) 82.4% (d) 85%

▼ *Solution Points*

91. A 3-phase, 400 V, 5 kW, star connected synchronous motor having an internal reactance of 100 Ω is operating at 50% load, unity power factor. Now, the excitation is increased by 1%. What will be the new load in per cent, if the power factor is to be kept same? Neglect all losses and consider linear magnetic circuit.
(a) 67.9% (b) 56.9% (c) 51% (d) 50%

Common Data for Questions 92, 93 and 94
A 4-pole, 50 Hz, synchronous generator has 48 slots in which a double layer winding is housed. Each coil has 10 turns and is short pitched by an angle to 36° electrical. The fundamental flux per pole is 0.025 Wb.

92. The line-to-line induced emf (in volt), for a three-phase star connection is approximately
(a) 808 (b) 888 (c) 1400 (d) 1538

93. The line-to-line induced emf (in volt), for a three-phase connection is approximately
(a) 1143 (b) 1332 (c) 1617 (d) 1791

94. The fifth harmonic component of phase emf (in volt), for a three-phase star connection is
(a) zero (b) 269 (c) 281 (d) 808

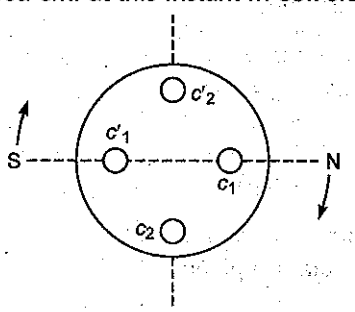
Statements for Linked Answer Questions 95 and 96
A 300 kVA transformer has 95% efficiency at full load 0.8 power factor lagging and 96% efficiency at half load, unity power factor.

95. The iron loss (P_i) and copper loss (P_c) in kW, under full load operation are
(a) $P_c = 4.12, P_i = 8.51$ (b) $P_c = 6.59, P_i = 9.21$
(c) $P_c = 8.51, P_i = 4.12$ (d) $P_c = 12.72, P_i = 3.07$

96. What is the maximum efficiency (in %) at unity power factor load?
(a) 95.1 (b) 96.2 (c) 96.4 (d) 98.1

GATE 2005

97. Two magnetic poles revolve around a stationary armature carrying two coils ($c_1 - c_1', c_2 - c_2'$) as shown in the figure. Consider the instant when the poles are in a position as shown. Identify the correct statement regarding the polarity of the induced emf at this instant in coil sides c_1 and c_2.



(a) ⊙ in c_1, no emf in c_2 (b) ⊗ in c_1, no emf in c_2
(c) ⊙ in c_2, no emf in c_1 (d) ⊗ in c_2, no emf in c_1

98. A 50 kW DC shunt motor is loaded to draw rated armature current at any given speed. When driven

▼ *Solution Points*

(i) at half the rated speed by armature voltage control and
(ii) at 1.5 times the rated speed by field control, the respective output powers delivered by the motor are approximately.

(a) 25 kW in (i) and 75 kW in (ii) (b) 25 kW in (i) and 50 kW in (ii)
(c) 50 kW in (i) and 75 kW in (ii) (d) 50 kW in (i) and 50 kW in (ii)

99. Match the following List I with List II and select the correct answer using the codes given below the lists.

List I	List II
P. Armature emf (E)	1. Flux (ϕ), speed (ω) and armature current (I_a)
Q. Developed torque (T)	2. ϕ and ω
R. Developed power (P)	3. ϕ and I_a
	4. I_a and ω
	5. I_a only

	P	Q	R
(a)	3	3	1
(b)	2	5	4
(c)	3	5	4
(d)	2	3	1

100. In relation to the synchronous machines, which one of the following statements is false?

(a) In salient pole machines, the direct-axis synchronous reactance is greater than the quadrature-axis synchronous reactance
(b) The damper bars help the synchronous motor self start
(c) Short circuit ratio is the ratio of the field current required to produce the rated voltage on open circuit to the rated armature current
(d) The V-curve of a synchronous motor represents the variation in the armature current with field excitation, at a given output power

101. Under no load condition, if the applied voltage to an induction motor is reduced from the rated voltage to half the rated value, then

(a) the speed decreases and the stator current increases
(b) both the speed and the stator current decrease
(c) the speed and the stator current remain practically constant
(d) there is negligible change in the speed but the stator current decreases

102. A three-phase cage induction motor is started by Direct-On-Line (DOL) switching at the rated voltage. If the starting current drawn is 6 times the full load current, and the full load slip is 4%, then ratio of the starting developed torque to the full load torque is approximately equal to

(a) 0.24 (b) 1.44 (c) 2.40 (d) 6.00

103. In a single-phase induction motor driving a fan load, the reason for having a high resistance rotor is to achieve

(a) low starting torque (b) quick acceleration
(c) high efficiency (d) reduced size

104. Consider the following statements :

Assertion (A) Under V/f control of induction motor, the maximum value of the developed torque remains constant over a wide range of speed in the sub-synchronous region.

Reason (R) The magnetic flux is maintained almost constant at the rated value by keeping the ratio V/f constant over the considered speed range.

▼ *Solution Points*

Of these statements, which one is correct?
(a) Both A and R are true and R is the correct explanation of A
(b) Both A and R are true but R is not the correct explanation of A
(c) A is true but R is false
(d) A is false but R is true

Statements for Linked Answer Questions 105 and 106
A 1000 kVA, 6.6 kV, 3-phase star connected cylindrical pole synchronous generator has a synchronous reactance of 20 Ω. Neglect the armature resistance and consider operation at full load and unity power factor.

105. The induced emf (line-to-line) is closed to
(a) 5.5 kV (b) 7.2 kV (c) 9.6 kV (d) 12.5 kV

106. The power (or torque) angle is closed to
(a) 13.9° (b) 18.3° (c) 24.6° (d) 33.0°

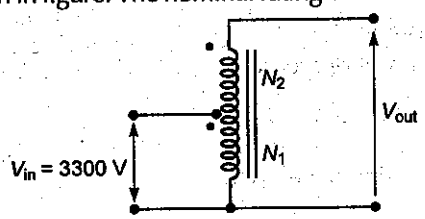
GATE 2004

107. The synchronous speed for the seventh space harmonic mmf wave of a 3-phase, 8-pole, 50 Hz induction machine is
(a) 107.14 rpm in forward direction (b) 107.14 rpm in reverse direction
(c) 5250 rpm in forward direction (d) 5250 rpm in reverse direction

108. A rotating electrical machine having its self-inductances of both the stator and the rotor windings, independent of the rotor position will be definitely not developed
(a) starting torque (b) synchronizing torque
(c) hysteresis torque (d) reluctance torque

109. The armature resistance of a permanent magnet DC motor is 0.8 Ω. At no load, the motor draws 1.5 A from a supply voltage of 25 V and runs at 1500 rpm. The efficiency of the motor while it is operating on load at 1500 rpm drawing a current of 3.5 A from the same source will be
(a) 48.0% (b) 57.1% (c) 59.2% (d) 88.8%

110. A 50 kVA, 3300/230 V single-phase transformer is connected as an auto transformer shown in figure. The nominal rating of the auto transformer will be



(a) 50.0 kVA (b) 53.5 kVA (c) 717.4 kVA (d) 767.4 kVA

111. The resistance and reactance of a 100 kVA, 11000/400 V, Δ-Y distribution transformer are 0.02 and 0.07 pu respectively. The phase impedance of the transformer referred to the primary is
(a) $(0.02 + j0.07)\,\Omega$ (b) $(0.55 + j1.925)\,\Omega$
(c) $(15.125 + j52.94)\,\Omega$ (d) $(72.6 + j254.1)\,\Omega$

112. A single-phase, 230 V, 50 Hz, 4-pole, capacitor-start induction motor has the following stand-still impedances
Main winding $Z_m = 6.0 + j4.0\,\Omega$
Auxiliary winding $Z_a = 8.0 + j6.0\,\Omega$

▼ *Solution Points*

The value of the starting capacitor required to produce 90° phase difference between the currents in the main and auxiliary windings will be

(a) 176.84 μF (b) 167.24 μF

(c) 265.26 μF (d) 280.86 μF

113. Two 3-phase, Y-connected alternators are to be paralleled to a set of common busbars. The armature has a per phase synchronous reactance of 1.7 Ω and negligible armature resistance. The line voltage of the first machine is adjusted to 3300 V and that of the second machine is adjusted to 3200 V. The machine voltages are in phase at the instant they are paralleled. Under this condition, the synchronizing current per phase will be

(a) 16.98 A (b) 29.41 A (c) 33.96 A (d) 58.82 A

114. A 400 V, 15 kW, 4-pole, 50 Hz, Y-connected induction motor has full load slip of 4%. The output torque of the machine at full load is

(a) 1.66 N-m (b) 95.50 N-m (c) 99.47 N-m (d) 624.73 N-m

115. For a 1.8°, 2-phase bipolar stepper motor, the stepping rate is 100 step/s. The rotational speed of the motor in rpm is

(a) 15 (b) 30 (c) 60 (d) 90

116. A 8-pole, DC generator has a simplex wave-wound armature containing 32 coils of 6 turns each. Its flux per pole is 0.06 Wb. The machine is running at 250 rpm. The induced armature voltage is

(a) 96 V (b) 192 V (c) 384 V (d) 768 V

117. A 400 V, 50 kVA, 0.8 PF leading Δ-connected, 50 Hz synchronous machine has a synchronous reactance of 2Ω and negligible armature resistance. The friction and windage losses are 2 kW and the core loss is 0.8 kW. The shaft is supplying 9 kW load at a power factor of 0.8 leading. The line current drawn is

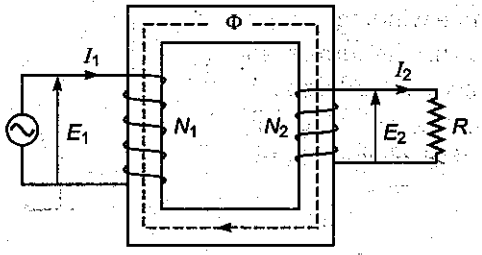
(a) 12.29 A (b) 16.24 A (c) 21.29 A (d) 36.88 A

118. A 500 MW, 3-phase Y-connected synchronous generator has a rated voltage of 21.5 kV at 0.85 PF. The line current when operating at full load rated conditions will be

(a) 13.43 kA (b) 15.79 kA (c) 23.25 kA (d) 27.36 kA

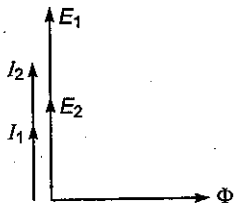
GATE 2003

119. Figure shows an ideal single-phase transformer. The primary and secondary coils are wound on the core as shown. Turns ratio $(N_1/N_2) = 2$. The correct phasors of voltages E_1, E_2, currents I_1, I_2 and core flux Φ are as shown in

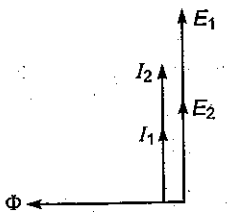


▼ *Solution Points*

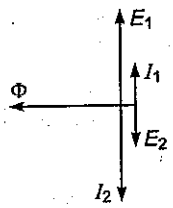
(A)



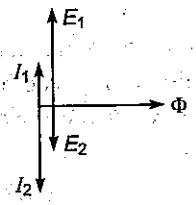
(B)



(C)



(D)

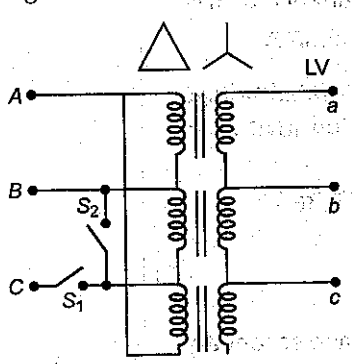


(a) Fig. (A) (b) Fig. (B) (c) Fig. (C) (d) Fig. (D)

120. To conduct load test on a DC shunt motor, it is coupled to a generator which is identical to the motor. The field of the generator is also connected to the same supply source as the motor. The armature of the generator is connected to a load resistance. The armature resistance is 0.02 pu. Armature reaction and mechanical losses can be neglected. With rated voltage across the motor, the load resistance across the generator is adjusted to obtain rated armature current in both motor and generator. The pu value of this load resistance is

(a) 1.0 (b) 0.98 (c) 0.96 (d) 0.94

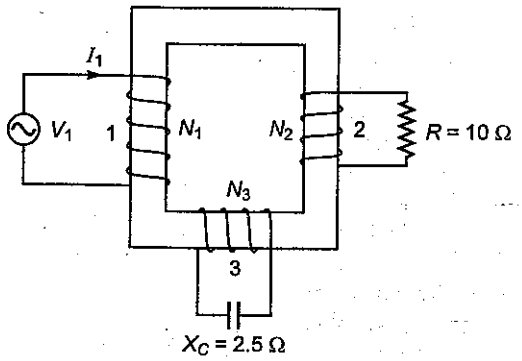
121. Figure shows a Δ-Y connected 3-phase distribution transformer used to step down the voltage from 11000 V to 415 V line-to-line. It has two switches S_1 and S_2. Under normal conditions S_1 is closed and S_2 is opened. Under certain special conditions S_1 is opened and S_2 is closed. In such a case, the magnitude of the voltage across the LV terminals a and c is



(a) 240 V (b) 480 V (c) 415 V (d) zero

122. Figure shows an ideal three-winding transformer. The three windings 1, 2, 3 of the transformer are wound on the same core as shown. The turns ratio $N_1 : N_2 : N_3$ is 4 : 2 : 1. A resistor of 10 Ω is connected across winding 2. A capacitor of reactance 2.5 Ω is connected across winding 3. Winding 1 is connected across a 400 V, AC supply. If the supply voltage phasor $V_1 = 400 \angle 0°$, the supply current phasor I_1 is given by

▼ *Solution Points*

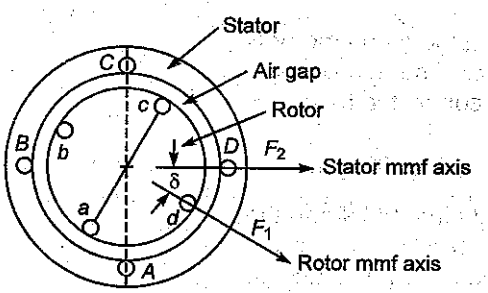


(a) $(-10 + j10)$ A
(b) $(-10 - j10)$ A
(c) $(10 + j10)$ A
(d) $(10 - j10)$ A

123. Following are some of the properties of rotating electrical machines:
P. Stator winding current is DC, rotor winding current is AC.
Q. Stator winding current is AC, rotor winding current is DC.
R. Stator winding current is AC, rotor winding current is AC.
S. Stator has salient poles and rotor has commutator.
T. Rotor has salient poles and slip rings, and stator is cylindrical.
U. Both stator and rotor have poly-phase windings DC machines.
Synchronous machines and Induction machines exhibit some of the above properties as given in the following table. Indicate the correct combination from this table.

DC machines	Synchronous machines	Induction machines
(a) P-S	Q-T	R-U
(b) Q-U	P-T	R-S
(c) P-S	R-U	Q-T
(d) R-S	Q-U	P-T

124. When stator and rotor windings of a 2-pole rotating electrical machine are excited, each would produce a sinusoidal mmf distribution in the air gap with peak values F_1 and F_2 respectively. The rotor mmf lags stator mmf by a space angle δ at any instant as shown in figure. Thus, half of stator and rotor surfaces will form one pole with the other half forming the second pole. Further, the direction of torque acting on the rotor can be clockwise or counter-clockwise. The table given ahead gives four sets of statements as regards poles and torque. Select the correct set corresponding to the mmf axes as shown in figure.



▼ *Solution Points*

Stator surface *ABC* forms	Stator surface *CDA* forms	Rotor surface *CDA* forms	Rotor surface *CDA* forms	Torque is
(a) North pole	South pole	North pole	South pole	Clockwise
(b) South pole	North pole	North pole	South pole	Counter-clockwise
(c) North pole	South pole	South pole	North pole	Counter-clockwise
(d) South pole	North pole	South pole	North pole	Clockwise

125. A 4-pole, 3-phase, double-layer winding is housed in a 36-slot stator for an AC machine with 60° phase spread. Coil span is 7 slot pitches. Number of slots in which top and bottom layers belong to different phases is
(a) 24 (b) 18 (c) 12 (d) zero

126. A 3-phase induction motor is driving a constant torque load at rated voltage and frequency. If both voltage and frequency are halved, following statements relate to the new condition. If stator resistance, leakage reactance and core loss are ignored.
P. The difference between synchronous speed and actual speed remains same.
Q. The air gap flux remains same.
R. The stator current remains same.
S. The pu slip remains same.
Among the above, correct statements are
(a) All (b) P, Q and R (c) Q, R and S (d) P and S

127. A single-phase induction motor with only the main winding excited would exhibit the following response at synchronous speed. Then
(a) rotor current is zero
(b) rotor current is non-zero and is at slip frequency
(c) forward and backward rotating fields are equal
(d) forward rotating field is more than the backward rotating field

128. A DC series motor driving an electric train faces a constant power load. It is running at rated speed and rated voltage. If the speed has to be brought down to 0.25 pu, the supply voltage has to be approximately brought down to
(a) 0.75 pu (b) 0.5 pu (c) 0.25 pu (d) 0.125 pu

GATE 2002

129. A 200 V, 2000 rpm, 10 A, separately excited DC motor has an armature resistance of 2Ω. Rated DC voltage is applied to both the armature and field winding of the motor. If the armature drawn 5A from the source, the torque developed by the motor is
(a) 4.30 N-m (b) 4.77 N-m (c) 0.45 N-m (d) 0.50 N-m

130. The rotor of a three-phase, 5 kW, 400 V, 50 Hz, slip ring induction motor is wound for 6 poles while its stator is wound for 4 poles. The approximate average no load steady state speed when this motor is connected to 400 V, 50 Hz supply is
(a) 1500 rpm (b) 500 rpm (c) zero (d) 1000 rpm

131. The flux per pole in a synchronous motor with the field circuit ON and the stator is disconnected from the supply is found to be 20 mWb. When the stator is connected to the rated supply with the field excitation unchanged,

▼ *Solution Points*

the flux per pole in the machine is found to be 25 mWb while the motor is running on no load. Assuming no load losses to be zero, the no load current down by the motor from the supply

(a) lags the supply voltage
(b) leads the supply voltage
(c) is in phase with the supply voltage
(d) is zero

GATE 2001

132. The hysteresis loop of a magnetic material has an area of 5 cm^2 with the scales given as 1 cm = 2 AT and 1 cm = 50 mWb. At 50 Hz, the total hysteresis loss is

(a) 15 W (b) 20 W (c) 25 W (d) 50 W

133. An electric motor with 'constant output power' will have a torque-speed characteristic in the form of a

(a) straight line through the origin
(b) straight line parallel to the speed axis
(c) circle about the origin
(d) rectangular hyperbola

134. A 3-phase transformer has rating of 20 MVA, 220 kV (star)-33 kV (delta) with leakage reactance of 12%. The transformer reactance (in ohm) referred to each phase of the LV delta connected side is

(a) 23.5 (b) 19.6 (c) 18.5 (d) 8.7

GATE 2000

135. A 3-phase delta/star transformer is supplied at 6000 V on the delta-connected side. The terminal voltage on the secondary side when supplying full load at 0.8 lagging power factor is 115 V. The equivalent resistance and reactance drops for the transformer are 1% and 5% respectively. The turns ratio of the transformer is

(a) 14 (b) 24 (c) 42 (d) 20

136. A 240 V DC series motor takes 40 A when giving its rated output at 1500 rpm. Its resistance is 0.3 Ω. The value of resistance which must be added to obtain rated torque at 1000 rpm is

(a) 6 Ω (b) 5.7 Ω (c) 2.2 Ω (d) 1.9 Ω

137. The power input to a 415 V, 50 Hz, 6-pole, 3-phase induction motor running at 975 rpm is 40 kW. The stator losses are 1 kW, and friction and windage losses total 2 kW. The efficiency of the motor is

(a) 92.5% (b) 98% (c) 90% (d) 88%

138. A single-phase, 2000 V alternator has armature resistance and reactance of 0.8 Ω and 4.94 Ω respectively. The voltage regulation of the alternator at 100 A load at 0.8 leading power factor is

(a) 7% (b) –8.9% (c) 14% (d) 0%

139. A permanent magnet DC commutator motor has a no-load speed of 6000 rpm when connected to a 120 V DC supply. The armature resistance is 2.5 Ω and other losses may be neglected. The speed of the motor with supply voltage of 60 V developing a torque 0.5 N-m, is

(a) 3000 rpm (b) 2673 rpm (c) 2836 rpm (d) 5346 rpm

Answers with Explanations

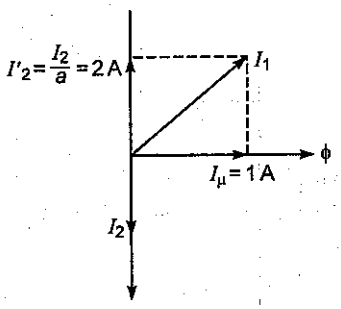
1. (c) It is a air core transformer so there will be no saturation. Hence, magnetizing current waveform will be sinusoidal.

2. (a) A negative sequence relay is commonly used to protect an alternator.

3. (a) It starts and controls the speed of DC shunt motor with armature resistance control.

4. (b)

5. (c) Here, transformation ratio $a = \frac{1}{2} = 0.5$



$$I'_2 = \frac{I_2}{a} = \frac{1}{0.5} = 2 \text{ A}$$

$$I_1 = \sqrt{(I'_2)^2 + (I_\mu)^2}$$

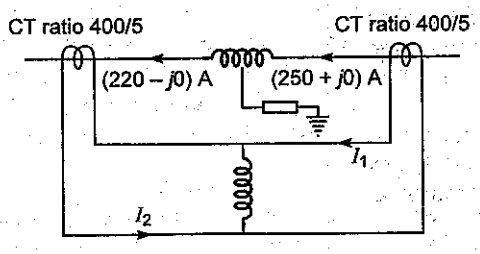
$$= \sqrt{2^2 + 1^2} = \sqrt{5}$$

$$= 2.24 \text{ A}$$

6. (a) The magnitude of starting current of motor will be

$$\frac{V}{\sqrt{(r_s + r_r)^2 + (x_s + x_r)^2}}$$

7. (c)



Current through operating coil $= \frac{(I_1 - I_2)}{a}$

where, a is CT ratio.

$$I = \frac{(250 - 220) \times 5}{400} = 0.375 \text{ A}$$

8. (d) Internal resistance of alternator

$$= \frac{2000 \text{ V}}{400 \text{ A}} = 5\,\Omega$$

$\therefore$ Internal voltage drop in alternator

$$= 5 \times 200 = 1000 \text{ V}$$

9. (c) We know that,

Mutual inductance $M = \sqrt{L_1 L_2}$

But inductance $L = \frac{N\phi}{I}$

Also $\phi = B \cdot A$

$\therefore$ $L \propto A$

$\therefore$ $M \propto \sqrt{A_1 A_2}$

$$\frac{M_1}{M_2} = \sqrt{\frac{20 \times 20}{10 \times 10}} = \sqrt{4} = 2$$

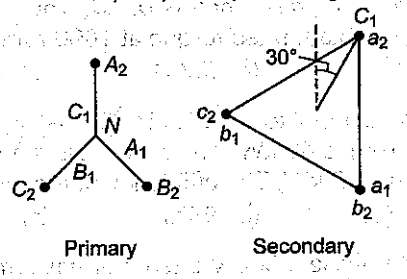
$\therefore$ $M_2 = \frac{M_1}{2}$

Therefore new mutual inductance will be halved of original mutual inductance.

10. (b) In the given graph, A is unstable because slip ≈ 0 and B is stable because slip ≈ 1.

11. (d) The distributed winding and short chording winding used in AC machine will result in reduction in both emf and harmonics.

12. (b) The given transformer connection will be represented by Yd_1 (30° phase lag).



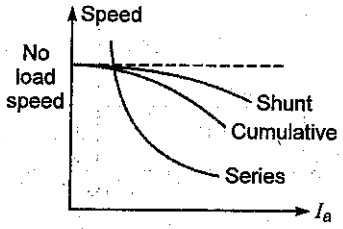
13. (c) The detent torque means minimum of the static torque with the phase winding unexcited.

14. (d) **15.** (a)

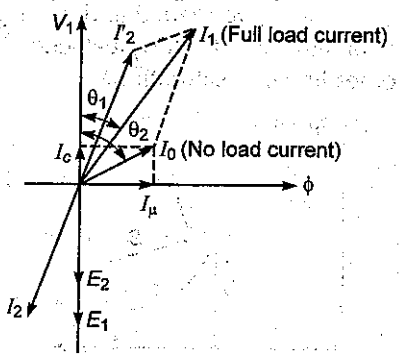
16. (c) The zero voltage regulation at full load is possible at leading power factor load.

17. (b) The speed and armature current characteristics of DC motor show that the speed of shunt motor is

almost constant at load. Therefore, shunt motor can provide zero speed regulation.



18. (d) In open circuit test, core losses are obtained and I_0 (no load current) is drawn at angle θ_2.
In short circuit test, copper losses are obtained and I_1 (full load current) is drawn at angle θ_1.
Here, $\theta_2 > \theta_1$
$\therefore \quad \cos\theta_2 < \cos\theta_1$



$\therefore$ In an open circuit test, current is drawn at low power factor.

19. (b) In a single-phase capacitor start induction motor, the direction of rotation can be changed by reversing the main winding terminal because magnetic flux direction will be reversed.

20. (b) In a DC machine, the effect of armature reaction under pole shoe can be neutralized by using compensating winding because compensating winding neutralizes the armature flux. While interpole winding is used for improving commutation, *i.e.*, sparkless commutation.

21. (d) The magnetizing reactance X_M has very high value to reduce magnetizing current.

$\because \quad X'_2 = a^2 X_2$

But in approximate equivalent circuit,

$$X_1 \approx a^2 X_2 \approx a^2 \cdot \frac{X'_2}{a^2}$$

$\therefore \quad X_1 \approx X'_2$

$\Rightarrow \quad X_1 \approx X'_2 << X_M$

22. (b) The phase difference of 30° can be introduced only in Δ-Y and Y-Δ connections of transformer windings.

23. (a) The region where slip $s > 1$, in this region braking takes place.

24. (b) Since, gross power output $= P_g(1-s)$
and air gap power $= P_g$
$\therefore$ Ratio of gross power output to air gap power $= (1-s)$

25. (a)

26. (b) We know that, at maximum efficiency,

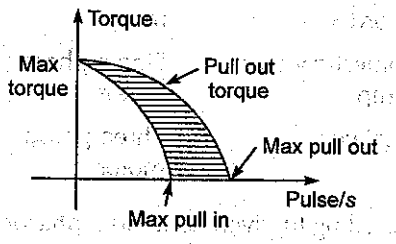
$$\% \text{ loading } x = \sqrt{\frac{W_i}{W_{Cu}}} = \sqrt{\frac{\text{Iron loss}}{\text{Copper loss}}}$$

$$\left[\begin{array}{l}\because \text{At max. efficiency} \\ \text{Copper loss} = \text{Iron loss} \\ x^2 W_{Cu} = W_i\end{array}\right]$$

$$= \sqrt{\frac{300}{600}} = 0.707$$

$$= 70.7\%$$

27. (c) In the stepper motor, the pull out torque has highest numerical value.



28. (c) The stepper motor has a permanent magnet rotor.

29. (d) The capacitor run single phase induction motor having highest power factor at full load because in this motor capacitor is permanently connected in series with auxiliary winding. At particular load, this auxiliary winding and capacitor give approximately 90° phase difference between two currents of main winding and auxiliary winding and motor works as balanced two phase motor and its backward rotating flux will be absent therefore motor gives better operating power factor and efficiency.

30. (c) For opposite (counter-clockwise) direction of rotation, any two phases can be interchanged, *i.e.*, *A-C-B*.

31. (d) We know that,
At maximum efficiency (η_{max}),
Copper loss = Iron loss
$\therefore \quad x^2 W_{Cu} = W_i$

Loading of transformer $x = \sqrt{\frac{W_i}{W_{Cu}}} = 1$

$\therefore \quad W_i = W_{Cu}$

But efficiency, $\eta = \frac{x \times kVA \times PF}{x \times kVA \times PF + x^2 . W_{Cu} + W_i}$

At full load, $90\% = \frac{1 \times kVA \times 1}{1 \times kVA \times 1 + 2W_i}$

$\therefore \quad W_i = 0.0555 \text{ kVA}$

At half load,

$$\eta = \frac{\frac{1}{2} \times kVA \times 1}{\frac{1}{2} \times kVA \times 1 + 0.01389 \text{ kVA} + 0.0555 \text{ kVA}} \quad \left[\because x = \frac{1}{2}\right]$$

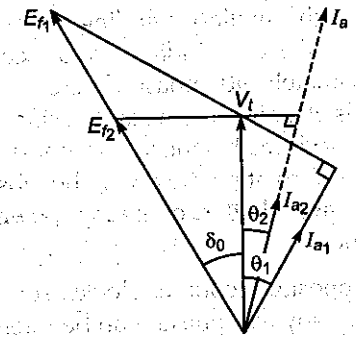
$$\eta = \frac{0.5}{0.5694} = 87.8\%$$

32. (c)

Food mixer	Universal motor
Cassette tape recorder	Permanent magnet DC motor
Domestic water pump	Single-phase induction motor
Escalator	Three-phase induction motor

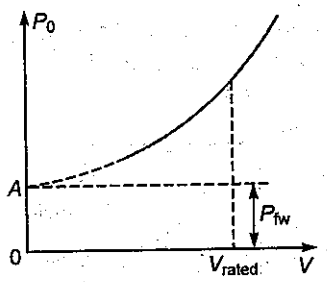
33. (d) According to given condition, phasor diagram,

Here, δ_0 = constant, since active power remains unchanged. To nullify the inductive current, power factor should be improved therefore, E_f should be reduced.



34. (a) Here, vertical distance PQ gives voltage drop across synchronous reactance.

35. (d) At the vertical axis $V=0$ and hence the intercept OA represents the voltage independent loss, *i.e.*, the loss due to friction and windage P_{fw}.

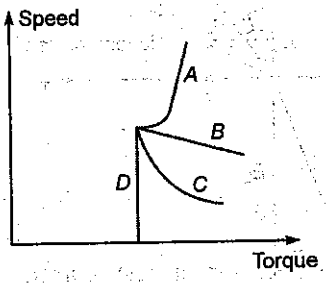


36. (a) Since, torque $T_c = \frac{3I_2^2 r_2}{\omega_s \cdot s}$

Here, frequency of motor is increasing so slip will be increased $\left(\text{since } N_s = \frac{120f}{p}\right)$.

As the slip increases therefore T_c decreases.

37. (b) Here, motor is overloaded and its magnetic circuit is saturated so its torque-speed characteristic becomes linear in saturation.



38. (d) Under saturation condition, current drawn from source will not be sinusoidal.

39. (a) The reading of voltmeter = 400 – (200 + 200) = 0

Since here, 200 V and 200 windings are in additive polarity and 400 V winding in subtractive polarity.

40. (a) To have minimum inrush current in a single-phase transformer, the switch should be closed at maximum supply voltage.

41. (b) A certain harmonic can be completely eliminated from the winding voltage by choosing a pitch for the coils that makes the pitch factor zero for that harmonic. In order to eliminate the nth harmonic voltage.

$\therefore \quad \cos\frac{n\alpha}{2} = 0$

$$\cos\frac{n\alpha}{2} = \cos 90^\circ \Rightarrow \frac{n\alpha}{2} = 90^\circ$$

$\therefore \quad \alpha = \frac{180^\circ}{n}$

$\therefore$ To eliminate 5th harmonic, the coil should be shorted by $\alpha = \frac{180^\circ}{5} = 36^\circ$ angle.

42. (d) The magnetizing curve for rated armature current will be as curve D.

43. (a) In a practical transformer with resistive load, the core flux is strictly constant with load changes.

44. (a) In a synchronous machine,

where, $X_d > X'_d > X''_d$

X_d = steady state d-axis synchronous reactance

X'_d = transient d-axis synchronous reactance

X''_d = sub-transient d-axis synchronous reactance

45. (a) The number of phases in rotor are same as stator phase.

46. (c) The compensating winding in a DC machine is located on pole shoes for avoiding the flashover at the commutator surface.

47. (d) By tapping the winding, output voltage remains constant in a constant voltage transformer.

48. (b) The phase sequence of the alternator will be reversed by reversing the direction of rotation but field current remains same.

49. (b) No. of rotor teeth $(T) = \dfrac{360}{\alpha \cdot n}$

n = no. of phases,

α = step angle

$$\therefore \quad T = \frac{360°}{1.8 \times 4} = \frac{360°}{7.2} = 50$$

50. (a) Synchronous speed $N_S = \dfrac{120f}{P} = \dfrac{120 \times 50}{6}$

$= 1000$ rpm

The speed of stator magnetic field $= 1000$ rpm (N_S)

Also, the speed of rotor magnetic field $= N_S = 1000$ rpm

Hence,

Relative speed between stator magnetic field and rotor magnetic field $= 1000 - 1000 = 0$

Rotor speed $= N_S - SN_S = 1000 \times -(0.05 \times 1000)$

$= 950$ rpm

$\therefore$ Rotor speed with respect to stator magnetic field $= 950 - 1000 = -50$ rpm

51. (b) $\because \tan\delta = \dfrac{I_a(X_q \cos\phi + R_a \sin\phi)}{V_t + I_a(X_q \sin\phi - R_a \cos\phi)}$

Here, $I_a = 1$pu

$V_t = 1$pu

$\phi = 0$ $\quad (\because \cos\phi = 1)$

$Xd = 1.2$pu and $X_q = 1$pu

$$\therefore \quad \tan\delta = \frac{1(1 \times 1 + 0)}{1 + 1(1 \times 0 - 0)} = 1$$

$$\therefore \quad \delta = \tan^{-1} 1 = 45°$$

52. (a) $\because \ \omega \propto \dfrac{E_b}{\phi}$

$$\Rightarrow \quad \frac{\omega_1}{\omega_2} = \frac{E_{b_1}}{E_b} \times \frac{\phi_2}{\phi_1} \qquad \ldots(i)$$

but $\phi_1 = 0.9\,\phi$

$\because$ Torque is constant.

$$I_{a_1}\phi_1 = I_{a_2}\phi_2$$

$$\therefore \quad I_{a_2} = I_{a_1}\frac{\phi_1}{\phi_2} = \frac{10 \times \phi_1}{\omega \cdot 9\,\phi_1}$$

$= 11.11$ A

From Eq. (i),

$$1 = \frac{\omega_1}{\omega_2} = \frac{220 - I_{a1}R_1}{220 - I_{a_2}(R_1 + R_{ex})} \times \frac{0.9\,\phi}{\phi_i}$$

$$1 = \frac{210 \times 0.9}{220 - 11.11(1 + R)}$$

$$\Rightarrow \quad 1 + R_{ex} = 2.79$$

$$\therefore \quad R_{ex} = 1.79\,\Omega$$

53. (c) In the given circuit, induction machine is running at speed of 1600 rpm but its synchronous speed is 1500 rpm. $\left[\because N_s = \dfrac{120f}{p}\right]$

Thus, induction machine acts as generator and DC machine acts as a motor.

54. (d) Per phase value of impedance at load

$$Z_{2ph} = (4 + j0)\,\Omega$$

impedance value referred to primary

$$Z'_{2ph} = (a)^2 Z_{2ph}$$

But $\quad a = \dfrac{1}{2}$

$$\therefore \quad Z'_{2ph} = \left(\frac{1}{2}\right)^2 (4 + j0)$$

$= (1 + j0)\,\Omega$

55. (b) At no load, $I_a = 0$, i.e., $V_t = E_1 = 200$ V

But at 1500 rpm,

$$E_1 = K\phi\omega_{m_1} \Rightarrow K\phi = \frac{E_1}{\omega_{m_1}} = \frac{200}{1500}$$

$\because$ Torque $T = \dfrac{30}{\pi} \times K\phi I_a$

$$\Rightarrow \quad 5 = \frac{30}{\pi} \times \frac{200}{1500} \times I_a \Rightarrow I_a = 3.93\ \text{A}$$

At 1400 rpm,

$$E_2 = K\phi\omega_{m_2} = \frac{200}{1500} \times 1400 = 186.67 \text{ V}$$

But in DC motor,

$$E = V - I_a R_a$$

$$186.67 = 200 - 3.93 \times R_a \Rightarrow R_a = 3.39 \approx 3.4\Omega$$

56. (b) Here, $T = 2.5$ N-m

But $$T = \frac{30}{\pi} K\phi I_a$$

$$2.5 = \frac{30}{\pi} \times \frac{200}{1500} \times I_a \Rightarrow I_a = 1.96 \text{ A}$$

$$E_a = V - I_a R_a = 200 - 1.96 \times 3.4 = 193.3 \text{ V}$$

57. (c) Impedance of auxiliary winding

$$Z_{aux} = r_a + j\omega L_a = 1 + j \times 2\pi \times 50 \times \frac{10}{\pi}$$

$$= 1 + j1000$$

Impedance of main winding

$$Z_{main} = r_m \times j\omega L_m = 0.1 + j \times 2\pi \times 50 \times \frac{0.1}{\pi}$$

$$= 0.1 + j10$$

Starting torque $= KI_a I_m \delta_m (I_a, I_m) \rightarrow \delta(0) = 0$

58. (a) Voltage waveform V_{AB} will be same as choice (a).

59. (b) The core flux waveform will be as flat topped with third harmonic because star current consists third harmonics in line current due to hysteresis saturation.

60. (a) The current waveform in the delta winding will be sinusoidal at fundamental frequency because third harmonic is not present in delta winding.

61. (d) $$V_{AB} = M\frac{dI}{dt}$$

$$= \frac{400}{\pi} \times 10^{-3} \times \frac{10}{5 \times 10^{-3}}$$

$$= \frac{800}{\pi} \text{ V}$$

62. (a) $$I(t) = 10 \sin(100\pi t) \text{ A}$$

$$V_{AB} = L\frac{dI(t)}{dt} \quad \text{but } L = \frac{400}{\pi} \text{ mH}$$

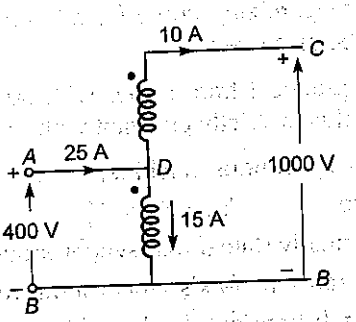
$$= \frac{400}{\pi} \times 10^{-3} \times \frac{d}{dt} I(t)$$

$$= \frac{400}{\pi} \times \pi \times 10^{-3} \times 1000 \times \cos(100\pi t)$$

$$V_{AB} = 400 \cos(100\pi t)$$

$$\therefore \quad V_m = 400 \text{ V}$$

63. (a)



64. (a) kVA rating will be same on both sides.

$$25 \text{ A} \times 400 \text{ V} = 10 \text{ A} \times 1000 \text{ V}$$

65. (b) Synchronous speed $N_s = \frac{120f}{p} = \frac{120 \times 50}{4}$

$$= 1500 \text{ rpm}$$

$$\text{Slip } s = \frac{N_s - N_r}{N_s} = \frac{1500 - 1425}{1500} = 0.05$$

$\therefore$ Resistance in backward branch

$$r_b = \frac{r_2}{2 - s} = \frac{7.8}{2 - 0.05} = 4\Omega$$

66. (C) Motor input power to stator

$$P_{in} = \sqrt{3} VI \cos\phi$$

$$= \sqrt{3} \times 400 \times 50 \times 0.8$$

$$= 27.71 \text{ kW}$$

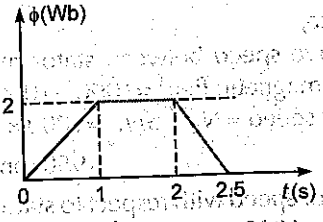
$\therefore$ Airgap power of motor = Motor input to stator – Stator copper loss – Coreloss

$$= 27.71 - 1.5 - 1.2$$

$$= 25.01 \text{ kW}$$

67. (b) Induced emf in secondary winding

$$e_{rs} = -N_2 \frac{d\phi}{dt}$$



According to graph,

For $0 < t < 1$; $\phi = 0.12t$

For $1 \le t \le 2$; $\phi = 0.12$

For $2 \le t \le 2.5$; $\phi = -0.24t + 0.6$

During 0-1 s, $e_{rs} = -200 \times 0.12 = -24$ V

During 1-2 s, $e_{rs} = -200 \times 0 = 0$

During 2-2.5 s, $e_{rs} = -200 \times (-0.24 + 0)$

$= 48$ V

68. (b) For maximum torque,

$$\text{Slip } s_m = \frac{R_r'}{X_{sm} + X_{rm}'}$$

But for starting torque $s_m = 1$

$$\therefore \quad R_r' = X_{sm} + X_{rm}'$$

$$1 = 2\pi f_m L_s + 2\pi f_m L_r'$$

$\therefore$ At maximum torque,

$$\text{Frequency } f_m = \frac{1}{2\pi(L_s + L_r')}$$

But $$L_s = \frac{X_s}{2\pi f} = \frac{1.5}{2\pi \times 50} = 4.77 \times 10^{-3}$$

$$L_r' = \frac{1.5}{2\pi \times 50} = 4.77 \times 10^{-3}$$

$$\therefore \quad f_m = \frac{1}{2\pi(4.77 \times 10^{-3} + 4.77 \times 10^{-3})}$$

$$= \frac{1}{0.06} = 16.67 \text{ Hz}$$

In constant $\frac{V}{f}$ control method,

$$\frac{V_1}{f_1} = \frac{400}{50} = 8$$

$$\frac{V_2}{f_2} = 8 \Rightarrow V_2 = f_2 \times 8$$

$$V_2 = 16.67 \times 8 = 133.3 \text{ V}$$

69. (a) Speed of stator magnetic field, *i.e.*, synchronous speed

$$N_s = \frac{120 f}{p} = \frac{120 \times 50}{4} = 1500 \text{ rpm}$$

Rotor speed = 1410 rpm

$\therefore$ Relative speed = 1500 − 1410 = 90 rpm in direction of rotation.

70. (c) Slip $s = \frac{N_s - N_r}{N_s} = \frac{1500 - 1410}{1500} = 0.06$

Total input power to induction motor

$$P_{in} = 1800 - 200 = 1600 \text{ W}$$

Power output of induction motor

$$P_o = (1 - S) \times P_{in}$$

$$= (1 - 0.06) \times 1600 = 1504 \text{ W}$$

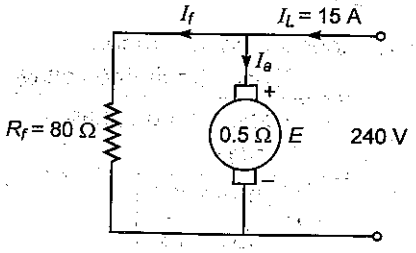
Since, all the losses are neglected, therefore, DC generator input = DC generator output = 1504 W

$$\therefore \quad I^2 R_{ex} = 1504$$

$$I^2 = \frac{1504}{10} = 150.4$$

$$\therefore \quad I = \sqrt{150.4} = 12.26 \text{ A}$$

71. (d)



Here, $$I_L = I_f + I_a$$

But $$I_f = \frac{240}{80} = 3 \text{ A}$$

$$\therefore \quad I_a = 15 - 3 = 12 \text{ A}$$

$$\because \quad E = X - I_a R_a$$

$$= 240 - 12 \times 0.5 = 234 \text{ V}$$

Voltage at plugging $= V + E = 240 + 234$

$$= 474 \text{ V}$$

72. (a) Current $I_{a_2} = 1.25 \times I_a = 1.25 \times 12$

$$= 15 \text{ A}$$

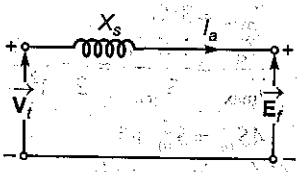
Let resistance R_{ex} is connected in series with R_a, at plugging, emf will be reversed.

$$\therefore \quad -E = V - I_{a_2}(R_a + R_{ex})$$

$$\frac{E + V}{I_{a_2}} = R_a + R_{ex} \Rightarrow \frac{474}{0.15} = 31.6$$

$$\therefore \quad R_{ex} = 31.6 - 0.5 = 31.1\Omega$$

73. (d)



$$\vec{E} = \vec{V}_t - j\vec{I}_a X_s$$

$$= 1\angle 0° - j0.6\,(1 + j0.1$$

$$= 1 - j0.6 = 1.17 \angle -30.96°$$

74. (a) $I_a = 1.2 I_a = 1.2 \times 0.6 = 0.72$ pu

$$(I_{a_2} \times s)^2 = E_f^2 + V_t^2 - 2E_f V_t \cos\delta$$

$$(0.72 \times 1)^2 = (1.17)^2 + (1)^2 - 2 \times 1.17 \times 1 \times \cos\delta$$

$$\cos\delta = 0.79 \Rightarrow \delta = 0.659°$$

$$\therefore \quad \sin\delta = 0.6123$$

$$\frac{V_t E_f}{X_s} \sin\delta = V_t I_{a_2} \cos\theta$$

$$\Rightarrow \quad \cos\theta = 0.995$$

$$E_f \cos\delta = 1.17 \times 0.79 = 0.924$$

$$E_f \cos\delta < V_t$$

Hence, power factor is 0.995, lagging.

75. (b)

76. (a) Synchronous impedance (line)

$$= \frac{\text{Open circuit line voltage}}{\sqrt{3} \times \text{Short circuit phase current}}$$

$$= \frac{415}{\sqrt{3}\left(\frac{100 \times 1000}{\sqrt{3} \times 415}\right)} = 1.722$$

77. (b) Step angle $\alpha = \frac{360°}{3 \times 20} = 6°$

78. (c) $\because$ Efficiency $\eta = \frac{\text{Output}}{\text{Input}} = \frac{\text{Output}}{\text{Output + Losses}}$

$$0.95 = \frac{50}{50 + \text{losses}}$$

$\therefore$ Losses $= 52.631 - 50 = 2.631$ kW

Now, KVA of autotransformer $= 50\left(\frac{1}{1-a}\right)$

But $a = \frac{500}{750} = \frac{2}{3}$

$\therefore$ $KVA_{auto} = 50 \times \frac{1}{(1-2/3)} = 150$

$\therefore$ $\eta_{auto} = \frac{150}{150 + 2.631} \times 100 = 98.276$

79. (d) Given, $T_{st} = 1.5\, T_{FL}$

and $T_{max} = 3\, T_{FL}$

$\therefore$ $\frac{T_{st}}{T_{max}} = \frac{1.5\, T_{FL}}{3\, T_{FL}} = \frac{1}{2} = \frac{2S_m}{1^2 + S_m^2}$

$\therefore$ $4S_m = S_m^2 + 1$

$\Rightarrow$ $4S_m^2 - 4S_m + 1 = 0$

$\therefore$ $S_m = 0.268 = 26.8\%$

80. (c) Here, $I_{st} = 7I_{FL}$

and slip at full load $S_{FL} = 5\% = 0.05$

When auto transformer starting is used,

$$\frac{T_{st}}{T_{FL}} = \left(\frac{I_{st}}{I_{FL}}\right)^2 \times x^2 \times S_{FL}$$

$$1.5 = (7)^2 \times x^2 \times 0.05$$

$\therefore$ $x = 78.25\%$

81. (b) When a star-delta starter is used,

$$\frac{T_{st}}{T_{FL}} = \frac{1}{3} \times \left(\frac{I_{st}}{I_{FL}}\right)^2 \times S_{FL}$$

$$= \frac{1}{3} \times 7^2 \times 0.05 = 0.816$$

82. (c) $$\frac{T_{st}}{T_{FL}} = \left(\frac{I_{st}}{I_{FL}}\right)^2 \times S_{FL}$$

$$0.5 = \left(\frac{I_{st}}{I_{FL}}\right)^2 \times 0.05$$

$\therefore$ $(I_{st})_{pu} = 3.16$

83. (d) Inductance of coil $L = \frac{\mu_0 N^2 A}{l}$

$$= \frac{4\pi \times 10^{-7} \times 400 \times 400 \times 16 \times 10^{-4}}{(1 \times 10^{-3})} = 0.3215$$

$X_L = 2\pi fL = 2 \times \pi \times 50 \times 0.3215 = 101.002\Omega$

$\therefore$ Current in inductor, $I = \frac{V}{X_L} = \frac{230}{101.002} = 2.277$

≈ 2.28 A

84. (a) Stored energy $E = \frac{1}{2}LI^2$

$$= \frac{1}{2} \times (0.3215) \times (2.28)^2$$

$$= 0.8356\, J$$

Force to reduce 1 mm air gap $= \frac{E}{d} = \frac{0.8356}{1 \times 10^{-3}}$

$= 832.29$ N

85. (a) In generator case,

$$E_1 = V + I_a R_a$$

$$= 200 + 20 \times 0.2 = 204 \text{ V}$$

In motor case, $E_2 = V - I_a R_a$

$$= 200 - 20 \times 0.2 = 196 \text{ V}$$

But $E = \frac{NP\phi Z}{60\, A}$

$\therefore$ $E \propto N\phi$

$\Rightarrow$ $\frac{E_1}{E_2} = \frac{N_1 \phi_1}{N_2 \phi_2}$

Consider $\phi_1 = 1$ then $\phi_2 = 1.1$

$$\frac{204}{196} = \frac{N_1}{N_2} \times \frac{1}{1.1}$$

$\Rightarrow$ $\frac{N_2}{N_1} = \frac{196}{204 \times 1.1} = 0.87$

86. (b) At zero power factor (lagging), the armature reaction will be demagnetizing nature.

87. (b) $\because$ PU leakage impedance $\propto \frac{1}{\text{kVA}}$

Let PU leakage impedance of 250 kVA transformer $= x$

$\therefore$ $500 \times 0.05 = 250 \times x$

$$x = \frac{500 \times 0.05}{250} = 0.10$$

88. (b) At 50 Hz,

$$N_s = \frac{120f}{p} = \frac{120 \times 50}{4} = 1500 \text{ rpm}$$

But $N_r = N_s(1-s)$

$1440 = 1500(1-s)$

$(1-s) = 0.96$

$s = 0.04$

At 30 Hz,

$$N_s = \frac{120 \times 30}{4} = 900 \text{ rpm}$$

$N_r = N_s(1-s) = 900(1-0.04)$

$= 864$ rpm

89. (a) $N_s = \frac{120 \times 50}{4} = 1500$ rpm

But starting torque

$$T_{st} = \frac{180}{2\pi N_s} \times \frac{V^2 . r'_2}{(r_1 + r'_2)^2 + X^2}$$

$$= \frac{180}{2\pi \times 1500} \times \frac{(400/\sqrt{3})^2 \times 0.5}{(1.5)^2 + (2.4)^2}$$

$= 63.58 \approx 63.6$ N-m

90. (b) In blocked rotor test, at full load

$$\text{Power loss} = \left(\frac{20}{15}\right)^2 \times 762 = 1354.6 \text{ W}$$

Total losses = 1354.6 + 1002

= 2356.6 W

Total input = Output + Total losses

= 10000 + 2356.6

= 12356.6 W

$$\therefore \%\ \text{efficiency}\ \eta = \frac{\text{Output}}{\text{Input}} \times 100\%$$

$$= \frac{10000}{12356.6} \times 100 = 81\%$$

91. (a) Full load current $I_{FL} = \frac{5 \times 10^3}{\sqrt{3} \times 400 \times 1} = 7.21$A

For synchronous motor,

$$E^2 = (V\cos\theta - I_a R_a)^2 + (V\sin\theta - I_a X_s)^2$$

$$\therefore \quad E_1 = \sqrt{(400/\sqrt{3})^2 + (10 \times 3.6)^2} = 233.729$$

If 0.01 increase in E_1 then E_1 becomes,

$E_2 = 233.729 \times 1.01 = 236$ V

$$I_a X = \sqrt{E_2^2 - V^2} = \sqrt{(236)^2 - (400/\sqrt{3})^2}$$

$= 48.93$

$$I_a = \frac{48.93}{10} = 4.893 \text{ A}$$

$$\%\ \text{load} = \frac{4.893}{7.21} \times 100 = 67.86\%$$

92. (c) Given, ϕ/pole = 0.025 Wb

Here, slot/pole $N = \frac{48}{4} = 12$

Slot/pole/phase $m = \frac{12}{3} = 4$

Slot angle $\beta = \frac{180}{12} = 15°$

$\because \quad E_{ph} = 4.44\phi F T_{ph} \cdot K_\omega$

But $K_\omega = K_p . K_d$

$$K_p = \cos\frac{\alpha}{2} = \cos\frac{36°}{2} = 0.95$$

$$K_d = \frac{\sin\frac{m\beta}{2}}{m\sin\frac{\beta}{2}} = \frac{\sin\left(\frac{4 \times 15}{2}\right)}{4\sin\left(\frac{15}{2}\right)} = 0.966$$

$$\therefore \quad E_{ph} = 4.44 \times 0.025 \times 50 \times 160 \times 0.966 \times 0.951$$

$= 808$ V

$E_{L-L} = \sqrt{3} \times 808 = 1400$ V

93. (c) In 2ϕ winding,

Slot/pole/phase = 6

$$T_{ph} = \frac{480}{2} = 240$$

$$\text{Slot angle}\ \beta = \frac{180 \times 4}{48} = 15°$$

$$K_p = \cos\frac{\alpha}{2} = \cos\frac{36}{2} = 0.951$$

$$K_d = \frac{\sin\left(\frac{6 \times 15}{2}\right)}{6\sin\frac{15}{2}} = 0.903$$

$$E_{ph} = 4.44\phi f K_p K_d \cdot T_{ph}$$

$$= 4.44 \times 0.025 \times 50 \times 0.903 \times 0.951 \times 240$$

$= 1143$ V

$E_{line} = \sqrt{2} \times 1143 = 1616.44$

≈ 1617 V

94. (a) Here, fifth harmonic component

$= E_m \cos(5 \times 18)$

$= E_m \cos 90°$ $\quad \left[\because \alpha = \frac{180}{5} = 36°\right]$

$= 0$

95. (c) $\because$ Efficiency $\eta = \dfrac{\text{kVA} \times \text{PF} \times x}{\text{kVA} \times x \times \text{PF} \times x^2 W_{Cu} \times W_i}$

x = loading of transformer

At full load and 8 PF

$$\therefore \quad 0.95 = \frac{\text{kVA} \times 0.8}{\text{kVA} \times 0.8 + W_{Cu} + W_i} \quad \ldots(i)$$

At half load and unity PF;

$$0.96 = \frac{\text{kVA} \times 0.5}{\text{kVA} \times 0.5 + 0.25\, W_{Cu} + W_i} \quad \ldots(ii)$$

Solving Eqs. (i) and (ii), we get

$W_{Cu} = 8.51$ W and $W_i = 4.12$ W

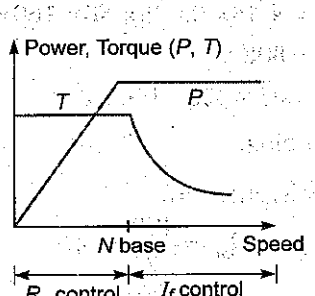
96. (b) $x = \sqrt{\dfrac{4.12}{8.51}} = 0.696$

$$\eta_{max} = \frac{0.696 \times 300}{0.696 \times 300 + 4.12 + 8.51 \times (0.696)^2}$$

$= 96.2\%$

97. (b) According to Fleming right hand rule $\otimes$ in c_1 and no emf will be in c_2.

98. (b)



(i) At half rated speed, by armature voltage control,

$$P \propto N$$

$$\therefore \quad P_2 = \frac{P_1}{2} = \frac{50}{2} = 25\,\text{kW}$$

(ii) At 1.5 times rated speed, by field control, power will remain same,

$$P_2 = P_1 = 50\ \text{kW}$$

99. (d)

Armature emf (E)	ϕ and ω
Developed torque (T)	ϕ and I_a
Developed power (P)	Flux (ϕ), speed (ω) and armature current (I_a)

Since, $E = K\phi\omega$

$T = K\phi I_a$

$P = E \cdot I_a = K\phi\omega I_a$

where, $K = \dfrac{PZ}{2\pi A} = \text{constant}$

100. (c) In synchronous machine, the d-axis synchronous reactance > q-axis synchronous reactance (in salient pole machine).

The damper winding is used to self starting of synchronous motor and V-curve is drawn between I_a and I_f at a given output power.

101. (b) By reducing the applied voltage to an induction motor, to half the rated value then its speed and stator current will be decreased.

102. (b) Here, $I_{st} = 6I_{FL}$

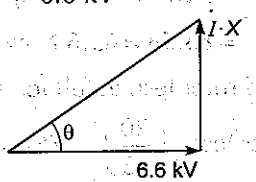
$$\therefore \quad \left(\frac{T_{st}}{T_{FL}}\right) = \left(\frac{I_{st}}{I_{FL}}\right)^2 S_{FL} = \left(\frac{6I_{FL}}{I_{FL}}\right)^2 \times 0.04$$

$$= 36 \times 0.04 = 1.44$$

103. (b) Since, high rotor resistance increases starting torque. Therefore, there will quick acceleration.

104. (a) Under V/f control of induction motor, by V/f constant, flux remains constant which keeps maximum torque constant.

105. (b) $I = \dfrac{1000\ \text{kVA}}{6.6\ \text{kV}} = 151.51$ A



$I \times R = 151.51 \times 20$

$= 3.03$ kV

$$E_{L-L} = \sqrt{(6.6)^2 + (3.03)^2} = 7.2\ \text{kV}$$

106. (b) Torque angle $\theta = \tan^{-1}\dfrac{3.03}{6.6} = 24.6°$

107. (a) Synchronous speed

$$N_s = \frac{120f}{p} = \frac{120 \times 50}{8}$$

$= 750$ rpm

The synchronous speed for seventh space harmonic mmf would be $N_s/7$ in forward direction $= \dfrac{750}{7} = 10714$ rpm in forward direction.

108. (b) With the synchronizing torque we tend to synchronize the relative rotor position with independent of rotor position. Since both rotor and stator windings have its self inductances with both independent of rotor position. Hence, it will not develop synchronizing torque.

109. (a) At no load condition,

No load losses $= 2.5\times 1.5 - (1.5)^2 \times 0.8$

$= 35.7$ W

At load condition, Variable loss

$= I^2R = 3.5^2 \times 0.8 = 9.8$ W

Total loss $= 35.7 + 9.8 = 45.5$ W

Total power $= 25\times 3.5 = 87.5$ W

Efficiency $\eta = \dfrac{87.5 - 45.5}{87.5} = 48\%$

110. (d) $V_{in} = 3300$ V

$V_{out} = 3300 + 230$

$= 3530$ V

$I_2 = \dfrac{50\times 10^3}{230} = 217.4$ A

kVA rating of auto transformer

$V_0 I_2 = 3530\times 217.4 = 767.42$ kVA

111. (d) PU impedance $= (0.02 + j0.07)$

Base impedance referred to primary $Z_{Base} = \dfrac{V_p^2}{V_p \cdot I_p}$

$= \dfrac{V_p^2}{(V_L I_L)/3} = \dfrac{11000}{(100\times 10^3)/3} = 3630\,\Omega$

$\therefore$ Phase impedance referred to primary

$Z = Z_{pu} \times Z_{Base}$

$= (0.2 + j0.7)\times(3630)$

$= (72.6 + j254.1)\,\Omega$

112. (a) Phase angle of main winding current

$\angle \vec{I}_m = \angle Z_m = -\angle(6 + j4)$

$= -\angle 33.7°$

When connect the capacitor with auxiliary winding, then the angle of this winding

$\angle \vec{I}_a = -\angle 8 + j6 - j/\omega C$

Here, $\theta = \angle \vec{I}_a - \vec{I}_m$

$90° = -\tan^{-1}\left(\dfrac{6 - 1/\omega c}{8}\right) - (-33.7°)$

$1/\omega c = 18$

$c = \dfrac{1}{18\times 2\times \pi\times 50} = 176.8$ mF

113. (a) Phase voltage of first machine

$E_{ph} = \dfrac{3300}{\sqrt{3}}$ V

Similarly, $E_{ph} = \dfrac{3200}{\sqrt{3}}$ V

Synchronizing current

$\vec{I}_c = \dfrac{\vec{E}_c}{\vec{Z}_{S_1} + \vec{Z}_{S_2}} = \dfrac{\vec{E}_{ph1} - \vec{E}_{ph2}}{Z_{S_1} + Z_{S_2}}$

$= \dfrac{100}{\sqrt{3}\,(1.7 + 1.7)} = \dfrac{100}{\sqrt{3}\times 3.4}$

$= 16.98$ A

114. (c) Synchronous speed $N_s = \dfrac{120\times f}{P}$

$= \dfrac{120\times 50}{4} = 1500$ rpm

Torque developed $= \dfrac{P}{\omega_s\,(1-s)}$

$= \dfrac{15\times 10^3}{\frac{2\pi N_s}{60}(1 - 0.04)}$

$= \dfrac{15\times 10^3\times 60}{157.07\,(1 - 0.04)}$

$= 99.47$ N-m

115. (b) Stepping rate $= 100$ step/s

Steps required for one revolution $= \dfrac{360°}{1.8} = 200$ step

$\therefore$ Time required for one revolution $= 2$ s

rev/s $= 1/2 = 0.5$ rps

rev/min $= 30$ rpm

116. (c) Induced volt $E = \dfrac{NP\phi Z}{60\,A}$

$Z =$ total number of inductors

$= 2\times 32\times 6 = 384$

$A = 2$ (wave winding)

$E = \dfrac{250\times 8\times 0.06\times 384}{60\times 2} = 384$ V

117. (c) Input power = Output power + Losses

$= 9$ kW $+ 2$ kW $+ 0.8$ kW

$= 11.8$ KW

or $\sqrt{3} V_L I_L = 11.8\times 10^3$ W

$I_L = \dfrac{11.8\times 10^3}{\sqrt{3}\times 400\times 0.8} = 21.29$ A

118. (b) Power $= 500$ MW

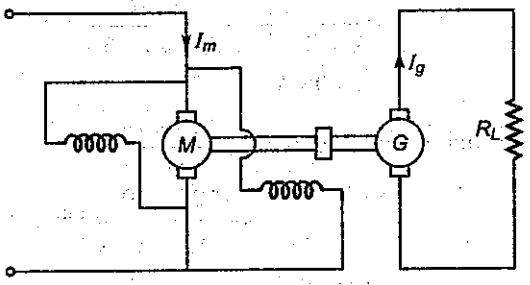
or $\sqrt{3} V_L I_L \cos\phi = 500$

$I_L = \dfrac{500}{\sqrt{3}\times 21.5\times 10^3\times 0.85}$

$= 15.79$ kA

119. (a) In the given circuit polarity of E_1, E_2 are same and currents I_1, I_2 flow in same direction but magnetic flux Φ always lagging 90° behind by induced EMF.

120. (c)



Back EMF of motor

$$E_m = V - I_m R_a$$
$$= 0.98$$

Because all values taken in pu values, there is no mechanical and armature reaction. Hence,
Back EMF of motor = Back EMF of generator

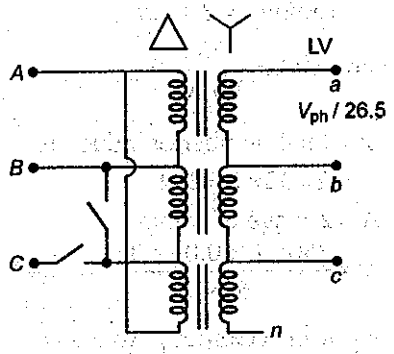
$$E_m = E_g = 0.98$$

but $$E_g = V + I_a R_a$$
$$V = 0.98 - 1 \times 0.2 = 0.96$$

but $$V = I_g R_c \Rightarrow 1 \times R_c = 0.96$$

Therefore, pu load resistance = 0.96

121. (d)



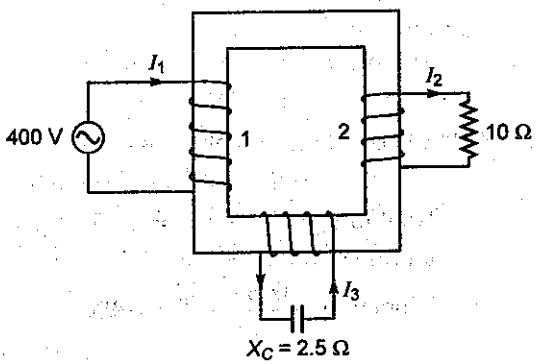
When S_1 is opened and S_2 is closed then turns ratio

$$K = \frac{V_1}{V_2} = \frac{11000}{415} = 26.5$$

Voltage in phase *a* will be opposite to that phase *c*, therefore

$$V_{ac} = \frac{V_{ph}}{26.5} - \frac{V_{ph}}{26.5} = 0 \text{ V}$$

122. (d) Here, $\frac{E_1}{E_2} = \frac{N_1}{N_2} = \frac{4}{2} = 2$



$$\therefore \quad E_2 = \frac{E_1}{2} = 200 \text{ V}$$

also $$\frac{E_1}{E_2} = \frac{N_1}{N_3}$$

$$\therefore \quad E_3 = \frac{E_1}{4} = 100 \text{ V}$$

$$\therefore \quad I_2 = \frac{E_2}{R} = \frac{200}{10} = 20 \text{ A}$$

$$I_3 = \frac{E_3}{X_L} = \frac{100}{-j\,2.5} = j\,40$$

We know that I_2 referred to primary side

$$\frac{I_1'}{I_2} = \frac{N_2}{N_1} = \frac{2}{4} \Rightarrow I_1' = \frac{I_2}{2} = \frac{20}{2} = 10 \text{ A}$$

Also I_3 referred to primary side

$$\frac{I_1''}{(-I_3)} = \frac{N_3}{N_1} = \frac{1}{4} \Rightarrow I_1'' = \frac{j\,40}{4} = -j10$$

$$\therefore \quad I_1 = I_1' + I_1'' = (10 - j\,10) \text{ A}$$

123. (a) **DC machine** Stator has salient pole and winding current is DC.
Rotor winding current is AC and it has commutator P-S.
Synchronous machine Stator is cylindrical and has armature winding, *i.e.*, rotor has salient poles and slip rings and rotor windings have field winding *i.e.*, DC current Q-T.
Induction machine Both rotor and stator windings have AC current and both may have polyphase winding R-U.

124. (c)

Stator surface *ABC* forms	Stator surface *CDA* forms	Rotor surface *abc* forms	Rotor surface *cda* forms	Torque surface
North pole	South pole	South pole	North pole	Counter-clockwise

125. (a) Pole pitch $= \frac{36}{4} = 9$ slot

$$\therefore \quad \text{Slot/pole/phase} = \frac{36}{4 \times 3} = 3$$

In one phase slots = 3
If it is chorded by 2 slots.
∴ In 3 → 2 different phases

$$36 \rightarrow \frac{36 \times 2}{3} = 24 \text{ different phases}$$

126. (b) We know that

$$V = 4.44 K_\omega N_{ph} \phi f$$

or $$\phi = \frac{1}{4.44 K_\omega N_{ph}} \times \frac{V}{f}$$

Since, ratio of V/f remains constant.
∴ ϕ_n will be constant. (air gap flux)

$$\because \quad S \propto \frac{1}{f}$$

Hence, pu slip will increase.

$$\text{Stator current } I_2 = \frac{S(f/f_0).V_0}{R_L}$$

As slip doubles and f halves.
∴ I_2 remains constant.

127. (d) Forward rotating field will be more than the backward rotating field when only the main winding is excited of 1-ϕ induction motor at synchronous speed.

128. (b) In series motor,

$$\text{Back emf } E = V - I(R_a + R_{se})$$

If $(R_a + R_{se}) = R$

Then, $E = V - IR$

where, $E = K_n \phi N$

$$N = \frac{V - IR}{K_n \phi}$$

In series motor, $\phi \propto I$
∴ At constant power load

$$E \times I = T \times \omega = \text{constant}$$

where, $T = K_a \phi I = K_a I^2$

If 'ω_n' decreases by (1/4) of its rated value, then torque will increase 4 times to maintain power constant.

$$\therefore \quad T \propto I^2$$

∴ Current increases 2 times so that supply voltage approximately brought down to 0.5 pu.

129. (b)

$$E_b = V - I_a R_a$$

$$= 200 - 10 \times 2 = 180 \text{ V}$$

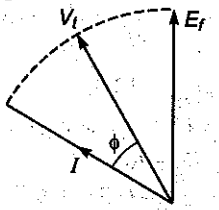
$$\because \quad E_b I_a = T\omega$$

$$\therefore \quad \text{Torque } T = \frac{E_b I_a}{\omega} = \frac{180 \times 10 \times 60}{2 \times 3.14 \times 2000}$$

$$= 4.77 \text{ N-m}$$

130. (c) Here, stator poles and rotor poles are not same. So, speed will be zero.

131. (b) When stator is not connected to supply, flux is 25 mWb and when stator is connected to supply flux is 20 mWb. Here, no load current leads by supply voltage.



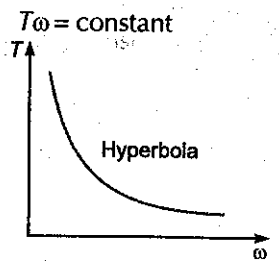
132. (c) Hysteresis loss

$$P_h = 2 \times 5 \times 50 \times f \times 10^{-3} \text{ W}$$

At 50 Hz,

$$P_h = 25 \text{ W}$$

133. (d) Power $= E_a I_a = T\omega$



134. (b) Power $P = 3VI$

$$\therefore \quad I = \frac{P}{3V}$$

$$Z_{base} = \frac{V_{base}}{I_{base}} = \frac{V(3V)}{P} = \frac{3V^2}{P}$$

$$Z_{pu} = \frac{Z_{actual}}{Z_{base}}$$

$$\Rightarrow \quad Z_{actual} = Z_{pu} \times Z_{base}$$

$$= (0.12) \times \left(\frac{3V^2}{P}\right)$$

$$= \frac{0.12 \times 3 \times (33 \times 10^3)^2}{20 \times 10^6}$$

$$\therefore \quad Z_{actual} = 19.6\,\Omega$$

135. (b) PU voltage regulation $= E_R \cos\phi_2 + E_x \sin\phi_2$

$$\frac{E_2 - V_2}{V_2} = 0.01 \times 0.8 + 0.05 \times 0.6$$

$$\therefore \quad \frac{E_2 - V_2}{V_2} = 0.038$$

$$E_2 = 1.038 \times 415 = 430.77$$

$$\frac{V_{L_1}}{V_{L_2}} = \frac{V_{P_1}}{\sqrt{3}V_{P_2}} = \frac{N_1}{\sqrt{3}N_2}$$

$$\therefore \quad \frac{N_1}{N_2} = \frac{\sqrt{3}V_{L_1}}{V_{L_2}} = \frac{\sqrt{3}\times 6000}{430.77} = 24.12$$

136. (d) $$E_2 = E_1 \times \frac{N_2}{N_1} \times \frac{I_{a_2}}{I_{a_1}} = 152\,\text{V}$$

Here, $I_{a_1} = I_{a_2} = 40$ A

$\therefore \quad 152 = 240 - I_{a_2} \times R_2 = 240 - 40 \times R_2$

$\therefore \quad R_{ex} = R_2 - R_1$

$= 2.2 - 0.3 = 1.9\,\Omega$

137. (c) Synchronous speed $N_s = \frac{120f}{p} = \frac{120\times 50}{6}$

$= 1000$ rpm

$$\text{Slip} = \frac{1000 - 975}{1000} = 0.025$$

$$\%\,\eta = \frac{36.025}{40} \times 100 = 90\%$$

138. (b) $V_0 = 1822\,\text{V}$

$$\%\ \text{Voltage regulation} = \frac{1822 - 2000}{2000} \times 100$$

$= -8.9\%$

139. (b) $E_b = V - I_a R_a$

$= 60 - 2.62 \times 2.5 = 53.45$ V

$N = \frac{53.45}{120} \times 6000 = 2673$ rpm

5

Power System

Syllabus

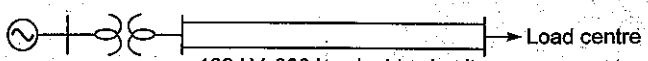
Power System *Basic power generation concepts; transmission line models and performance; cable performance, insulation; corona and radio interference; distribution systems; per unit quantities; bus impedance and admittance matrices; load flow; voltage control; power factor correction; economic operation; symmetrical components; fault analysis; principles of over-current, differential and distance protection; solid state relays and digital protection; circuit breakers; system stability concepts, swing curves and equal area criterion; HVDC transmission and FACTS concepts.*

GATE 2011 *(1 Mark Questions)*

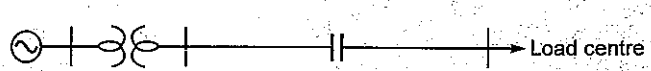
▼ *Solution Points*

1. For enhancing the power transmission in a long EHV transmission line, the most preferred method is to connect a
(a) series inductive compensator in the line
(b) shunt inductive compensator at the receiving end
(c) series capacitive compensator in the line
(d) shunt capacitive compensator at the sending end

2. A nuclear power station of 500 MW capacity is located at 300 km away from a load centre. Select the most suitable power evacuation transmission configuration among the following options.

(a) 

132 kV, 300 km double circuit

(b) 

132 kV, 300 km single circuit with 40% series capacitor compensation

(c)

400 kV, 300 km single circuit

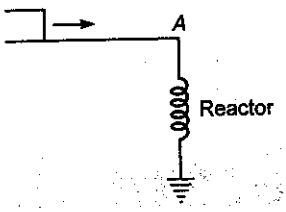
(d)

400 kV, 300 km double circuit

▼ *Solution Points*

GATE 2010

3. Consider a stop voltage wave of magnitude 1pu travelling along a lossless transmission line that terminates in a reactor. The voltage magnitude across the reactor at the instant the travelling wave reaches the reactor is



(a) –1 pu (b) 1 pu (c) 2 pu (d) 3 pu

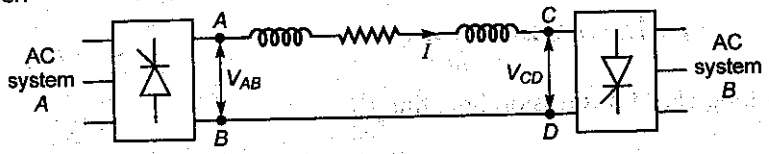
4. Consider two buses connected by an impedance of $(0 + j5)\,\Omega$. The bus 1 voltage is $100\angle 30°$ V, and bus 2 voltage is $100\angle 0°$ V. The real and reactive powers supplied by bus 1, respectively, are

(a) 1000 W, 268 VAR (b) –1000 W, –134 VAR
(c) 276.9 W, –86.7 VAR (d) –276.9 W, 56.7 VAR

5. A three-phase, 33 kV oil circuit breaker is rated 1200 A, 2000 MVA, 3 s. The symmetrical breaking current is

(a) 1200 A (b) 3600 A (c) 35 kA (d) 104.8 kA

6. Power is transferred from system *A* to system *B* by an HVDC link as shown in the figure. If the voltage V_{AB} and V_{CD} are as indicated in the figure, and $I > 0$, then



(a) $V_{AB} < 0, V_{CD} < 0, V_{AB} > V_{CD}$ (b) $V_{AB} > 0, V_{CD} > 0, V_{AB} < V_{CD}$
(c) $V_{AB} > 0, V_{CD} > 0, V_{AB} > V_{CD}$ (d) $V_{AB} > 0, V_{CD} < 0$

GATE 2009

7. Out of the following plant categories

(i) Nuclear (ii) Run-of-river
(iii) Pump storage (iv) Diesel

The base load power plants are

(a) (i) and (ii) (b) (ii) and (iii)
(c) (i), (ii) and (iv) (d) (i), (iii) and (iv)

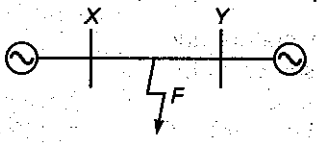
8. For a fixed value of complex power flow in a transmission line having a sending end voltage *V*, the real power loss will be proportional to

(a) V (b) V^2 (c) $1/V^2$ (d) $1/V$

GATE 2008

▼ *Solution Points*

9. A two machine power system is shown below. Transmission line XY has positive sequence impedance of $Z_1\ \Omega$ and zero sequence impedance of $Z_0\ \Omega$.



An 'a' phase to ground fault with zero fault impedance occurs at the centre of the transmission line. Bus voltage at X and line current from X to F for the phase 'a', are given by V volt and I ampere, respectively. Then, the impedance measured by the ground distance relay located at the terminal X of line XY will be given by

(a) $\frac{Z_1}{2}\Omega$ (b) $\frac{Z_0}{2}\Omega$ (c) $\frac{(Z_0+Z_1)}{2}\Omega$ (d) $\frac{V_a}{I_a}\Omega$

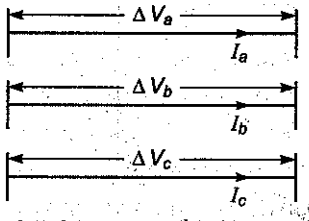
10. An extra high voltage transmission line of length 300 km can be approximated by a lossless line having propagation constant $\beta = 0.00127$ rad/km. Then the percentage ratio of line length to wavelength will be given by

(a) 24.24% (b) 12.12% (c) 19.05% (d) 6.06%

11. A 3-phase transmission line is shown in the figure. Voltage drop across the transmission line is given by the following equation

$$\begin{bmatrix}\Delta V_a\\ \Delta V_b\\ \Delta V_c\end{bmatrix} = \begin{bmatrix}Z_s & Z_m & Z_m\\ Z_m & Z_s & Z_m\\ Z_m & Z_m & Z_s\end{bmatrix}\begin{bmatrix}I_a\\ I_b\\ I_c\end{bmatrix}$$

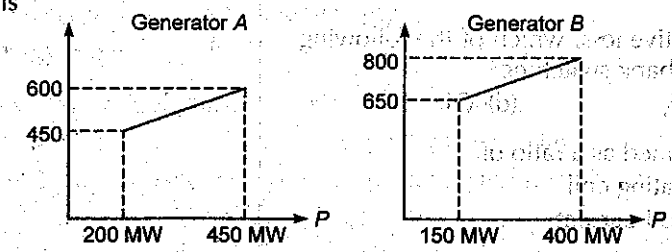
Shunt capacitance of the line can be neglected. If the line has positive sequence impedance of 15 Ω and zero sequence impedance of 48 Ω, then the values of Z_s and Z_m will be



(a) $Z_s = 31.5\ \Omega, Z_m = 16.5\ \Omega$ (b) $Z_s = 26\ \Omega, Z_m = 11\ \Omega$
(c) $Z_s = 16.5\ \Omega, Z_m = 31.5\ \Omega$ (d) $Z_s = 11\ \Omega, Z_m = 26\ \Omega$

GATE 2007

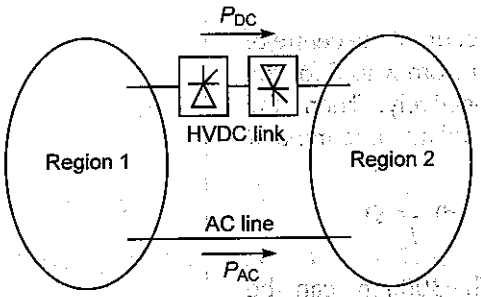
12. The incremental cost curves in ₹/MW hour for two generators supplying a common load of 700 MW are shown in the figures. The maximum and minimum generation limits are also indicated. The optimum generation schedule is



▼ *Solution Points*

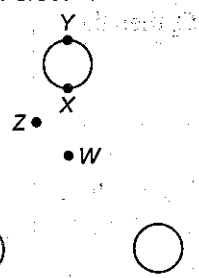
(a) Generator A : 400 MW, Generator B : 300 MW
(b) Generator A : 350 MW, Generator B : 350 MW
(c) Generator A : 450 MW, Generator B : 250 MW
(d) Generator A : 425 MW, Generator B : 275 MW

13. Two regional systems, each having several synchronous generators and loads are interconnected by an AC line and a HVDC link as shown in the figure. Which of the following statements is true in the steady state?



(a) Both regions need not have the same frequency
(b) The total power flow between the regions ($P_{AC} + P_{DC}$) can be changed by controlling the HVDC converters alone
(c) The power sharing between the AC line and the HVDC link can be changed by controlling the HVDC converters alone
(d) The direction of power flow in the HVDC link (P_{DC}) cannot be reversed

14. Consider a bundled conductor of an overhead line, consisting of three identical sub-conductors placed at the corners of an equilateral triangle as shown in the figure. If we neglect the charges on the other phase conductors and ground, and assume that spacing between sub-conductors is much larger than their radii, the maximum electric field intensity is experienced at



(a) point X (b) point Y (c) point Z (d) point W

GATE 2006

15. The concept of an electrically short, medium, and long line is primarily based on the
(a) nominal voltage of the line (b) physical length of the line
(c) wavelength of the line (d) power transmitted over the line

16. Keeping in view the cost and overall effectiveness, which of the following circuit breakers is best suited for capacitor bank switching?
(a) Vacuum (b) Air blast (c) SF_6 (d) Oil

17. In a biased differential relay, the bias is defined as a ratio of
(a) number of turns of restraining and operating coil
(b) operating coil current and restraining coil current

(c) fault current and operating coil current
(d) fault current and restraining coil current

18. An HVDC link consists of rectifier, inverter, transmission line and other equipments. Which one of the following is true for this link?
(a) The transmission line produces/supplies reactive power
(b) The rectifier consumes reactive power and the inverter supplies reactive power from/to the respective connected AC systems
(c) Rectifier supplies reactive power and the inverted consumes reactive power to/from the respective connected AC systems
(d) Both the converters (rectifier and inverter) consume reactive power from the respective connected AC systems

GATE 2005

19. The pu parameters for a 500 MVA machine on its own base are
inertia $M = 20$ pu; reactance $X = 2$ pu
The pu values of inertia and reactance on 100 MVA common base, respectively, are
(a) 4, 0.4 (b) 100, 10 (c) 4, 10 (d) 100, 0.4

20. An 800 kV transmission line has a maximum power transfer capacity of P. If it is operated at 400 kV with the series reactance unchanged, then new maximum power transfer capacity is approximately
(a) P (b) $2P$ (c) $P/2$ (d) $P/4$

21. The insulation strength of an EHV transmission line is mainly governed by
(a) load power factor (b) switching over voltages
(c) harmonics (d) corona

22. High Voltage DC (HVDC) transmission is mainly used for
(a) bulk power transmission over very long distances
(b) interconnecting two systems with the same nominal frequency
(c) eliminating reactive power requirement in the operation
(d) minimizing harmonics at the converter stations

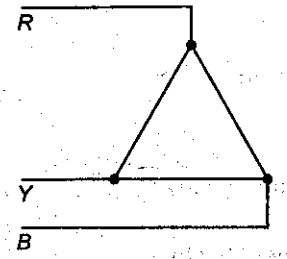
GATE 2004

23. For a linear electromagnetic circuit, which of the following statements is true?
(a) Field energy is equal to the co-energy
(b) Field energy is greater than the co-energy
(c) Field energy is lesser than the co-energy
(d) Co-energy is zero

24. The rated voltage of a 3-phase power system is given as
(a) rms phase voltage (b) peak phase voltage
(c) rms line to line voltage (d) peak line to line voltage

▼ *Solution Points*

25. The phase sequence of the 3-phase system shown in figure is



(a) RYB (b) RBY (c) BRY (d) YBR

26. In thermal power plants, the pressure in the working fluid cycle is developed by

(a) condenser (b) super heater
(c) feed water pump (d) turbine

27. For harmessing low variable waterheads, the suitable hydraulic turbine with high percentage of reaction and runner adjustable varies is

(a) Kaplan (b) Francis (c) Pelton (d) Impeller

28. The transmission line distance protection relay having the property of being inherently directional is

(a) impedance relay (b) mho relay
(c) ohm relay (d) reactance relay

GATE 2003

29. Bundled conductors are mainly used in high voltage overhead transmission lines to

(a) reduce transmission line losses
(b) increase mechanical strength of the line
(c) reduce corona
(d) reduce sag

30. A power system consists of 300 buses out of which 20 buses are generator buses, 25 buses are the ones with reactive power support and 15 buses are the ones with fixed shunt capacitors. All the other buses are load buses. It is proposed to perform a loadflow analysis for the system using Newton-Raphson method. The size of the Newton-Raphson Jacobian matrix is

(a) 553×553 (b) 540×540 (c) 555×555 (d) 554×554

31. Choose two appropriate auxiliary components of a HVDC transmission system from the following.

P. DC line inductor
Q. AC line inductor
R. Reactive power sources
S. Distance relays on DC line
T. Series capacitance on AC line

(a) P and Q (b) P and R (c) Q and S (d) S and T

32. Total instantaneous power supplied by a 3-phase AC supply to a balanced *RL* load is

(a) zero (b) constant
(c) pulsating with zero average (d) pulsating with non-zero average

▼ *Solution Points*

▼ *Solution Points*

33. The interrupting time of a circuit breaker is the period between the instants of
(a) initiation of short circuit and the arc extinction on an opening operation
(b) energizing of the trip circuit and the arc extinction on an opening operation
(c) initiation of short circuit and the parting of primary arc contacts
(d) energizing of the trip circuit and the parting of primary arc contacts

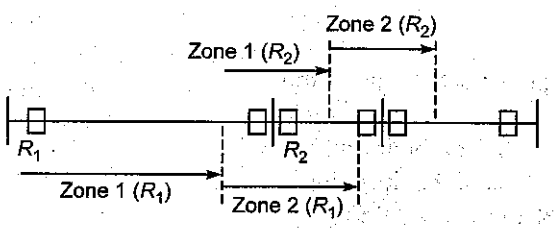
GATE 2002

34. Consider a long, two-wire line composed of solid round conductors. The radius of both conductors is 0.25 cm and the distance between their centres is 1 m. If this distance is doubled, then the inductance per unit length
(a) doubles
(b) halves
(c) increases but does not double
(d) decreases but does not halve

35. Consider a power system with three identical generators. The transmission losses are negligible. One generator (G_1) has a speed governor which maintains its speed constant at the rated value, while the other generators (G_2 and G_3) have governors with a droop of 5%. If the load of the system is increased, then in steady state
(a) generations of G_2 and G_3 are increased equally while generation of G_1 is unchanged
(b) generation of G_1 alone is increased while generations of G_2 and G_3 are unchanged
(c) generations of G_1, G_2 and G_3 are increased equally
(d) generations of G_1, G_2 and G_3 are increased in the ratio 0.5 : 0.25 : 0.25

36. A long wire composed of a smooth round conductor runs above and parallel to the ground (assumed to be a large conducting plane). A high voltage exists between the conductor and the ground. The maximum electric stress occurs at
(a) the upper surface of the conductor
(b) the lower surface of the conductor
(c) the ground surface
(d) midway between the conductor and the ground

37. Consider the problem of relay coordination for the distance relays R_1 and R_2 on adjacent lines of a transmission system (figure). The Zone 1 and Zone 2 settings for both the relays are indicated on the diagram. Which of the following indicates the correct time settings for the Zone 2 of relays R_1 and R_2



(a) $TZ2_{R_1} = 0.6$ s, $TZ2_{R_2} = 0.3$ s
(b) $TZ2_{R_1} = 0.3$ s, $TZ2_{R_2} = 0.6$ s
(c) $TZ2_{R_1} = 0.3$ s, $TZ2_{R_2} = 0.3$ s
(d) $TZ2_{R_1} = 0.1$ s, $TZ2_{R_2} = 0.3$ s

▼ *Solution Points*

GATE 2001

38. A lossless radial transmission line with surge impedance loading
(a) takes negative VAR at sending end and zero VAR at receiving end
(b) takes positive VAR at sending end and zero VAR at receiving end
(c) has flat voltage profile and unity power factor at all points along it
(d) has sending end voltage higher than receiving end voltage and unity power factor at sending end

GATE 2000

39. In a thermal power plant, the feed water coming to the economiser is heated using
(a) HP steam (b) LP steam
(c) direct heat in the furnace (d) flue gases

40. For given base voltage and base volt-ampere, the per unit impedance value of an element is x. What will be the per unit impedance value of this element when the voltage and volt-ampere bases are both doubled?
(a) $0.5x$ (b) $2x$
(c) $4x$ (d) x

41. In an inverse definite minimum time, electromagnetic type over-current relay, the minimum time feature is achieved because of
(a) saturation of the magnetic circuit
(b) proper mechanical design
(c) appropriate time delay element
(d) electromagnetic damping

42. Out of the considerations (i) to (iv) listed below,
(i) no distance limitation related to steady state stability
(ii) no reactive power requirement from the system at the two terminals
(iii) no substantial effect on fault level of the two systems at the terminals inspite of the interconnection
(iv) no corona problems
The considerations which constitute advantages of HVDC transmission are
(a) all of the above (b) (i) and (iii)
(c) (iii) and (iv) (d) (ii) and (iv)

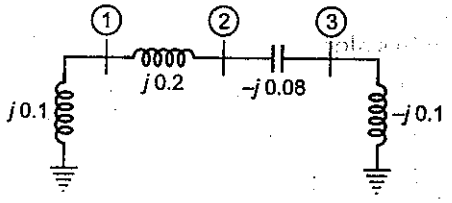
43. In a 3-step distance protection, the reach of the three zones of the relay at the beginning of the first line typically extends upto
(a) 100% of the first line, 50% of the second line and 20% of the third line
(b) 80% of the first line, 50% of the second line and 20% of the third line
(c) 80% of the first line, 20% of second line and 10% of the third line
(d) 50% of first line, 50% of second line and 20% of the third line

GATE 2011

(2 Marks Questions)

▼ *Solution Points*

44. A three-bus network is shown in the figure below indicating the per unit impedances of each element.



The bus admittance matrix, *Y*-bus, of the network is

(a) $j\begin{bmatrix} 0.3 & -0.2 & 0 \\ -0.2 & 0.12 & 0.08 \\ 0 & 0.08 & 0.02 \end{bmatrix}$ (b) $j\begin{bmatrix} -15 & 5 & 0 \\ 5 & 7.5 & -12.5 \\ 0 & -12.5 & 2.5 \end{bmatrix}$

(c) $j\begin{bmatrix} 0.1 & 0.2 & 0 \\ 0.2 & 0.12 & -0.08 \\ 0 & -0.08 & 0.10 \end{bmatrix}$ (d) $j\begin{bmatrix} -10 & 5 & 0 \\ 5 & 7.5 & 12.5 \\ 0 & 12.5 & -10 \end{bmatrix}$

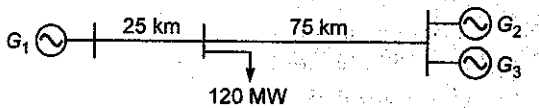
45. Solution of the variables x_1 and x_2 for the following equations is to be obtained by employing the Newton-Raphson iterative method

$$10x_2 \sin x_1 - 0.8 = 0$$
$$10x_2^2 - 10x_2 \cos x_1 - 0.6 = 0$$

Assuming the initial values $x_1 = 0.0$ and $x_2 = 1.0$, the Jacobian matrix is

(a) $\begin{bmatrix} 10 & -0.8 \\ 0 & -0.6 \end{bmatrix}$ (b) $\begin{bmatrix} 10 & 0 \\ 0 & 10 \end{bmatrix}$ (c) $\begin{bmatrix} 0 & -0.8 \\ 10 & -0.6 \end{bmatrix}$ (d) $\begin{bmatrix} 10 & 0 \\ 10 & -10 \end{bmatrix}$

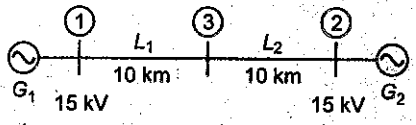
46. A load centre of 120 MW derives power from two power stations connected by 220 kV transmission lines of 25 km and 75 km as shown in the figure below. The three generators G_1, G_2 and G_3 are of 100 MW capacity each and have identical fuel cost characteristics. The minimum loss generation schedule for supplying the 120 MW load is



(a) $P_1 = 80$ MW + Losses, $P_2 = 20$ MW, $P_3 = 20$ MW
(b) $P_1 = 60$ MW, $P_2 = 30$ MW + Losses, $P_3 = 30$ MW
(c) $P_1 = 40$ MW, $P_2 = 40$ MW, $P_3 = 40$ MW + Losses
(d) $P_1 = 30$ MW + Losses, $P_2 = 45$ MW, $P_3 = 45$ MW

Statements for Linked Answer Questions 47 and 48

Two generator units G_1 and G_2 are connected by 15 kV line with a bus at the mid-point as shown below.



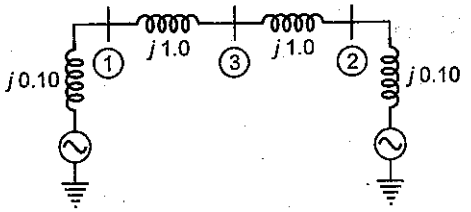
$G_1 = 250$ MVA, 15 kV, positive sequence reactance $X = 25\%$ on its own base

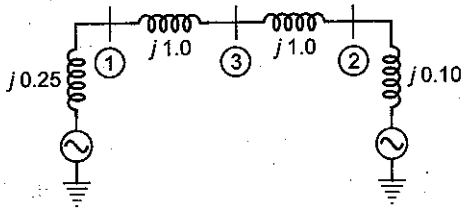
$G_2 = 100$ MVA, 15 kV, positive sequence reactance $X = 10\%$ on its own base

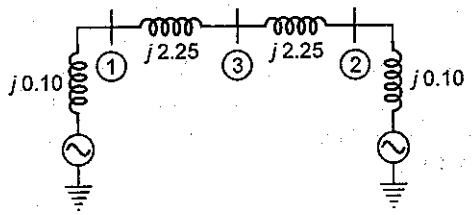
L_1 and $L_2 = 10$ km, positive sequence reactance $X = 0.225\,\Omega$/km

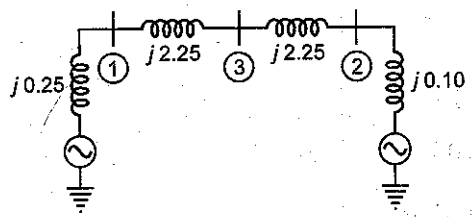
▼ *Solution Points*

47. For the above system, the positive sequence diagram with the per unit values on the 100 MVA common base is

(a)


(b)


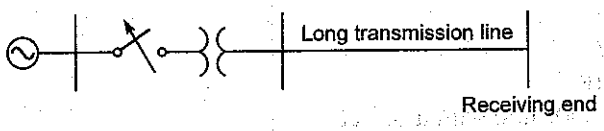
(c)


(d)


48. In the above system, the three-phase fault MVA at the bus 3 is
(a) 82.55 MVA (b) 85.11 MVA (c) 170.91 MVA (d) 181.82 MVA

GATE 2010

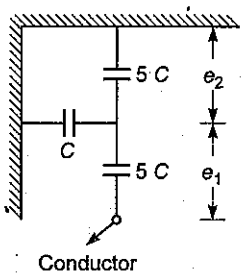
49. A 50 Hz synchronous generator is initially connected to a long lossless transmission line which is open circuited at the receiving end. With the field voltage held constant, the generator is disconnected from the transmission line. Which of the following may be said about the steady state terminal voltage and field current of the generator?



(a) The magnitude of terminal voltage decreases and the field current does not change
(b) The magnitude of terminal voltage increases and the field current does not change
(c) The magnitude of terminal voltage increases and the field current increases
(d) The magnitude of terminal voltage does not change and the field current decreases

▼ *Solution Points*

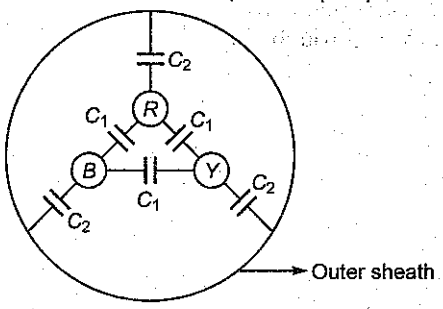
50. Consider a three-phase, 50 Hz, 11 kV distribution system. Each of the conductors is suspended by an insulator string having two identical porcelain insulators. The self capacitance of the insulator is 5 times the shunt capacitance between the link and the ground as shown in the figure. The voltages across the two insulators are



(a) $e_1 = 3.74\,kV, e_2 = 2.61\,kV$ (b) $e_1 = 3.46\,kV, e_2 = 2.89\,kV$

(c) $e_1 = 6.0\,kV, e_2 = 4.23\,kV$ (d) $e_1 = 5.5\,kV, e_2 = 5.5\,kV$

51. Consider a three-core, three-phase, 50 Hz, 11 kV cable whose conductors are denoted as *R*, *Y* and *B* in the figure. The inter-phase capacitance (C_1) between each pair of conductors is 0.2 μF and the capacitance between each line conductor and the sheath is 0.4 μF. The per-phase charging current is



(a) 2.0 A (b) 2.4 A (c) 2.7 A (d) 3.5 A

52. For the power system in the figure below, the specifications of the components are the following.

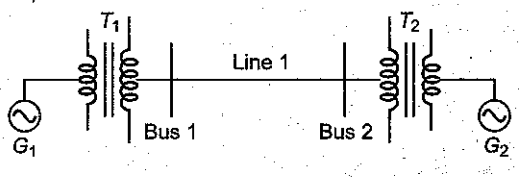
G_1 : 25 kV, 100 MVA, $X = 9\%$

G_2 : 25′ kV, 100 MVA, $X = 9\%$

T_1 : 25 kV/220 kV, 90 MVA, $X = 12\%$

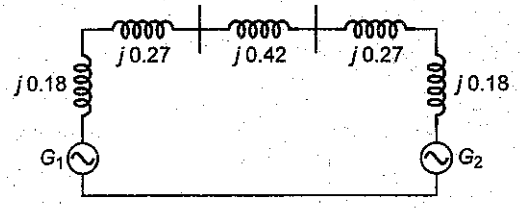
T_2 : 220 kV/25 kV, 90 MVA, $X = 12\%$

Line 1 : 220 kV, $X = 150\,\Omega$



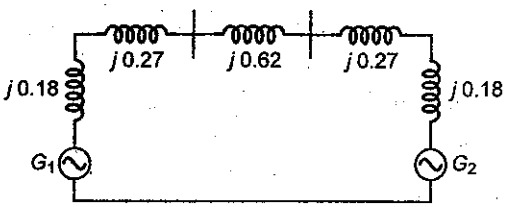
Choose 25 kV as the base voltage at the generator G_1 and 200 MVA as the MVA base. The impedance diagram is

(a)

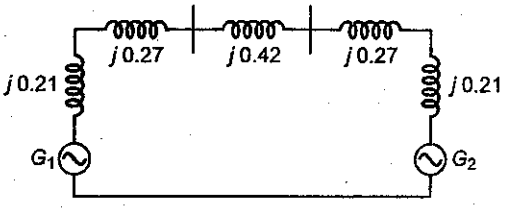


▼ *Solution Points*

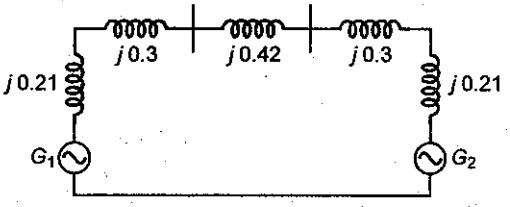
(b)



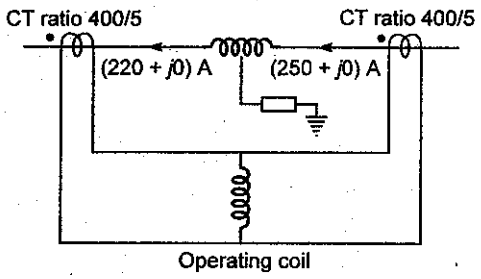
(c)



(d)

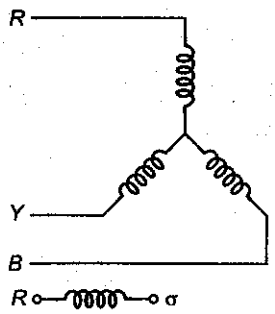


53. Consider a stator winding of an alternator with an internal high resistance ground fault. The currents under the fault condition are as shown in the figure. The winding is protected using a differential current scheme with current transformers of ratio 400/5 A as shown. The current through the operating coil is

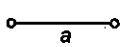


(a) 0.1875 A (b) 0.2 A (c) 0.375 A (d) 60 kA

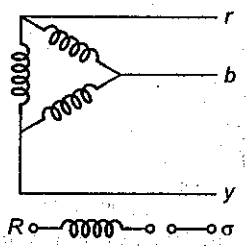
54. The zero sequence circuit of the three-phase transformer shown in the figure is

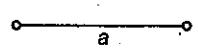


(a)

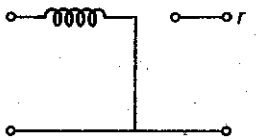


(b)

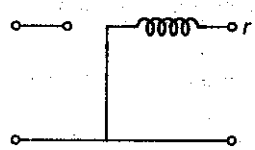




(c)



(d)



GATE 2009

▼ *Solution Points*

55. The generators are feeding a load of 100 MW. The details of the generators are

	Rating (MW)	Efficiency (%)	Regulation (pu) on 100 MVA base
Generator 1	100	20	0.02
Generator 2	100	30	0.04
Generator 3	100	40	0.03

In the event of increased load power demand, which of the following will happen?

(a) All the generators will share equal power
(b) Generator 3 will share more power compared to generator 1
(c) Generator 1 will share more power compared to generator 2
(d) Generator 2 will share more power compared to generator 3

56. A 500 MW, 21 kW, 50 Hz, 3-phase, 2-pole synchronous generator having a rated PF =0.9, has a moment of inertia of 27.5×10^3 kg-m^2. The inertia constant will be

(a) 2.44 MJ/MVA (b) 2.71 MJ/MVA (c) 4.88 MJ/MVA (d) 5.42 MJ/MVA

57. For the *Y*-bus matrix of a 4-bus system given in per unit, the buses having shunt elements are

$$Y_{BUS} = j\begin{bmatrix} -5 & 2 & 2.5 & 0 \\ 2 & -10 & 2.5 & 4 \\ 2.5 & 2.5 & -9 & 4 \\ 0 & 4 & 4 & -8 \end{bmatrix}$$

(a) 3 and 4 (b) 2 and 3 (c) 1 and 2 (d) 1, 2 and 4

58. Match the items in List I with the List II and select the correct answer using the codes given below the lists.

List I (To)	List II (Use)
A. Improves power factor	1. Shunt reactor
B. Reduces the current ripples	2. Shunt capacitor
C. Increases the power flow in line	3. Series capacitor
D. Reduces the Fernatti effect	4. Series reactor

Codes

	A	B	C	D
(a)	2	3	4	1
(b)	2	4	3	1
(c)	4	3	1	2
(d)	4	1	3	2

59. Match the items in List I with the items in List II and select the correct answer using the codes given below the lists.

List I Type of transmission line	List II Type of distance relay preferred
A. Short line	1. Ohm relay
B. Medium line	2. Reactance relay
C. Long line	3. Mho relay

Codes

	A	B	C
(a)	2	1	3
(b)	3	2	1
(c)	1	2	3
(d)	1	3	2

▼ *Solution Points*

GATE 2008

60. Voltage phasors at the two terminals of a transmission line of length 70 km have a magnitude of 1.0 per unit but are 180° out of phase. Assuming that the maximum load current in the line is $\frac{1}{5}$th of minimum 3-phase fault current, which one of the following transmission line protection schemes will not pick up for this condition?

(a) Distance protection using mho relays with zone 1 set to 80% of the line impedance
(b) Directional over-current protection set to pick up at 1.25 times the maximum load current
(c) Pilot relaying system with directional comparison scheme
(d) Pilot relaying system with segregated phase comparison scheme

61. A lossless transmission line having Surge Impedance Loading (SIL) of 2280 MW is provided with a uniformly distributed series capacitive compensation of 30%. Then, SIL of the compensated transmission line will be

(a) 1835 MW (b) 2280 MW (c) 2725 MW (d) 3257 MW

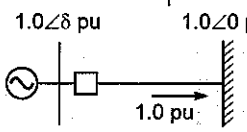
62. A lossless power system has to serve a load of 250 MW. There are two generators (G_1 and G_2) in the system with cost curves C_1 and C_2 respectively defined as follows

$$C_1(P_{G_1}) = P_{G_1} + 0.055 \times P_{G_1}^2$$
$$C_1(P_{G_2}) = 3P_{G_2} + 0.03 \times P_{G_2}^2$$

where P_{G_1} and P_{G_2} are the MW injections from generators G_1 and G_2 respectively. Then, the minimum cost dispatch will be

(a) $P_{G_1} = 250$ MW; $P_{G_2} = 0$ MW (b) $P_{G_1} = 150$ MW; $P_{G_2} = 100$ MW
(c) $P_{G_1} = 100$ MW; $P_{G_2} = 150$ MW (d) $P_{G_1} = 0$ MW; $P_{G_2} = 250$ MW

63. A lossless single machine infinite bus power system is shown below.



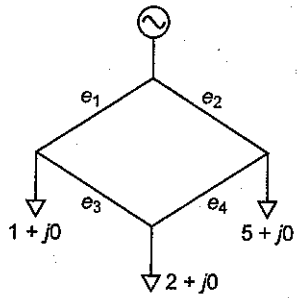
The synchronous generator transfers 1.0 per unit of power to the infinite bus. Critical clearing time of circuit breaker is 0.28 s. If another identical synchronous generator is connected in parallel to the existing generator and each generator is scheduled to supply 0.5 per unit of power. Then the critical clearing time of the circuit breaker will

(a) reduce to 0.14 s
(b) reduce but will be more than 0.14 s
(c) remain constant at 0.28 s
(d) increase beyond 0.28 s

64. Single line diagram of a 4-bus single source distribution system is shown below. Branches e_1, e_2, e_3 and e_4 have equal impedances. The load current values indicated in the figure are in per unit.

▼ *Solution Points*

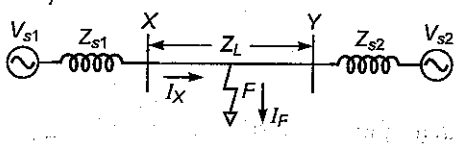
Distribution company's policy requires radial system operation with minimum loss. This can be achieved by opening of the branch



(a) e_1 (b) e_2 (c) e_3 (d) e_4

Common Data for Questions 65, 66 and 67

Consider a power system shown below :



Given that

$$V_{s1} = V_{s2} = 1.0 + j0.0 \text{ pu};$$

The positive sequence impedance are $Z_{s1} = Z_{s2} = 0.001 + j0.01$ pu and $Z_L = 0.006 + j0.06$ pu.

3-phase base MVA = 100

Voltage base = 400 kV (line to line)

Nominal system frequency = 50 Hz

The reference voltage for phase *a* is defined as $V(t) = V_m \cos(\omega t)$

A symmetrical three-phase fault occurs at centre of the line, *i.e.*, point *F* at time t_0. The positive sequence impedance from source S_1 to point *F* equals $0.004 + j0.04$ pu. The waveform corresponding to phase *a* fault current from bus *X* reveals that decaying DC offset current is negative and in magnitude at its maximum initial value. Assume that the negative sequence impedances are equal to positive sequence impedances, and the zero sequence impedances are three times positive sequence impedances.

65. The instant (t_0) of the fault will be

(a) 4.682 ms (b) 9.667 ms (c) 14.667 ms (d) 19.667 ms

66. The rms value of the AC component of fault current (I_s) will be

(a) 3.59 kA (b) 5.07 kA (c) 7.18 kA (d) 10.15 kA

67. Instead of the three-phase fault, if a single line to ground fault occurs on phase *a* at point *F* with zero fault impedance, then the rms value of the AC component of fault current (I_s) for phase *a* will be

(a) 4.97 pu (b) 7.0 pu (c) 14.93 pu (d) 29.85 pu

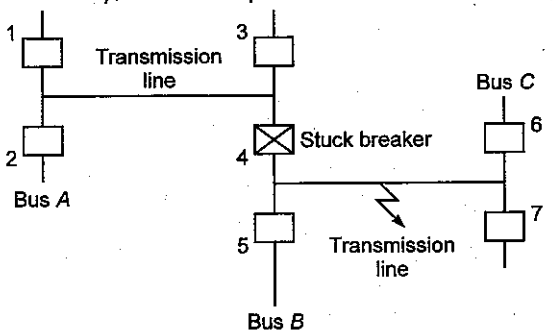
GATE 2007

68. The total reactance and total susceptance of a lossless overhead EHV line, operating at 50 Hz, are given by 0.045 pu and 1.2 pu respectively. If the velocity of wave propagation is 3×10^5 km/s, then the approximate length of the line is

(a) 122 km (b) 172 km (c) 222 km (d) 272 km

▼ *Solution Points*

69. Conside the protection system shown in the figure below. The circuit breakers, numbered from 1 to 7 are of identical type. A single line to ground fault with zero fault impedance occurs at the mid-point of the line (at point *F*), but circuit breaker 4 fails to operate (stuck breaker). If the relays are coordinated correctly, a valid sequence of circuit breaker operations is



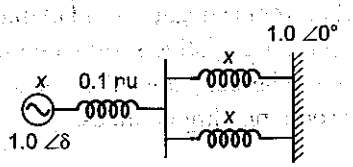
(a) 1, 2, 6, 7, 3, 5 (b) 1, 2, 5, 6, 7, 3
(c) 5, 6, 7, 3, 1, 2 (d) 5, 1, 2, 3, 6, 7

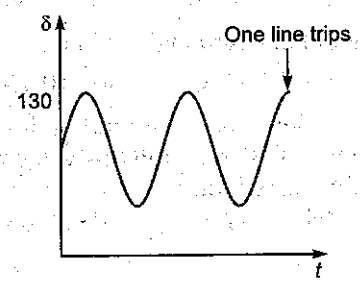
70. A three-phase balanced star connected voltage source with frequency ω rad/s is connected to a star connected balanced load which is purely inductive. The instantaneous line currents and phase to neutral voltages are denoted by (I_x, I_y, I_z) and $(V'_{sa}, V'_{ba}, V_{cn})$ respectively and their rms values are denoted by V and I.

If $R = [V_{an}, V_{bn}, V_{cn}]\begin{bmatrix} 0 & \frac{1}{\sqrt{3}} & -\frac{1}{\sqrt{3}} \\ -\frac{1}{\sqrt{3}} & 0 & \frac{1}{\sqrt{3}} \\ \frac{1}{\sqrt{3}} & -\frac{1}{\sqrt{3}} & 0 \end{bmatrix}\begin{bmatrix} I_a \\ I_b \\ I_c \end{bmatrix}$, then the magnitude of R is

(a) 3 *VI* (b) *VI* (c) 0.7 *VI* (d) 0

71. Consider a synchronous generator connected to an infinite bus by two identical parallel transmission lines. The transient reactance x of the generator is 0.1 pu and the mechanical power input to it is constant at 1.0 pu. Due to some previous disturbance, the rotor angle (δ) is undergoing an undamped oscillation, with the maximum value of $\delta(t)$ equal to 130°. One of the parallel lines trips due to relay maloperation at an instant when $\delta(1) = 130°$ as shown in the figure. The maximum value of the per unit line reactance x, such that the system does not lose synchronism subsequent to this tripping is





(a) 0.87 (b) 0.74 (c) 0.67 (d) 0.54

72. Consider the two power systems shown in Fig. *A* below, which are initially not interconnected, and are operating in steady state at the same frequency. Separate loadflow solutions are computed individually for the two systems, corresponding to this scenario. The bus voltage phasors so obtained are

▼ *Solution Points*

indicated on Fig. *A*. These two isolated systems are now interconnected by a short transmission line as shown in Fig. *B*, and it is found that $P_1 = P_2 = Q_1 = Q_2 = 0$

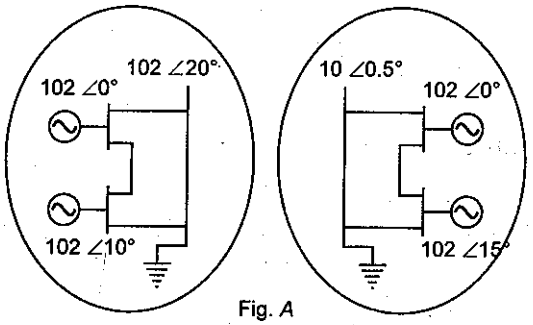


Fig. *A*

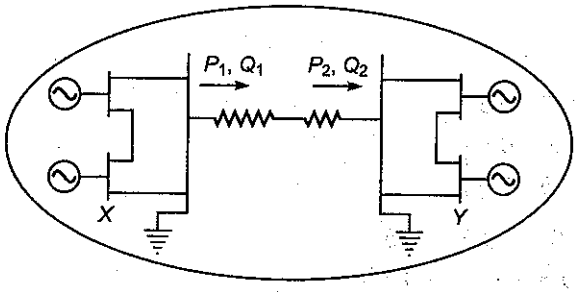


Fig. *B*

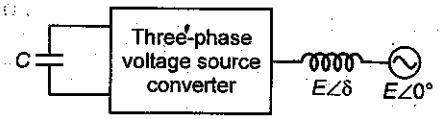
The bus voltage phase angular difference between generator bus *X* and generator bus *Y* after the interconnection is

(a) 10° (b) 25° (c) –30° (d) 30°

73. A 230 V (phase), 50 Hz, three-phase, 4-wire system has a phase sequence *ABC*. A unity power factor load of 4 kW is connected between phase *A* and neutral *N*. It is desired to achieve zero neutral current through the use of a pure inductor and a pure capacitor in the other two phases. The value of inductor and capacitor are

(a) 72.95 mH in phase *C* and 139.02 μF in phase *B*
(b) 72.95 mH in phase *B* and 139.02 μF in phase *C*
(c) 42.12 mH in phase *C* and 240.79 μF in phase *B*
(d) 42.12 mH in phase *B* and 240.79 μF in phase *C*

74. The figure below shows a three-phase self-commutated voltage source converter connected to a power system. The converter's DC bus capacitor is marked at *C* in the figure. The circuit is initially operating in steady state with $\delta = 0$ and the capacitor DC voltage is equal to V_{DC0}. You may neglect all losses and harmonics. What action should be taken to increase the capacitor DC voltage slowly to a new steady state value?



(a) Make δ positive and maintain it at a positive value
(b) Make δ positive and return it to its original value
(c) Make δ negative and maintain it at negative value
(d) Make δ negative and return it to its original value

▼ *Solution Points*

75. Suppose we define a sequence transformation between *a-b-c* and *p-n-o* variables as follows

$$\begin{bmatrix} f_a \\ f_b \\ f_c \end{bmatrix} = k \begin{bmatrix} 1 & 1 & 1 \\ \alpha^2 & \alpha & 1 \\ \alpha & \alpha^2 & 1 \end{bmatrix} \begin{bmatrix} f_p \\ f_n \\ f_0 \end{bmatrix}.$$

where, $\alpha = e^{j\frac{2\pi}{3}}$ and k is a constant. Now, if it is given that

$$\begin{bmatrix} V_p \\ V_n \\ V_0 \end{bmatrix} = \begin{bmatrix} 0.5 & 0 & 0 \\ 0 & 0.5 & 0 \\ 0 & 0 & 2.0 \end{bmatrix} \begin{bmatrix} I_p \\ I_n \\ I_0 \end{bmatrix} \text{ and}$$

$$\begin{bmatrix} V_a \\ V_b \\ V_c \end{bmatrix} = Z \begin{bmatrix} I_a \\ I_b \\ I_c \end{bmatrix}, \text{ then}$$

(a) $Z = \begin{bmatrix} 1.0 & 0.5 & 0.75 \\ 0.75 & 1.0 & 0.5 \\ 0.5 & 0.75 & 1.0 \end{bmatrix}$ (b) $Z = \begin{bmatrix} 1.0 & 0.5 & 0.5 \\ 0.5 & 1.0 & 0.5 \\ 0.5 & 0.5 & 1.0 \end{bmatrix}$

(c) $Z = 3k^2 \begin{bmatrix} 1.0 & 0.75 & 0.5 \\ 0.5 & 1.0 & 0.75 \\ 0.75 & 0.5 & 1.0 \end{bmatrix}$ (d) $Z = \frac{k^2}{3} \begin{bmatrix} 1.0 & -0.5 & -0.5 \\ -0.5 & 1.0 & -0.5 \\ -0.5 & -0.5 & 1.0 \end{bmatrix}$

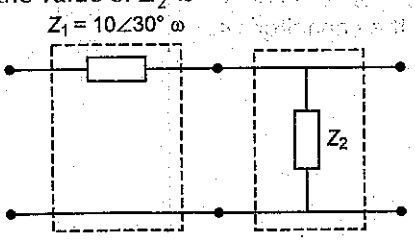
76. An isolated 50 Hz synchronous generator is rated at 15 MW which is also the maximum continuous power limit of its prime mover. It is equipped with a speed governer with 5% droop. Initially, the generator is teeding three loads of 4 MW each at 50 Hz. One of these loads is programmed to trip permanently, if the frequency falls below 48 Hz. If an additional load of 3.5 MW is connected then the frequency will settle down to

(a) 49.417 Hz (b) 49.917 Hz (c) 50.083 Hz (d) 50.583 Hz

77. A 400 V, 50 Hz three-phase balanced source supplies power to a star connected load whose rating is $12\sqrt{3}$ kVA, 0.8 PF (lag). The rating (in AR) of the delta connected (capacitive) reactive power bank necessary to bring the PF to unity is

(a) 28.78 (b) 21.60 (c) 16.60 (d) 12.47

78. Two networks are connected in cascade as shown in the figure with the usual notations, the equivalent A, B, C and D constants are obtained. Given that, $C = 0.025 \angle 45°$, the value of Z_2 is



(a) $10 \angle 30° \Omega$ (b) $40 \angle -45° \Omega$ (c) 1Ω (d) zero

GATE 2006

▼ *Solution Points*

79. The A, B, C, D constants of a 220 kV line are $A = D = 0.94\angle 1°, B = 130\angle 73°$, $C = 0.001\angle 90°$. If the sending end voltage of the line for a given load delivered at nominal voltage is 240 kV, then % voltage regulation of the line is

(a) 5 (b) 9 (c) 16 (d) 21

80. Three identical star resistors of 1.0 pu are connected to an unbalanced 3-phase supply. The load neutral is isolated. The symmetrical components of the line voltages in pu are $V_{ab1} = X\angle\theta_1, V_{ab_2} = Y\angle\theta_2$. If all the pu calculations are with the respective base values, the phase to neutral sequence voltages are

(a) $V_{an_1} = X\angle(\theta_1 + 30°), V_{an_2} = Y\angle(\theta_2 - 30°)$

(b) $V_{an_1} = X\angle(\theta_1 - 30°), V_{an_2} = Y\angle(\theta_2 + 30°)$

(c) $V_{an_1} = \frac{1}{\sqrt{3}} X\angle(\theta_1 - 30°), V_{an_2} = \frac{1}{\sqrt{3}} Y\angle(\theta_2 - 30°)$

(d) $V_{an_1} = \frac{1}{\sqrt{3}} X\angle(\theta_1 - 60°), V_{an_2} = \frac{1}{\sqrt{3}} Y\angle(\theta_2 - 60°)$

81. A generator is connected through a 20 MVA, 13.8/138 kV step-down transformer to a transmission line. At the receiving end of the line a load is supplied through a step down transformer of 10 MVA, 138/69 kV rating. A 0.72 pu load, evaluated on load side transformer ratings as base values, is supplied from the above system. For system base values of 10 MVA and 69 kV in load circuit, the value of the load (in per unit) in generator circuit will be

(a) 36 (b) 1.44 (c) 0.72 (d) 0.18

82. The Gauss-Seidel load flow method has following disadvantages. Tick the incorrect statement.

(a) Unreliable convergence
(b) Slow convergence
(c) Choice of slack bus affects convergence
(d) A good initial guess for voltage is essential for convergence

Common Data for Questions 83 and 84

A generator feeds power to an infinite bus through a double circuit transmission line. A 3_3 phase fault occurs at the middle point of one of the lines. The infinite bus voltage is 1 pu, the transient internal voltage of the generator is 1.1 pu and the equivalent transfer admittance during fault is 0.8 pu. The 100 MVA generator has an inertia constant of 5 MJ/MVA and it was delivering 1.0 pu power prior of the fault with rotor power angle of 30°. The system frequency is 50 Hz.

83. The initial accelerating power (in pu) will be

(a) 1.0 (b) 0.6 (c) 0.56 (d) 0.4

84. If the initial accelerating power is X pu, the initial acceleration in elect-deg/s^2, and the inertia constant in MJ-s/elect-deg respectively will be

(a) 31.4 X, 18 (b) 1800 X, 0.056
(c) X/1800, 0.056 (d) X/31.4, 18

Statements for Linked Answer Questions 85 and 86

For a power system the admittance and impedance matrices for the fault studies are as follows

$$Y_{bus} = \begin{bmatrix} -j8.75 & j1.25 & j2.50 \\ j1.25 & -j6.25 & j2.50 \\ j2.50 & -j2.50 & -j5.00 \end{bmatrix}$$

$$Z_{bus} = \begin{bmatrix} j0.16 & j0.08 & j0.12 \\ j0.08 & j0.24 & j0.16 \\ j0.12 & j0.16 & j0.34 \end{bmatrix}$$

The pre-fault voltages are 1.0 pu at all the buses. The system was unloaded prior to the fault. A solid 3-phase fault takes place at bus 2.

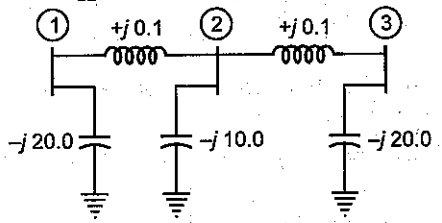
85. The post fault voltages at buses 1 and 3 in per unit respectively are
(a) 0.24, 0.63 (b) 0.31, 0.76 (c) 0.33, 0.67 (d) 0.67, 0.33

86. The per unit fault feeds from generators connected to buses 1 and 2 respectively are
(a) 1.20, 2.51 (b) 1.55, 2.61 (c) 1.66, 2.50 (d) 5.00, 2.50

87. The parameters of a transposed overhead transmission line are given as Self reactance $x_s = 0.4\Omega$/km and Mutual reactance $x_m = 0.1\Omega$/km.
The positive sequence reactance x_1 and zero sequence reactance x_0, respectively, in Ω/km are
(a) 0.3, 0.2 (b) 0.5, 0.2 (c) 0.5, 0.6 (d) 0.3, 0.6

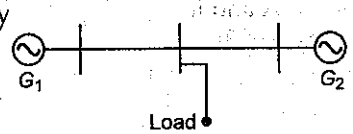
88. At an industrial sub-station with a 4 MW load, a capacitor of 2 MVAR is installed to maintain the load power factor at 0.97 lagging. If the capacitor goes out of service, the load power factor becomes
(a) 0.85 (b) 1.00 (c) 0.80 lag (d) 0.90 lag

89. The network shown in the given figure has impedances in pu as indicated. The diagonal element Y_{22} of the bus admittance matrix Y_{BUS} of the network is



(a) $-j19.8$ (b) $+j20.0$ (c) $+j0.2$ (d) $-j19.95$

90. A load centre is at an equidistant from the two thermal generating stations G_1 and G_2 as shown in the figure. The fuel cost characteristics of the generating stations are given by



$F_1 = a + bP_1 + cP_1^2$ ₹/h

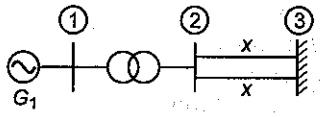
$F_2 = a + bP_2 + 2cP_2^2$ ₹/h

where, P_1 and P_2 are the generations in MW of G_1 and G_2 respectively.
For most economic generation to meet 300 MW of loads P_1 and P_2 respectively, are
(a) 150, 150 (b) 100, 200 (c) 200, 100 (d) 175, 125

▼ *Solution Points*

Points ▼ Solution Points

91. A generator with constant 1.0 pu terminal voltage supplies power through a step-up transformer of 0.12 pu reactance and a double-circuit line to an infinite bus bar as shown in the figure. The infinite bus voltage is maintained at 1.0 pu Neglecting the resistances are susceptances of the system, the steady state stability power limit of the system is 6.25 pu. If one of the double-circuit is tripped, then resulting steady state stability power limit in pu will be



(a) 12.5 pu (b) 3.125 pu (c) 10.0 pu (d) 5.0 pu

Statements for Linked Answer Questions 92 and 93

At a 220 kV sub-station of a power system, it is given that the three-phase fault level is 4000 MVA and single-line to ground fault level is 5000 MVA. Neglecting the resistance and the shunt susceptances of the system.

92. The positive sequence driving point reactance at the bus is
(a) 2.5 Ω (b) 4.033 Ω (c) 5.5 Ω (d) 12.1 Ω

93. The zero sequence driving point reactance at the bus is
(a) 2.2 Ω (b) 4.84 Ω (c) 18.18 Ω (d) 22.72 Ω

GATE 2004

94. A 800 kV transmission line is having per phase line inductance of 1.1 mH/km and per phase line capacitance of 11.68 nF/km. Ignoring the length of the line, its ideal power transfer capability in MW is
(a) 1204 MW (b) 1504 MW (c) 2085 MW (d) 2606 MW

95. A 110 kV, single core coaxial, XLPE insulated power cable delivering power at 50 Hz, has a capacitance of 125 nF/km. If the dielectric loss tangent of XLPE is 2×10^{-4}, then dielectric power loss in this cable in W/km is
(a) 5.0 (b) 31.7 (c) 37.8 (d) 189.0

96. A lightning stroke discharges impulse current of 10 kA (peak) on a 400 kV transmission line having surge impedance of 250 Ω. The magnitude of transient over-voltage travelling waves in either direction assuming equal distribution from the point of lightning strike will be
(a) 1250 kV (b) 1650 kV (c) 2500 kV (d) 2900 kV

97. The generalized circuit constants of a 3-phase, 220 kV rated voltage, medium length transmission line are

$$A = D = 0.936 + j0.016 \approx 0.936\,(0.98°)\text{ !}$$
$$B = 33.5 + j138 = 142.0\,(76.4°)\text{ !}\Omega$$
$$C = (-5.18 + j9.14)\times10^{-4}\,\Omega$$

If the load at the receiving end is 50 MW at 220 kV with a power factor of 0.9 lagging, then magnitude of line to line sending end voltage should be
(a) 133.23 kV (b) 220.00 kV (c) 230.78 kV (d) 246.30 kV

98. A new generator having $E_g = 1.4\,(30°)$ pu [equivalent to $(1.212 + j0.70)$ pu] and synchronous reactance X of 1.0 pu on the system base, is to be

▼ *Solution Points*

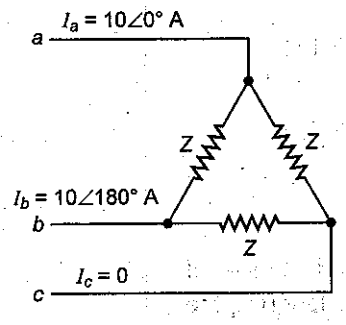
connected to a bus having voltage V_1 in the existing power system. This existing power system can be represented by Thevenin's voltage $E_{Th} = 0.9\ 0°$!pu in series with Thevenin's impedance $Z_{Th} = 0.25\ 0°$!pu. The magnitude of the bus voltage V_t of the system in pu will be

(a) 0.990 (b) 0.973 (c) 0.963 (d) 0.900

99. A 3-phase generator rated at 110 MVA, 11 kV is connected through circuit breakers to a transformer. The generator is having direct axis sub-transient reactance $X_t'' = 19\%$, transient reactance $X_a' = 36\%$ and synchronous reactance =130%. The generator is operating at no load and rated voltage when a three-phase short circuit fault occurs between the breakers and the transformer. The magnitude of initial symmetrical rms current in the breakers will be

(a) 4.44 kA (b) 22.20 kA (c) 30.39 kA (d) 38.45 kA

100. A 3-phase transmission line supplies Δ-connected load Z. The conductor 'c' of the line develops an open circuit fault as shown in figure. The currents in the lines are as shown on the diagram. The +ve sequence current component in line 'a' will be



(a) 5.78 −30°! (b) 5.78 90°! (c) 6.33 90°! (d) 10.00 (−30°)!

101. A 500 MVA, 50 Hz, 3-phase turbo-generator produces power at 22 kV. Generator is Y-connected and its neutral is solidly grounded. Its sequence reactances are $X_1 = X_2 = 0.15$ and $X_a = 0.05$ pu. It is operating at rated voltage and disconnected from the rest of the system (no load). The magnitude of the sub-transient line current for single line to ground fault at the generator terminal in pu will be

(a) 2.851 (b) 3.333 (c) 6.667 (d) 8.553

102. A 50 Hz, 4-pole, 500 MVA, 22 kV turbo-generator is delivering rated MVA at 0.8 power factor. Suddenly a fault occurs reducing is electric power output by 40%. Neglect losses and assume constant power input to the shaft. The accelerating torque in the generator in MNm at the time of the fault will be

(a) 1.528 (b) 1.018 (c) 0.848 (d) 0.509

103. A hydraulic turbine having rated speed of 250 rpm is connected to a synchronous generator. In order to produce power at 50 Hz, the number of poles required in the generator are

(a) 6 (b) 12 (c) 16 (d) 24

Answers with Explanations

1. (c)

2. (d) The most suitable configuration as choice (d) because it has long transmission line so voltage should be 400 kV and double circuit should be preferred.

3. (c)

4. (a) $P_1 = \frac{V_1 V_2}{X}\sin\delta = \frac{100\times 100}{5}\sin(30°) = 1000\ \text{W}$

$$Q = \frac{V_1 V_2}{X} - \frac{V_1 V_2}{X}\cos(\delta)$$

$$= \frac{100\times 100}{5} - \frac{100\times 100}{5}\cos(30°)$$

$$= 2000 - 2000\times 0.866 = 267.95\ \text{VAR}$$

5. (c) The symmetrical breaking current

$$I_{\text{symmetrical}} = \frac{2000}{\sqrt{3}\times 33} = 35\ \text{kA}$$

6. (c) As we know that, the current flows from high potential point to low potential point as well as power has same property so that for transferring power from system A to system B, V_{AB} should be greater than V_{CD} ($V_{AB} > V_{CD}$ and $V_{AB} > 0, V_{CD} > 0$).

7. (c) In pump storage scheme, the peak load power plants are used.

8. (c) The voltage $V = |V| < \delta$

and current $I = |I| < (\delta - \theta)$

Complex power, $S = VI^*$

$$= |V||I| < \theta$$

$$\Rightarrow \quad |I| = S/|V|$$

The power loss $= I^2R$

$$= \frac{S^2}{|V|^2}R$$

$$\propto \frac{1}{|V|^2}$$

9. (d) In distance relay, the voltage is restraining quantity and current is operating quantity. And relay of this type is also called an impedance relay as line impedance is proportional to distance.

$$\text{Impedance} = \frac{\text{Voltage}}{\text{Current}} = \frac{V_a}{I_a}\Omega$$

10. (d) Given, $\beta = 0.00127$

The wavelength

$$\lambda = \frac{2\pi}{\beta} = \frac{2\pi}{0.00127} = 4947.39\ \text{km}$$

$$\text{Ratio \%} = \frac{\text{Line length}}{\text{Wavelength}} = \frac{300\ \text{km}}{4947.39\ \text{km}}\times 100\% = 6.06\%$$

11. (b) $\begin{bmatrix}\Delta V_a\\ \Delta V_b\\ \Delta V_c\end{bmatrix} = \begin{bmatrix}Z_s & Z_m & Z_m\\ Z_m & Z_s & Z_m\\ Z_m & Z_m & Z_s\end{bmatrix}\begin{bmatrix}I_a\\ I_b\\ I_c\end{bmatrix}$

Positive sequence impedance

$$= Z_s - Z_m = 15\,\Omega \qquad \ldots(\text{i})$$

Zero sequence impedance

$$= Z_s + 2Z_m = 48\,\Omega \qquad \ldots(\text{ii})$$

Solving Eqs. (i) and (ii), we get

$$Z_s = 26, Z_m = 11$$

12. (c) From given figure,

Maximum incremental cost of generator $A = 600$

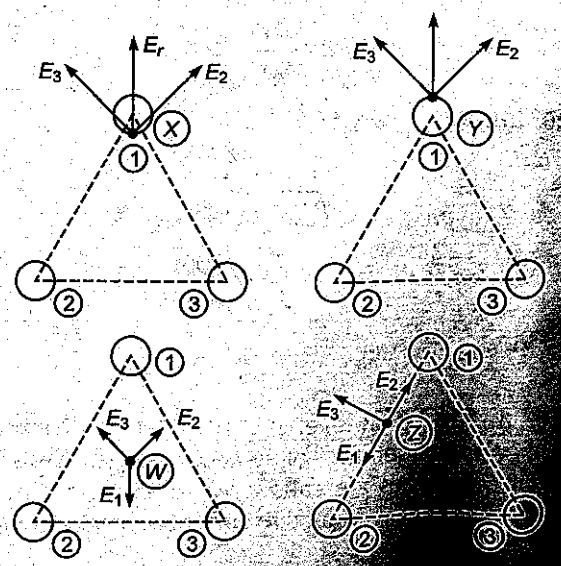
Minimum incremental cost of generator $B = 650$

As maximum incremental cost of generator $A <$ minimum incremental cost of generator B.

Hence, for optimum generation schedule generator A has to operate at 450 MW and generator B has to operate at $(700 - 450) = 250$ MW.

13. (c) Control on DC voltage is exercised by converter control angle. Hence, power sharing between AC line and HVDC link can be changed by controlling the HVDC converter only.

14. (a)



The electric field intensity at (*W*) and (*Z*) will be less as some components get cancelled whereas for point (*X*) and (*Y*) it will be nearly same but for *X* the distance will be small. Hence, electric field will be maximum.

15. (b) The concept of electrically short, medium and long line is primarily based on the physical length of the line.
For physical length less than 80 km → short line
Between 80 km to 160 km → medium line
Above 160 km → long line.

16. (a) Because of short gap and excellent recovery characteristics of vacuum breakers, they can be used where the switching frequency is high and required to be reliable and maintenance required is also least. Hence, suitable for capacitor bank switching.

17. (b) In biased differential relay restraining coil is called a biased coil as it provides additional flux and ratio of differential operating current to average restraining current is called bias.

18. (b) In HVDC link, the rectifier consumes reactive power and the inverter supplies reactive power from/to the respective connected system.

19. (d) $$M_{\text{pu new}} = M_{\text{pu old}} \times \frac{\text{Old rating MVA}}{\text{New rating MVA}}$$

$$= 20 \times \frac{500}{100} = 100 \text{ pu}$$

$$X_{\text{pu new}} = X_{\text{pu old}} \times \frac{\text{New rating MVA}}{\text{Old rating MVA}}$$

$$= 2 \times \frac{100}{500} = 0.4$$

20. (d) As $P_{max} \propto V^2$

$$\Rightarrow \quad \frac{P_{1,\max}}{P_{2,\max}} = \frac{V_1^2}{V_2^2}$$

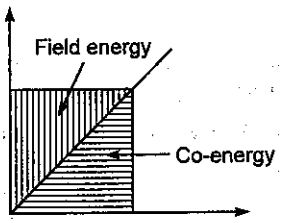
$$\Rightarrow \quad \frac{P}{P_2} = \frac{(800)^2}{(400)^2} = 4$$

$$\Rightarrow \quad P_2 = \frac{P}{4}$$

21. (b) Insulation level of EHV system is determined from consideration of switching surges. Switching over voltages are caused by switching of lines with no load/low inductive current or short circuit etc.

22. (a) HVDC transmission is mainly economical for long distance bulk power transmission by overhead lines (reduce tower and cable cost).

23. (a) For linear electromagnetic circuit.



Field energy = Co-energy.

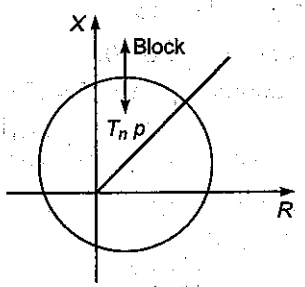
24. (c) The rated voltage of the three-phase power system is given as rms line-to-line voltage.

25. (b) The phase sequence of the given system is RBY.

26. (c) In thermal power plant, condensate is recirculated by feed water pump. Hence, pressure in the working fluid cycle is developed by feed water pump.

27. (a) Kaplan turbine is suitable for low variable waterhead (3-15 head), also called run-of-river and pondage station with head of 61 m. This type has an axial flow rotor with variable pitch blade.

28. (b) The impedance relay is inherently non-directional *i.e.*, it does not distinguish between $Z < \theta$ and $Z < (\theta + \pi)$. To make impedance relay see in one direction only, a modified impedance relay called mho relay results if a directional relay is restrained by voltage.



R-X diagram of mho relay

29. (c) In bundled conductor increased diameter results in reduced electrical stress at conductor surface for a given voltage so that the line is corona-free.

30. (b)

31. (a) The DC line inductor and reactive power sources are the auxiliary components of a HVDC transmission.

32. (b) Total instantaneous power supplied is

$$P = V_a I_a + V_b I_b + V_c I_c$$
$$= [V_m \sin \omega t][I_m \sin(\omega t - \phi)] + [V_m \sin(\omega t - 120)]$$
$$= I_m \sin(\omega t - 120 - \phi)$$
$$= VI\,[\cos\phi - \cos(2\omega t - \phi) + \cos\phi - \cos(2\omega t - 240 - \phi)]$$
$$+ \cos\phi - \cos(2\omega t + 240 - \phi)]$$
$$P = 3VI\cos\phi$$

It implies that total instantaneous power is constant.

33. (b) The interrupting time of a circuit breaker is the time elapsed between the instant of closure of trip circuit and the instant of final current zero, *i.e.*, the arc excitation on an opening operation. It is of the order of few cycle.

34. (c) The inductance per unit length $= 2\times10^{-7} \ln\frac{D}{r}$

Hence, $\frac{L_2}{L_1} = \frac{\ln 2D/r}{\ln D/r}$

i.e. $L_2 > L_1$

increased but doesn't double.

35. (b) If load is increased generation of G_1 increases while G_2 and G_3 are unchanged.

36. (b) The maximum electrical stress occurs at the lower surface of the conductor.

37. (a)

38. (c) Surge Impedance Loading (SIL) of transmission line is defined as the power delivered by line to purely resistive load equal in value to the surge impedance of the line, *i.e.*, has flat voltage profile and unity power factor at all point along it.

39. (d) In a thermal power plant, the feed water coming to the economiser is heated using flue gasses.

40. (a) $X_{new} = X_{old} \times \frac{(XVA)_{new}}{(KVA)_{old}} \times \frac{(KV)^2_{old}}{(KV)^2_{new}}$

$$= X \cdot \frac{2}{4} = 0.5x$$

41. (c) This type of relay is not instantaneous and provided delay with delaying element such as drag magnet, back-stop arrangement etc.

42. (b) Reactive power compensation required at two terminal and corona problem exist in HVDC transmission.

43. (c) In a 3-step distant protection, the reach of the three zones of the relay at begining of the first time typically extends upto 80% of the first line, 20% of the second line and 10% of the third line.

44. (b) Here, $Y_{11} = \frac{1}{j0.1} + \frac{1}{j0.2} = -j15$

$$Y_{12} = \frac{1}{j0.2} = -j5$$
$$Y_{13} = 0$$

45. (b) Here, $10x_2 \sin x_1 - 0.5 = 0$ ...(i)

$$10x_2^2 - 10x_2 \cos x_1 - 0.6 = 0 \quad \text{...(ii)}$$

$$j = \begin{bmatrix} \frac{\partial 1}{\partial x_1} & \frac{\partial 1}{\partial x_2} \\ \frac{\partial 11}{\partial x_1} & \frac{\partial 11}{\partial x_2} \end{bmatrix} \text{ at } x_1 = 0 \text{ and } x_2 = 1$$

$$j = \begin{bmatrix} 10 & 0 \\ 0 & 10 \end{bmatrix}$$

46. (a) $\because$ Loss $\propto P^2$

$\therefore$ Loss $\propto$ Length

47. (a) Here, $MVA_{base} = 100$ MVA

$$kV_{base} = 15\,kV$$
$$X_{L_1} = 0.225\times10 = 2.25\,\Omega$$
$$X_{L_2} = 0.225\times = 2.25\,\Omega$$

For generator G_1,

$$X_{G_1} = 0.25\times\frac{100}{250} = j\,0.1\,pu$$

For generator G_2,

$$X_{G_2} = 0.1\times\left(\frac{100}{100}\right) = 0.1\,pu$$

For transmission lines L_1 and L_2,

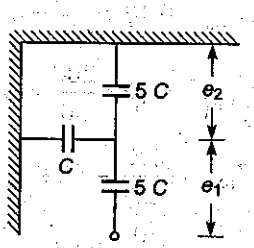
$$Z_{base} = \frac{(15)^2}{100} = 2.25\,\Omega$$
$$X_{TL_1} = \frac{2.25}{2.25} = 1\,pu$$
$$X_{TL_2} = 1\,pu$$

48. (d) $X_{Th} = 1.1 \| 1.1 = 0.55$

$$\text{Fault MVA} = \frac{\text{Base MVA}}{X_{Th}} = \frac{100}{0.55}$$
$$= 181.82\,\text{MVA}$$

49. (b)

50. (b)



$$\text{Let } m = \frac{\text{Capacitance per insulator}}{\text{Capacitance to ground}} = \frac{5C}{C} = 5$$

Then we can write

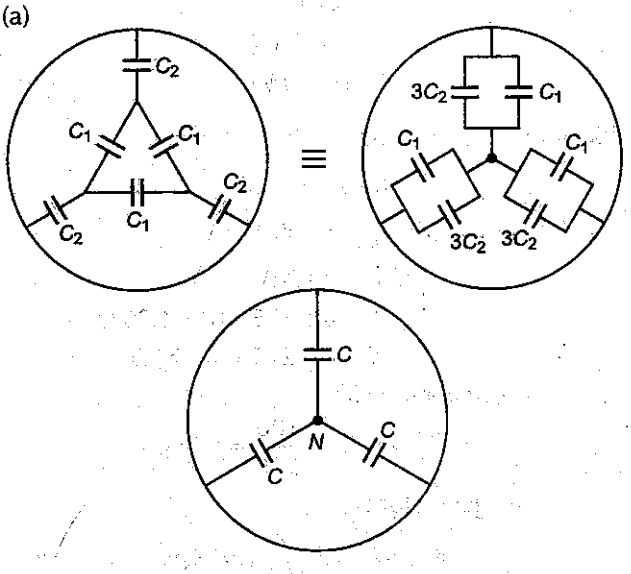
$$e_1 = \left(1+\frac{1}{m}\right)e_2 = \left(1+\frac{1}{5}\right)e_2 = 1.2e_2$$

Given, $e_1 + e_2 = \frac{11}{\sqrt{3}} \Rightarrow 1.2e_2 + e_2 = \frac{11}{\sqrt{3}}$

$\Rightarrow \quad e_2 = 2.89$ kV

$e_1 = 1.2e_2 = 1.2 \times 2.89 = 3.46$ kV

51. (a)



$C = C_1 + 3C_2 = 0.4 + 3 \times 0.2 = 1\mu F$

Given, $V_{Ph} = \frac{11}{\sqrt{3}} \times 10^3$

$$I_{Ph} = V_{Ph}\,\omega C = \frac{11}{\sqrt{3}} \times 10^3 \times 2 \times \pi \times 50 \times 1 \times 10^{-6}$$

$$\simeq 2 \text{ A}$$

52. (b) Given, base voltage 25 kV and 200 MVA

$$X_{g1\,pu} = 0.09 \times \frac{200}{100} = 0.18$$

$$X_{g2\,pu} = 0.09 \times \frac{200}{100} = 0.18$$

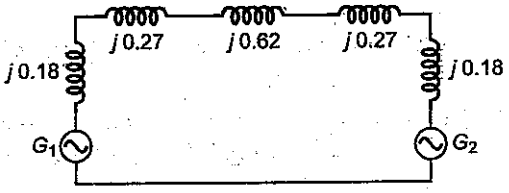
$$X_{T_1\,pu} = 0.12 \times \frac{200}{90} = 0.27$$

$$X_{T_2\,pu} = 0.12 \times \frac{200}{90} = 0.27$$

$$Z_{base} = \frac{(220)^2}{200} = 242\Omega$$

$$X_{line\,pu} = \frac{150}{242} = 0.62$$

Hence, impedance diagram



53. (c) $I_{operating} = \frac{250}{400/5} - \frac{220}{400/5}$

$= 0.375$ A

54. (c)

55. (c) Since, generator having better regulation will share more power. Therefore, generator 1 will share more power than generator 2.

56. (a) $G = \frac{P}{\cos\phi} = \frac{500}{0.9}$ MW

$= 555.56$ MVA

$$\text{KE stored} = \frac{1}{2}M \times \left(\frac{2\pi N}{60}\right)^2$$

$$= \frac{1}{2} \times 27.5 \times 10^3 \times \left(\frac{2\pi \times 3000}{60}\right)^2$$

$= 1357.07$ MJ

$$A = \frac{KE}{G} = \frac{1357.07}{555.56}$$

$= 2.44$ MJ/MVA

57. (c) The admittance matrix

$$Y_{BUS} = \begin{bmatrix} Y_{11} & Y_{12} & \cdots & Y_{1n} \\ Y_{21} & Y_{22} & \cdots & Y_{2n} \\ \vdots & \vdots & \vdots & \vdots \\ Y_{n1} & Y_{n2} & \cdots & Y_{nn} \end{bmatrix}_{n \times n}$$

$$= 9\begin{bmatrix} 5 & 2 & 2.5 & 0 \\ 2 & -10 & 2.5 & 4 \\ 2.5 & 2.5 & -9 & 4 \\ 0 & 4 & 4 & -8 \end{bmatrix}$$

The diagonal elements of Y_{BUS} are self-admittance and given by

$$Y_{11} = Y_{10} + Y_{12} + Y_{13}$$

$$Y_{22} = Y_{20} + Y_{12} + Y_{23} + Y_{24}$$

$$Y_{33} = Y_{30} + Y_{13} + Y_{23} + Y_{34}$$

$$Y_{44} = Y_{40} + Y_{24} + Y_{34}$$

The off-diagonal elements of Y_{BUS} are transfer admittance and given by

$$Y_{12}=Y_{21}=-Y_{12}$$
$$Y_{13}=Y_{31}=-Y_{13}$$
$$Y_{14}=Y_{41}=0$$
$$Y_{23}=Y_{32}=-Y_{23}$$
$$Y_{24}=Y_{42}=-Y_{24}$$
$$Y_{34}=Y_{43}=-Y_{34}$$

Now,

$$Y_{10}=Y_{11}-Y_{12}-Y_{13}-Y_{14}$$
$$=-5j+2j+2.5j+0j=-0.5j$$
$$Y_{20}=Y_{22}-Y_{12}-Y_{23}-Y_{24}$$
$$=-10j+2j+2.5j+4j=-1.5j$$
$$Y_{30}=Y_{33}-Y_{31}-Y_{32}-Y_{34}$$
$$=-9j+2.5j+2.5j+4j=0$$
$$Y_{40}=Y_{44}-Y_{41}-Y_{42}-Y_{43}$$
$$=-8j-0+4j+4j=0$$

if $Y_{ik}=0$. The i th bus and k th bus are not connected to ϕ. Hence, the buses 1 and 2 are connects and hence, having shunt elements.

58. (b) By placing a capacitor in parallel with the load, the receiving end power factor improves (from 0.707 lag + 0.911 lag). By connecting reactor in series, the current ripples can be reduced. Series capacitor increases power flow in line and shunt reactor reduces the Fernatti effect.

59. (c) Short line → Ohm relay

Medium line → Reactance relay

Long line → Mho relay

60. (a) Distance protection using mho relays with Zone 1 set to 80% of line impedance will not provide protection.

61. (b) Surge Impedance Loading (SIL) of a transmission line is defined as the power delivered by a line to purely resistive load equal in value to surge impedance of line and capacitve compensation for series capacitance is defined as $X_C/X_L=0.3$ that has no effect on SIL. Hence, for compensated transmitted line SIL is 2280 MW.

62. (c) $C_1(P_{G_1})=P_{G_1}+0.055\times P_{G_1}^2$

$$\frac{dC_1}{dP_{G_1}}=1+0.055\times 2P_{G_1}=1+0.11P_{G_1}$$

$$C_2(P_{G_2})=3P_{G_2}+0.03P_{G_2}^2$$

$$\frac{dC_2}{dP_{G_2}}=3+0.06P_{G_2}$$

For minimum cost dispatch,

$$\frac{dC_1}{dP_{G_1}}=\frac{dC_2}{dP_{G_2}}$$

$$\Rightarrow \quad 1+0.11P_{G_1}=3+0.06P_{G_2}$$

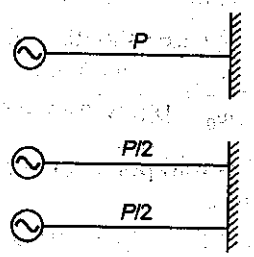
$$\Rightarrow \quad 0.11P_{G_1}-0.06P_{G_2}=2 \qquad ...(i)$$

and $\quad P_{G_1}+P_{G_2}=250\text{ MW}$ ...(ii)

Solving Eqs. (i) and (ii), we get

$$P_{G_1}=100\text{ MW}$$
$$P_{G_2}=150\text{ MW}$$

63. (b) As per question,

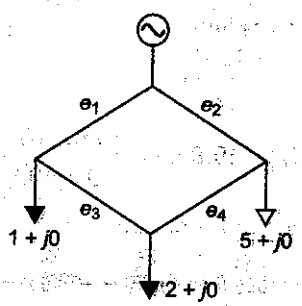


Critical clearing time $=t_{cr}=\frac{0.28}{\sqrt{2}}=0.1979$ s

i.e., reduced but will be more than 0.14 s.

64. (d) Let impedance of each line will be R.

on removing e_1,



Losses $=8^2R+3^2R+1^2R=74R$

on removing e_2,

Losses $=8^2R+7^2R+5^2R=138R$

On removing e_3,

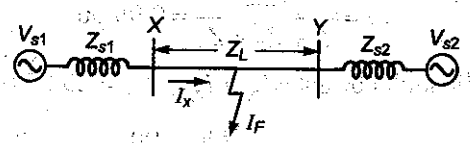
Losses $=1^2R+7^2R+2^2R=54$

On removing e_4,

Losses $=3^2R+2^2R+5^2R=38R$

For radial system operation with minimum loss can be obtained by removing e_4.

65. (a)



$V=V_m\cos\omega t$

The current $I(t) = Ae^{-(R/L)t} + \dfrac{\sqrt{2}V}{|Z|}\cos(\omega t - \theta)$

At $t = t_0; I(t) = 0$

$$0 = Ae^{-(R/L)t} = \frac{\sqrt{2}V}{|Z|}\cos(\omega_0 t - \theta)$$

$$Ae^{-(R/L)t} = \frac{\sqrt{2}V}{|Z|}\cos(\omega_0 t - \theta)$$

Maximum value at t_0 will be at

$$\omega_0 t = \theta$$

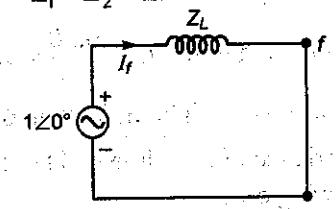
Given, $Z = 0.004 + j0.004$

$$Z = |Z| \angle\theta = 0.0402 \angle 84.3°$$

$$t = \frac{\theta}{\omega_0} = \frac{84.3° \times \pi}{180° \times 2 \times \pi \times 50} = 4.682 \text{ ms}$$

66. (c) The point F is mid-point, so same impedance from both the sides.

$$Z_1 = Z_2 = Z/2 = 0.201 \angle 84.3°$$

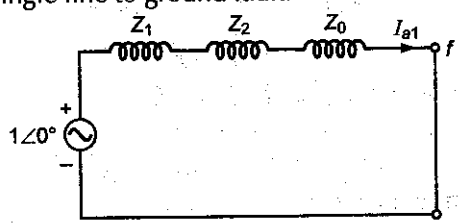


Hence for 3-ϕ fault,

$$I_f = \frac{1\angle 0°}{Z_1}$$

$$|I_f| = 49.8 \text{ pu} = \frac{49.8 \times 100}{\sqrt{3} \times 400} = 7.18 \text{ kA}$$

67. (a) Single line to ground fault.



$$Z_1 = Z_2 = \frac{Z}{2} = 0.201 \angle 84.3°$$

$$Z_0 = 3Z_1 = 0.603 \angle 84.3°$$

The current

$$I_a = \frac{1\angle 0°}{Z_1 + Z_2 + Z_3} = 9.95 \text{ pu}$$

The fault current $= I_{a_1} + I_{a_2} + I_{a_0} = 29.85$ pu

$$= \frac{29.85 \times 100}{\sqrt{3} \times 400} = 4.97 \text{ kA}$$

68. (b) Given, $X_L = 2\pi fL = 0.045$ pu

$$\Rightarrow \quad L = \frac{0.045}{2\pi f} = \frac{0.045}{2\pi \times 50} = 1.43 \times 10^{-4} \text{ H}$$

$$X_C = \frac{1}{2\pi fC} = 1.2 \text{ pu}$$

$$\Rightarrow \quad C = \frac{1}{2\pi f \times 1.2} = 2.6 \times 10^{-3} \text{ F}$$

Length of line $= V_x \times \sqrt{LC}$

$$= 3 \times 10^5 \times \sqrt{1.43 \times 10^{-4} \times 2.6 \times 10^{-3}}$$

$$= 182 \text{ km} \simeq 172 \text{ km}$$

69. (c) In given protection system, if the fault occurs at F and circuit breaker 4 fails to operate, then the relays 1, 2 and 3 are in backup protection. Due to fault at F relays in that section operate first *i.e.*, 5, 6, 7 and after that relays in back protection *i.e.*, 3, 1, 2. Hence, sequence is 5, 6, 7, 3, 1, 2.

70. (a) Given,

$$R = [V_{an}\ V_{bn}\ V_{cn}] \begin{bmatrix} 0 & 1/\sqrt{3} & -1/3 \\ -1/\sqrt{3} & 0 & 1/\sqrt{3} \\ 1/\sqrt{3} & -1/\sqrt{3} & 0 \end{bmatrix} \begin{bmatrix} I_a \\ I_b \\ I_c \end{bmatrix}$$

$$= \left[\frac{V_{an}}{\sqrt{3}}(I_b - I_c) + \frac{V_{bn}}{\sqrt{3}}(I_c - I_a) + \frac{V_{cn}}{\sqrt{3}}(I_a - I_b)\right]$$

Magnitude of R is determinant of R-matrix $= 3VI$

71. (c) For system does not lose synchronisation,

$$P_m = P_e = \frac{EV}{X_{\text{total}}}\sin\delta_1$$

$$1 = \frac{1 \times 1}{X_{\text{total}}}\sin 130°$$

$$X_{\text{total}} = 0.77$$

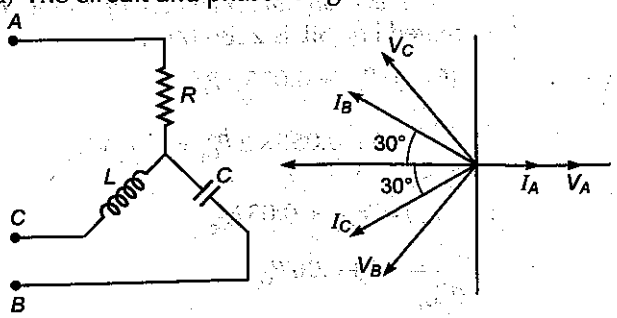
$$0.1 + X = 0.77$$

$$X = 0.67$$

72. (a) There is no energy transfer after the interconnection, so both bus bars should have the same voltage and phase angle.
Hence, phase difference

$$\theta = 30° - 20° = 10°$$

73. (a) The circuit and phasor diagrams are



$$I_A = \frac{4000}{230} = 17.39 \text{ A}$$

Consider choice (b).

$$X_L = 2\pi \times 50 \times 72.95 \times 10^{-3} = 22.918$$

$$X_C = \frac{10^6}{314 \times 139.02} = 22.918$$

As $X_L = X_C$

$$\therefore \quad I_B = I_C = \frac{23.0}{22.91} \simeq 1$$

$$I_A = I_B \cos 30° + I_C \cos 30°$$

$$= 10\frac{\sqrt{3}}{2} + 10\frac{\sqrt{3}}{2}$$

$$= 10\sqrt{3} = 17.32 \text{ A}$$

74. (d) To increase the capacitor DC voltage slowly to a new steady state value, make δ negative and return it to its original value.

75. (b) Here,
$$\begin{bmatrix} V_p \\ V_n \\ V_0 \end{bmatrix} = \begin{bmatrix} 0.5 & 0 & 0 \\ 0 & 0.5 & 0 \\ 0 & 0 & 2 \end{bmatrix} \begin{bmatrix} I_p \\ I_n \\ I_0 \end{bmatrix}$$

where,
$$\begin{bmatrix} V_a \\ V_b \\ V_c \end{bmatrix} = [Z] \begin{bmatrix} I_a \\ I_b \\ I_c \end{bmatrix}$$

$$\begin{bmatrix} I_a \\ I_b \\ I_c \end{bmatrix} = K \begin{bmatrix} 1 & 1 & 1 \\ \alpha^2 & \alpha & 1 \\ \alpha & \alpha^2 & 1 \end{bmatrix} \begin{bmatrix} I_p \\ I_n \\ I_0 \end{bmatrix} = KA \begin{bmatrix} I_p \\ I_n \\ I_0 \end{bmatrix}$$

Similarly, $V_{Ph} = K[A]\, V_{sequence}$

$$= K[A] \begin{bmatrix} 0.5 & 0 & 0 \\ 0 & 0.5 & 0 \\ 0 & 0 & 2 \end{bmatrix} [I_{sequence}]$$

$$= K[A] \begin{bmatrix} 0.5 & 0 & 0 \\ 0 & 0.5 & 0 \\ 0 & 0 & 2 \end{bmatrix} \frac{[A^{-1}]}{K} I_{Ph}$$

$$\therefore \quad V_{Ph} = A \begin{bmatrix} 0.5 & 0 & 0 \\ 0 & 0.5 & 0 \\ 0 & 0 & 2 \end{bmatrix} [A^{-1}]$$

$$= \frac{1}{3} \begin{bmatrix} 1 & 1 & 1 \\ \alpha^2 & \alpha & 1 \\ \alpha & \alpha^2 & 1 \end{bmatrix} \begin{bmatrix} 0.5 & 0 & 0 \\ 0 & 0.5 & 0 \\ 0 & 0 & 2 \end{bmatrix} \begin{bmatrix} 1 & \alpha & \alpha^2 \\ 1 & \alpha^2 & \alpha \\ 1 & 1 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0.5 & 0.5 \\ 0.5 & 1 & 0.5 \\ 0.5 & 0.5 & 1 \end{bmatrix}$$

76. (a) Here, change in frequency w.r.t. to power

$$\left(\frac{5}{15}\right) \times 3.5 = 1.16\%$$

$$\Delta f = 0.583$$

$$f = 50 - 0.583 = 49.417 \text{ Hz}$$

77. (d) $P_L = 12 \times \sqrt{3} \times 0.8 = 16.627$ kW

$$Q_L = 12 \times \sqrt{3} \times 0.6 = 12.47 \text{ kW}$$

For setting unity (pf), we have to add capacitance reactive power (12.47) kW.

78. (b)
$$\begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} V_2 \\ I_2 \end{bmatrix}$$

$$B = \left.\frac{V_1}{I_2}\right|_{V_2 = 0} ; \qquad C = \left.\frac{I_1}{V_2}\right|_{I_2 = 0}$$

$$C = \frac{\dfrac{V_1}{(Z_1 + Z_2)}}{\dfrac{V_1}{Z_1 + Z_2} \times Z_2} = \frac{1}{Z_2}$$

$$Z_2 = \frac{1}{C} = \frac{1}{0.025 \angle 45°} = (40 \angle 45°)\,\Omega$$

79. (c) Voltage regulation $= \dfrac{V_{R(NL)} - V_{R(FL)}}{V_{R(FL)}}$

From *ABCD* parameter, we can write

$$V_S = AV_R + BI_R$$

$$I_S = CV_R + DI_R$$

$V_{R(NL)}$ means $I_R = 0$

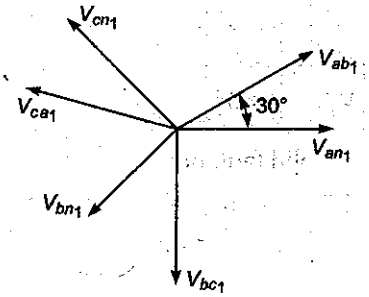
$$V_{R(NL)} = \left.\frac{V_S}{A}\right|_{I_R = 0} = \frac{240}{0.94}$$

$$V_{R(FL)} = V_S = 220$$

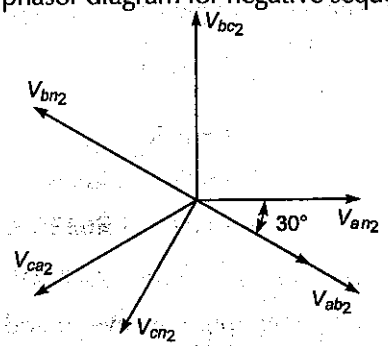
$$\therefore \text{ Voltage regulation} = \frac{240/0.94 - 220}{220} \times 100\%$$

$$= 16\%$$

80. (b) The phasor diagram for positive sequence.



The phasor diagram for negative sequence.



81. (a) The value of the load (in per unit) in generator circuit will be

$$Z = 0.72 \times \left(\frac{69}{13.8}\right)^2 \times \frac{20}{10}$$

$$= 36 \text{ pu}$$

82. (d) In Gauss-Siedel method instead of updating voltage at every step of an iteration, updating is carried out at the end of complete iteration, the process is known as Gauss interative method. It converges much slower and may sometimes fail to do so.

83. (c) The accelerating power

$$P_a = P_s - P_m \sin\delta$$

$$= P_s - \frac{EV}{X}\sin\delta$$

$$= 1 - \frac{1.1 \times 1}{1/0.8}\sin 30°$$

$$= 0.56 \text{ pu}$$

84. (b) The initial acceleration (in pu)

$$\alpha = \frac{P_a}{M} = \frac{X(\text{pu}) \times S}{\frac{S+f}{180f}}$$

$$= \frac{180 \times 50 \times X \times 100}{5 \times 100}$$

$$= 1800\, X$$

Inertia constant = 1/18 = 0.056

85. (d) $V_{\text{bus}} = \begin{bmatrix} V_1^\circ \\ V_2^\circ \\ V_3^\circ \end{bmatrix} = \begin{bmatrix} 1\angle 0° \\ 1\angle 0° \\ 1\angle 0° \end{bmatrix}$

At bus S, solid fault occurs

$$z(f) = 0$$

$$r = 2$$

Fault current

$$I_f = \frac{V_r^\circ}{Z_{22} + Z_f}$$

$$= \frac{1\angle 0°}{j\,24} = -j\,4$$

$$V_1(f) = V_1^\circ - Z_{12} I_f = 1\angle 0° - j\,0.08\,(-j\,4)$$

$$= 1 - 0.32 = 0.68$$

$$V_3(f) = V_3^\circ - Z_{32} I_f = 1\angle 0° - j\,0.16\,(-j\,4)$$

$$= 1 - 0.64 = 0.36$$

86. (c) $I_{12}(f) = Y_{12}(V_1^f - V_2^f)$

$$= j\,(1.25)\,(0.68 - 0) = j\,1.25 \times 0.68 = j\,0.85$$

$$I_{13}(f) = Y_{13}(V_1^f - V_3^f) = j\,2.5 \times (0.68 - 0.34) = j0.85$$

$$I_1(f) = j0.85 + j0.85 = j1.7 \approx j1.66$$

87. (d) Given, self reactance

$$x_s = 0.4\,\Omega/\text{km}$$

Mutual reactance

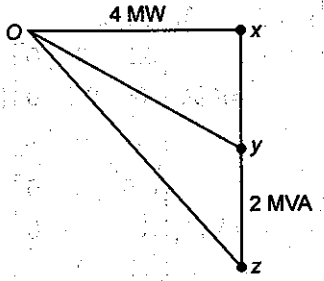
$$x_m = 0.1\Omega/\text{km}$$

Positive sequence reactance,

$$x_l = x_s - x_m = 0.4 - 0.1 = 0.3\,\Omega/\text{km}$$

Zero sequence reactance, $x_0 = 2x_l = 0.6\,\Omega/\text{km}$

88. (c) The phasor diagram



$$\theta_1 = \cos^{-1}(0.97) = 14.07$$

From the phasor diagram,

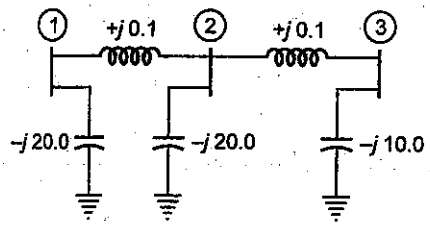
$$xy = 0x \tan\theta_1 = 1.0025 \text{ MVAR}$$

Hence, $xz = xy + yz = 3.0025$ MVAR

$$\theta_2 = \tan^{-1}\left(\frac{3.0025}{4}\right) = 36.89$$

Power factor = $\cos\theta_2 = 0.8$ lag

89. (d)



We can write

$$I_1 = V_1 Y_{11} + (V_1 - V_2) Y_{12}$$
$$= j0.05 V_1 - j10 (V_1 - V_2)$$
$$= -j9.95 V_1 + j10 V_2$$
$$I_2 = (V_2 - V_1) Y_{21} + (V_2 - V_3) Y_{23}$$
$$= j10 V_1 - j9.9 V_2 - j0.1 V_3$$
$$Y_{22} = Y_{11} + Y_{23} + Y_2 = -j9.95 - j9.9 - 0.1j$$
$$= -j19.95$$

90. (c) Given,

$$F_1 = a + bP_1 + cP_1^2 \text{ ₹/h}$$
$$F_2 = a + bP_2 + 2cP_2^2 \text{ ₹/h}$$

For economic generation.

$$\frac{dF_1}{dP_1} = \frac{dF_2}{dP_2}$$

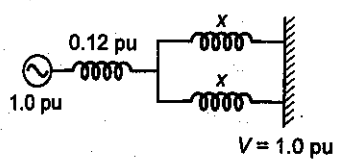
$$\Rightarrow \quad b + 2cP_1 + b + 4cP_2$$

$$\Rightarrow \quad P_1 = 2P_2$$

Given, $P_1 + P_2 = 300$

$$3P_2 = 300$$
$$P_2 = 100 \text{ MW}$$
$$P_1 = 200 \text{ MW}$$

91. (d) Neglecting resistance and susceptances, the circuit will be



$$P_{ini} = \frac{1.0 \times 1.0}{0.12 + 0.5x} = 6.25$$

$$x = 0.08\,\Omega$$

When one of the double circuit is tripped,

$$P_{new} = \frac{1.0 \times 1.0}{0.12 + x} = \frac{1}{0.12 + 0.08} = \frac{1}{0.2} = 5 \text{ pu}$$

92. (d) The single line current $I_1 = \frac{4000}{220} = 18.18$ A

$$\text{Positive sequence driving point reactance} = \frac{220}{18.18}$$
$$= 12.1\Omega$$

93. (a)

94. (c) Surge impedance $Z_c = \sqrt{\frac{L}{C}} = \sqrt{\frac{1.1 \times 10^{-3}}{11.68 \times 10^{-9}}}$

$$= 306.88\,\Omega$$

$$\text{The power transfer capability} = \frac{(800 \text{ kV})^2}{Z_C}$$

$$= \frac{(800)^2}{306.88} = 2085 \text{ MW}$$

95. (d) Given, dielectric loss tangent $= \tan\delta = 2 \times 10^{-4}$

$$\text{Dielectric power loss} = V^2\, \omega c \tan\delta$$
$$= (110 \text{ kV})^2 \times (2\pi \times 50)\,(125 \times 10^{-9})\,(2 \times 10^{-4})$$
$$= 189 \text{ W/km}$$

96. (a) As there is equal distribution both sides, the impulse current of 10 kA will equally divided in both directions. Hence, current in each direction is 5 kA. Then, magnitude of transient over-voltage

$$= 5 \times 250 = 1250 \text{ kV}$$

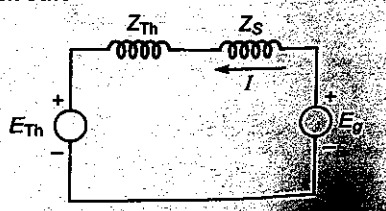
97. (c) From *ABCD* matrix,

$$V_s = AV_r + BI_r$$
$$I_s = CV_r + DI_r$$

Hence, sending end voltage

$$V_s = AV_r + BI_r$$
$$I_r = \frac{50 \times 10^6}{\sqrt{3} \times 220 \times 10^3 \times 0.9} = 130.31 \angle -25.8°$$
$$V_s = (0.936 \angle 0.98°)\left(\frac{220 \text{ kV}}{\sqrt{3}}\right) \angle 0°$$
$$+ (42 \angle 76.4°)\,(130.31 \angle -25.8°)$$
$$= V_s = 130 + j16.3$$
$$|V_s| = 227 \text{ kV}$$

98. (b) The circuit



$$I = \frac{E_g - E_{Th}}{Z_{Th} + X_s} = \frac{1.4\angle 30° - (0.9)\angle 0°}{1.25\,j}$$

$$= 0.56 - 0.2496\,j$$

The bus voltage $V_t = E_g - IX_s$

$$= (1.212 + j0.7) - (0.56 - j0.2496) \times j$$

$$= 0.973$$

99. (c) The current $I_{sc} = \frac{1}{x''_d} = \frac{1}{0.19\,j} = -j5.26$ pu

Base current $I_B = \frac{110 \times 10^6}{\sqrt{3} \times 11 \times 10^3}$

$$= 5773.67 \text{ A}$$

The symmetrical rms current $= I_B \cdot I_{sc} = 30.37$ kA

100. (a) The positive sequence current component

$$I_{a_1} = \frac{1}{3}(I_a + \alpha I_b + \alpha^2 I_c)$$

$$= \frac{1}{3}(10\angle 0° + 10\angle 300° + 0)$$

$$= \frac{1}{3}(10 + 5 - 8.66j) = 5.78\angle -30°$$

101. (d) Sub-transient line current $= \frac{1}{X_2 + X_1 + X_0}$ pu

$$= \frac{1}{0.35\,j} = -2.857j$$

Sub-transient line current $= 3I_{a1} = 8.57$

102. (a) $P = 500 \times 0.8 = 400$ MW

After fault, $P = 0.6 \times 400 = 240$ MW

$$\text{Torque} = \frac{240}{2\pi \times \frac{1500}{60}} = 1.528 \text{ MW}$$

103. (d) No. of poles $= \frac{120f}{n} = \frac{120 \times 50}{250} = 24$

6

Control Systems

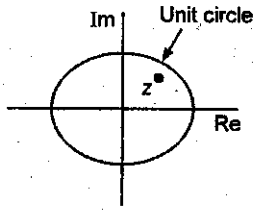
SYLLABUS

Control Systems *Basic control system components; block diagrammatic description, reduction of block diagrams. Open-loop and closed-loop (feedback) systems and stability analysis of these systems. Signal flow graphs and their use in determining transfer functions of systems; transient and steady state analysis of LTI control systems and frequency response. Tools and techniques for LTI control system analysis: root loci, Routh-Hurwitz criterion, Bode and Nyquist plots. Control system compensators : elements of lead and lag compensation, elements of Proportional-Integral-Derivative (PID) control. State variable representation and solution of state equation of LTI control systems.*

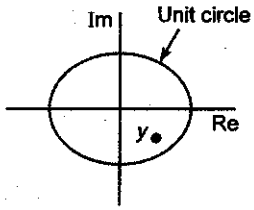
GATE 2011 *(1 Mark Questions)*

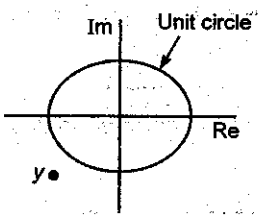
▼ *Solution Points*

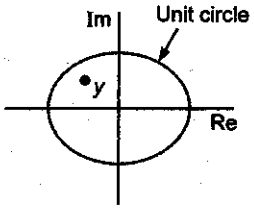
1. A point z has been plotted in the complex plane as shown in figure below.

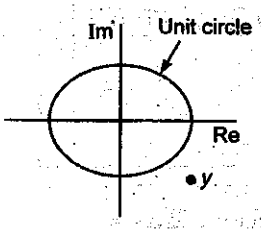


The plot of the complex number $y = \frac{1}{z}$ is

(a)


(b)


(c)


(d)


▼ *Solution Points*

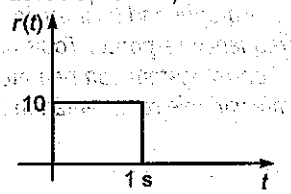
2. An open-loop system represented by the transfer function $G(s) = \dfrac{(s-1)}{(s+2)(s+3)}$ is
(a) stable and of the minimum phase type
(b) stable and of the non-minimum phase type
(c) unstable and of the minimum phase type
(d) unstable and of the non-minimum phase type

3. The frequency response of a linear system $G(j\omega)$ is provided in the tabular form below.

$\lvert G(j\omega)\rvert$	1.3	1.2	1.0	0.8	0.5	0.3
$\angle G(j\omega)$	– 130°	– 140°	– 150°	– 160°	– 180°	– 200°

The gain margin and phase margin of the system are
(a) 6 dB and 30° (b) 6 dB and – 30°
(c) – 6 dB and 30° (d) – 6 dB and – 30°

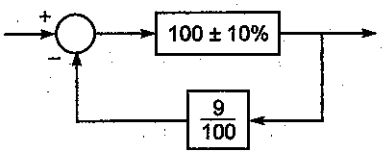
4. The steady state error of a unity feedback linear system for a unit step input is 0.1. The steady state error of the same system, for a pulse input $r(t)$ having a magnitude of 10 and a duration of 1 s, as shown in the figure is



(a) 0 (b) 0.1 (c) 1 (d) 10

GATE 2010

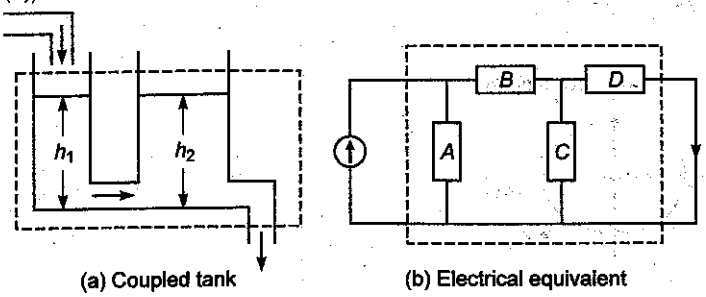
5. As shown in the figure, a negative feedback system has an amplifier of gain 100 with ±10% tolerance in the forward path, and an attenuator of value 9/100 in the feedback path. The overall system gain is approximately



(a) 10 ± 1% (b) 10 ± 2% (c) 10 ± 5% (d) 10 ± 10%

6. For the system $2/(s+1)$, the approximate time taken for a step response to reach 98% of its final value is
(a) 1 s (b) 2 s (c) 4 s (d) 8 s

7. If the electrical circuit of Fig. (b) is an equivalent of the coupled tank system of Fig. (a), then



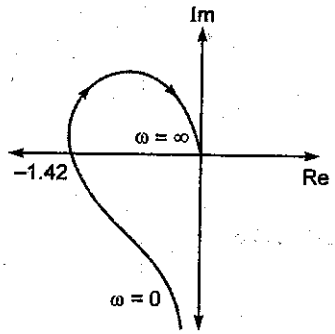
(a) Coupled tank (b) Electrical equivalent

(a) A, B are resistances and C, D are capacitances
(b) A, C are resistances and B, D are capacitances
(c) A, B are capacitances and C, D are resistances
(d) A, C are capacitances and B, D are resistances

▼ *Solution Points*

GATE 2009

8. The polar plot of an open-loop stable system is shown below. The closed-loop system is

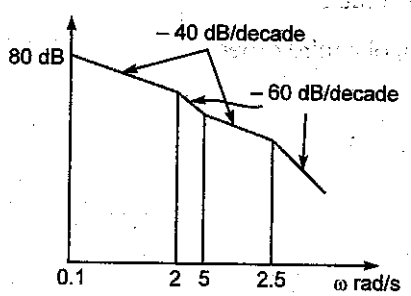


(a) always stable
(b) marginally stable
(c) unstable with one pole on the RH s-plane
(d) unstable with two poles on the RH s-plane

9. The first two rows of Routh's tabulation of a third order equation are as follows. $\begin{matrix} s^3 & 2 & 2 \\ s^2 & 4 & 4 \end{matrix}$. This means there are

(a) two roots at $s = \pm j$ and one root in right half s-plane
(b) two roots at $s = \pm j\,2$ and one root in left half s-plane
(c) two roots at $s = \pm j\,2$ and one root in right half s-plane
(d) two roots at $s = \pm j$ and one root in left half s-plane

10. The asymptotic approximation of the log magnitude *vs* frequency plot of a system containing only real poles and zeros is shown. Its transfer function is



(a) $\dfrac{10(s+5)}{s(s+2)(s+25)}$

(b) $\dfrac{1000(s+5)}{s^2(s+2)(s+25)}$

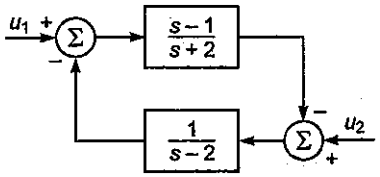
(c) $\dfrac{100(s+5)}{s(s+2)(s+25)}$

(d) $\dfrac{80(s+5)}{s^2(s+2)(s+25)}$

▼ *Solution Points*

GATE 2007

11. The system shown in the figure is



(a) stable
(b) unstable
(c) conditionally stable
(d) stable for input u_1 but unstable for input u_2

GATE 2006

12. For a system with the transfer function $H(s) = \dfrac{3(s-2)}{4s^2 - 2s + 1}$, the matrix A in the state-space form $\dot{x} = Ax + Bu$ is equal to

(a) $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ -1 & 2 & -4 \end{bmatrix}$ (b) $\begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -1 & 2 & -4 \end{bmatrix}$

(c) $\begin{bmatrix} 0 & 1 & 0 \\ 3 & -2 & 1 \\ 1 & -2 & 4 \end{bmatrix}$ (d) $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ -1 & 2 & -4 \end{bmatrix}$

GATE 2005

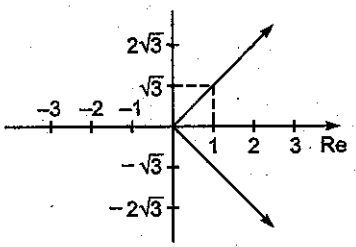
13. A system with zero initial condition has the closed-loop transfer function

$$T(s) = \frac{s^2 + 4}{(s+1)(s+4)}$$

The system output is zero at the frequency

(a) 0.5 rad/s (b) 1 rad/s (c) 2 rad/s (d) 4 rad/s

14. Figure shows the root locus plot (location of poles not given) of a third order system whose open-loop transfer function is



(a) k/s^3 (b) $\dfrac{k}{s^2(s+1)}$

(c) $\dfrac{k}{s(s^2+1)}$ (d) $\dfrac{k}{s(s^2-1)}$

▼ *Solution Points*

15. The gain margin of a unity feedback control system with open-loop transfer function

$$G(s)=\frac{(s+1)}{s^2}, \text{ is}$$

(a) 0 (b) $\frac{1}{\sqrt{2}}$ (c) $\sqrt{2}$ (d) ∞

GATE 2004

16. The Nyquist plot of loop transfer function $G(s)\,H(s)$ of a closed-loop control system passes through the point $(-1, j0)$ in the $G(s)H(s)$ plane. The phase margin of the system is

(a) 0° (b) 45° (c) 90° (d) 180°

17. Consider the function $F(s)=\frac{5}{s(s^2+3s+2)}$, where $F(s)$ is the Laplace transform of the function $f(t)$. The initial value of $f(t)$ is equal to

(a) 5 (b) $\frac{5}{2}$ (c) $\frac{5}{3}$ (d) 0

18. For a tachometer, if $\theta(t)$ is the rotor displacement in radian, $e(t)$ is the output voltage and K_t is the tachometer constant in V/rad/s, then the transfer function $\frac{E(s)}{Q(s)}$ will be

(a) $K_t s^2$ (b) $\frac{K_t}{s}$ (c) $K_t s$ (d) K_t

GATE 2003

19. A lead compensator used for a closed-loop controller has the following transfer function

$$\frac{K\left(1+\frac{s}{a}\right)}{\left(1+\frac{s}{b}\right)}$$

For such a lead compensator,

(a) $a<b$ (b) $b<a$ (c) $a>Kb$ (d) $a<Kb$

20. A second order system starts with an initial condition of $\begin{bmatrix}2\\3\end{bmatrix}$ without any external input. The state transition matrix for the system is given by $\begin{bmatrix}e^{-2t} & 0\\ 0 & e^{-t}\end{bmatrix}$. The state of the system at the end of 1 s is given by

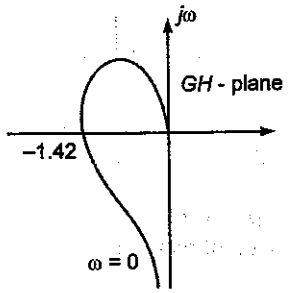
(a) $\begin{bmatrix}0.271\\1.100\end{bmatrix}$ (b) $\begin{bmatrix}0.135\\0.368\end{bmatrix}$

(c) $\begin{bmatrix}0.271\\0.736\end{bmatrix}$ (d) $\begin{bmatrix}0.135\\1.100\end{bmatrix}$

▼ *Solution Points*

GATE 2001

21. The polar plot of a type 1, 3-pole, open-loop system is shown in figure. The closed-loop system is



(a) always stable
(b) marginally stable
(c) unstable with one pole on the right half s-plane
(d) unstable with two poles on the right half s-plane

22. Given the homogeneous state-space equation

$$\dot{x} = \begin{bmatrix} -3 & 1 \\ 0 & -2 \end{bmatrix} x$$

The steady state value of $x_{st} = \lim_{t \to \infty} x(t)$, given the initial state value of $x(0) = [10\ -10]^T$, is

(a) $x_{st} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$ (b) $x_{st} = \begin{bmatrix} -3 \\ -2 \end{bmatrix}$ (c) $x_{st} = \begin{bmatrix} -10 \\ 10 \end{bmatrix}$ (d) $x_{st} = \begin{bmatrix} \infty \\ \infty \end{bmatrix}$

GATE 2000

23. Ratio of the rotor reactance X to the rotor resistance R for a two-phase servomotor
(a) is equal to that of a normal induction motor
(b) is less than that of a normal induction motor
(c) is greater than that of a normal induction motor
(d) may be less or greater than that of a normal induction motor

24. Feedback control systems are
(a) insensitive to both forward and feedback path parameter changes
(b) less sensitive to feedback path parameter changes than to forward path parameter changes
(c) less sensitive to forward path parameter changes than to feedback path parameter changes
(d) equally sensitive to forward and feedback path parameter changes

25. A unity feedback system has open-loop transfer function $G(s)$. The steady state error is zero for
(a) step input and type 1 $G(s)$ (b) ramp input and type 1 $G(s)$
(c) step input and type 0 $G(s)$ (d) ramp input and type 0 $G(s)$

26. A linear time-invariant system initially at rest, when subjected to a unit step input, gives a response $y(t) = te^{-t}, t > 0$. The transfer function of the system is

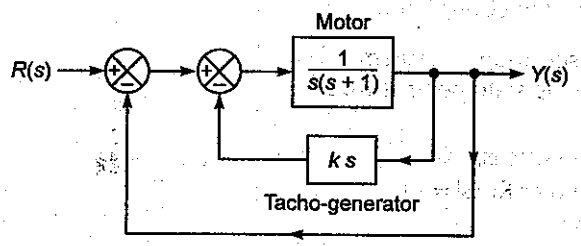
(a) $\frac{1}{(s+1)^2}$ (b) $\frac{1}{s(s+1)^2}$ (c) $\frac{s}{(s+1)^2}$ (d) $\frac{1}{s(s+1)}$

GATE 2011

(2 Marks Questions)

▼ *Solution Point*

27. A two-loop position control system is shown below.



The gain k of the tacho-generator influences mainly the

(a) peak overshoot

(b) natural frequency of oscillation

(c) phase shift of the closed-loop transfer function at very low frequencies ($\omega \to 0$)

(d) phase shift of the closed-loop transfer function at very high frequencies ($\omega \to \infty$)

28. The open-loop transfer function $G(s)$ of a unity feedback control system is given as

$$G(s) = \frac{k\left(s + \frac{2}{3}\right)}{s^2(s+2)}$$

From the root locus, it can be inferred that when k tends to positive infinity,

(a) three roots with nearly equal real parts exist on the left half of the s-plane

(b) one real root is found on the right half of the s-plane

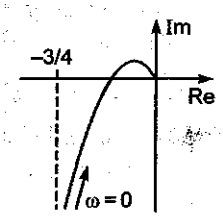
(c) the root loci cross the $j\omega$-axis for a finite value of k; $k \neq 0$

(d) three real roots are found on the right half of the s-plane

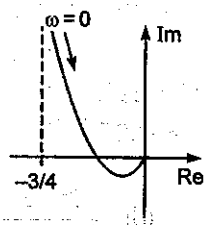
GATE 2010

29. The frequency response of $G(s) = 1/[s(s+1)(s+2)]$ plotted in the complex $G(j\omega)$ plane (for $0 < \omega < \infty$) is

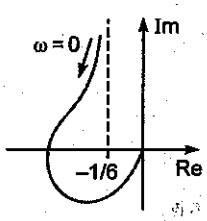
(a)



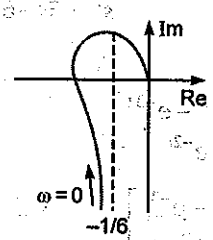
(b)



(c)



(d)



▼ *Solution Points*

30. The system $\dot{x} = Ax + Bu$ with $A = \begin{bmatrix} -1 & 2 \\ 0 & 2 \end{bmatrix}, B = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$ is

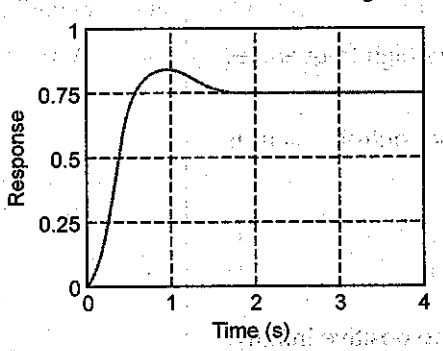
(a) stable and controllable (b) stable but uncontrollable
(c) unstable but controllable (d) unstable and uncontrollable

31. The characteristic equation of a closed-loop system is $s(s+1)(s+3) + k(s+2) = 0, k > 0$. Which of the following statements is true?

(a) Its roots are always real
(b) It cannot have a breakaway point in the range $-1 < \text{Re}[s] < 0$
(c) Two of its roots tend to infinity along the asymptotes $\text{Re}[s] = -1$
(d) It may have complex roots in the right half plane

GATE 2009

32. The unit step response of a unity feedback system with open-loop transfer function $G(s) = K/((s+1)(s+2))$ is shown in the figure. The value of K is



(a) 0.5 (b) 2 (c) 4 (d) 6

33. The open-loop transfer function of a unity feedback system is given by $G(s) = (e^{-0.1s})/s$. The gain margin of this system is

(a) 11.95 dB (b) 17.67 dB (c) 21.33 dB (d) 23.9 dB

Common Data for Questions 34 and 35

A system is described by the following state and output equations

$$\frac{dx_1(t)}{dt} = -3x_1(t) + x_2(t) + 2u(t)$$

$$\frac{dx_2(t)}{dt} = -2x_2(t) + u(t)$$

$$y(t) = x_1(t)$$

where, $u(t)$ is the input and $y(t)$ is the output.

34. The system transfer function is

(a) $\frac{s+2}{s^2+5s-6}$ (b) $\frac{s+3}{s^2+5s+6}$ (c) $\frac{2s+5}{s^2+5s+6}$ (d) $\frac{2s-5}{s^2+5s-6}$

35. The state transition matrix of the above system is

(a) $\begin{bmatrix} e^{-3t} & 0 \\ e^{-2t} + e^{-3t} & e^{-2t} \end{bmatrix}$ (b) $\begin{bmatrix} e^{-3t} & e^{-2t} - e^{-3t} \\ 0 & e^{-2t} \end{bmatrix}$

(c) $\begin{bmatrix} e^{-3t} & e^{-2t} + e^{-3t} \\ 0 & e^{-2t} \end{bmatrix}$ (d) $\begin{bmatrix} e^{3t} & e^{-2t} - e^{-3t} \\ 0 & e^{-2t} \end{bmatrix}$

GATE 2008

▼ *Solution Points*

36. The transfer functions of two compensators are given below.

$$C_1 = \frac{10(s+1)}{(s+10)}, C_2 = \frac{s+10}{10(s+1)}$$

Which one of the following statements is correct?

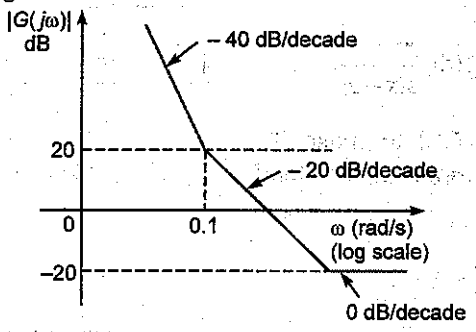
(a) C_1 is a lead compensator and C_2 is a lag compensator

(b) C_1 is a lag compensator and C_2 is a lead compensator

(c) Both C_1 and C_2 are lead compensators

(d) Both C_1 and C_2 are lag compensators

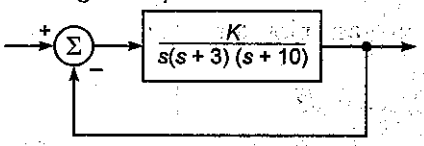
37. The asymptotic Bode magnitude plot of a minimum phase transfer function is shown in the figure. This transfer function has



(a) three poles and one zero (b) two poles and one zero

(c) two poles and two zeros (d) one pole and two zeros

38. Figure shows a feedback system, where $K > 0$. The range of K for which the system is stable will be given by



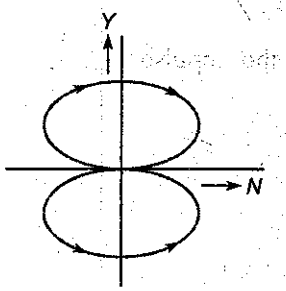
(a) $0 < K < 30$ (b) $0 < K < 39$ (c) $0 < K < 390$ (d) $K > 390$

39. The transfer function of a system is given as $\frac{100}{s^2 + 20s + 100}$. This system is

(a) an overdamped system (b) an underdamped system

(c) a critically damped system (d) an unstable system

40. Two sinusoidal signals $p(\omega_1 t) = A \sin \omega_1 t$ and $q(\omega_2 t)$ are applied to X and Y inputs of a dual channel CRO. The Lissajous figure displayed on the screen is shown below. The signal $q(\omega_2 t)$ will be represented as



(a) $q(\omega_2 t) = A \sin \omega_2 t, \omega_2 = 2\omega_1$ (b) $q(\omega_2 t) = A \sin \omega_2 t, \omega_2 = \frac{\omega_1}{2}$

(c) $q(\omega_2 t) = A \cos \omega_2 t, \omega_2 = 2\omega_1$ (d) $q(\omega_2 t) = A \cos \omega_2 t, \omega_2 = \frac{\omega_1}{2}$

Statements for Linked Answer Questions 41 and 42

The state-space equation of a system is described by

$$\dot{x} = Ax + Bu$$
$$y = Cx$$

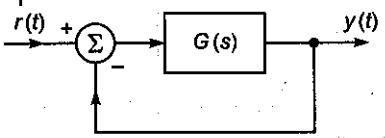
where, x is state vector, u is input, y is output and

$$A = \begin{bmatrix} 0 & 1 \\ 0 & -2 \end{bmatrix}, B = \begin{bmatrix} 0 \\ 1 \end{bmatrix}, C = [1 \;\; 0].$$

41. The transfer function $G(s)$ of this system will be

(a) $\frac{s}{(s+2)}$ (b) $\frac{s+1}{s(s-2)}$ (c) $\frac{s}{(s-2)}$ (d) $\frac{1}{s(s+2)}$

42. A unity feedback is provided to the above system $G(s)$ to make it a closed-loop system as shown in figure. For a unit step input $r(t)$, the steady state error in the output will be



(a) 0 (b) 1 (c) 2 (d) ∞

GATE 2007

43. If $x = \text{Re}\, G(j\omega)$, and $y = \text{Im}\, G(j\omega)$ then for $\omega \to 0$, the Nyquist plot for $G(s) = 1/s(s+1)(s+2)$ becomes asymptotic to the line

(a) $x = 0$ (b) $x = -3/4$ (c) $x = y - 1/6$ (d) $x = y/\sqrt{3}$

44. The system $900/s(s+1)(s+9)$ is to be compensated such that its gain cross-over frequency becomes same as its uncompensated phase cross-over frequency and provides a 45° phase margin. To achieve this, one may use

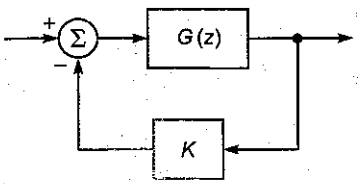
(a) a lag compensator that provides an attenuation of 20 dB and a phase lag of 45°at the frequency of $3\sqrt{3}$ rad/s

(b) a lead compensator that provides an amplification of 20 dB and a phase lead of 45° at the frequency of 3 rad/s

(c) a lag-lead compensator that provides an amplification of 20 dB and a phase lag of 45° at the frequency of $\sqrt{3}$ rad/s

(d) a lag-lead compensator that provides an attenuation of 20 dB and phase lead of 45° at the frequency of 3 rad/s

45. Consider the discrete-time system shown in the figure where the impulse response of $G(z)$ is $g(0) = 0, g(1) = g(2) = 1, g(3) = g(4) = \ldots = 0$,



▼ *Solution Points*

This system is stable for range of values of K

(a) $[-1, 1/2]$ (b) $[-1, 1]$ (c) $[-1/2, 1]$ (d) $[-1/2, 2]$

46. If the loop gain K of a negative feedback system having a loop transfer function $K(s+3)/(s+8)^2$ is to be adjusted to induce a sustained oscillation, then

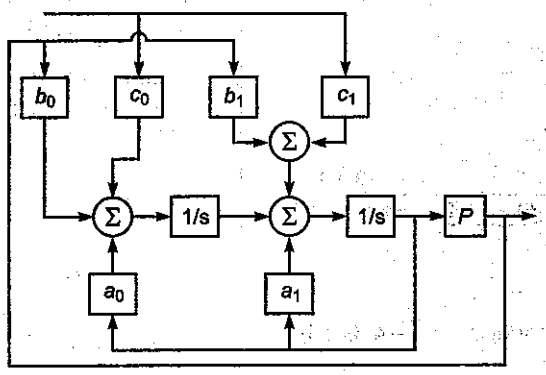
(a) the frequency of this oscillation must be $4/\sqrt{3}$ rad/s

(b) the frequency of this oscillation must be 4 rad/s

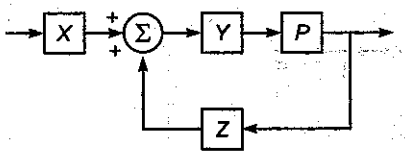
(c) the frequency of this oscillation must be 4 or $4/\sqrt{3}$ rad/s

(d) such a K does not exist

47. The system shown in figure below.



can be reduced to the form



with

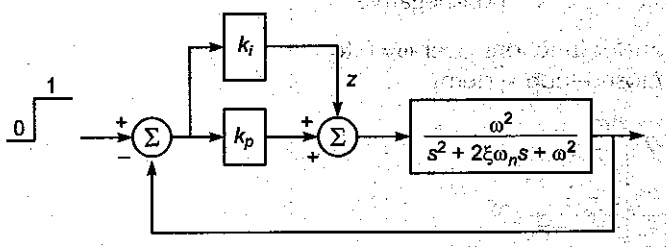
(a) $X = c_0 s + c_1, Y = 1/(s^2 + a_0 s + a_1), \quad Z = b_0 s + b_1$

(b) $X = 1, Y = (c_0 s + c_1)/(s^2 + a_0 s + a_1), Z = b_0 s + b_1$

(c) $X = c_1 s + c_0, Y = (b_1 s + b_0)/(s^2 + a_1 s + a_0), Z = 1$

(d) $X = c_1 s + c_0, Y = 1/(s^2 + a_1 s + a_0), Z = b_1 s + b_0$

48. Consider the feedback control system shown below which is subjected to a unit step input. The system is stable and has the following parameters $k_p = 4, k_i = 10, \omega = 500$ and $\xi = 0.7$. The steady state value of z is



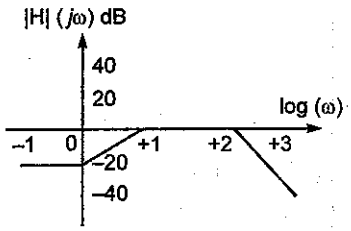
(a) 1 (b) 0.25 (c) 0.1 (d) 0

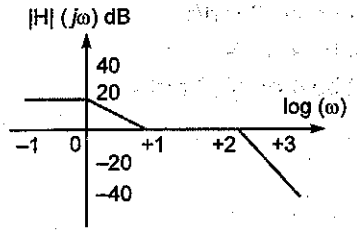
GATE 2006

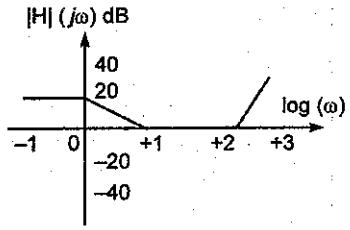
▼ Solution Poi

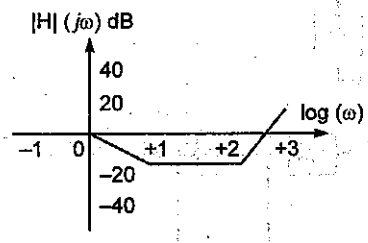
49. The Bode magnitude plot of

$$H(j\omega) = \frac{10^4(1+j\omega)}{(10+j\omega)(100+j\omega)^2} \text{ is}$$

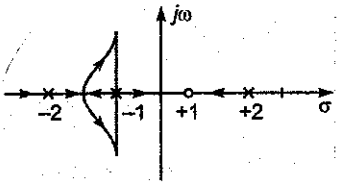
(a)


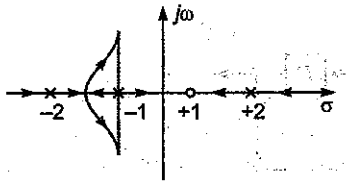
(b)


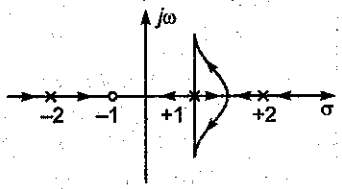
(c)


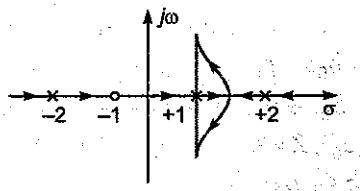
(d)


50. A closed-loop system has the characteristic function $(s^2-4)(s+1) + K(s-1) = 0$. Its root locus plot against K is

(a)


(b)


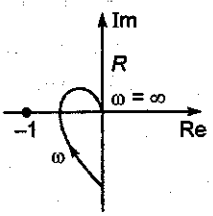
(c)


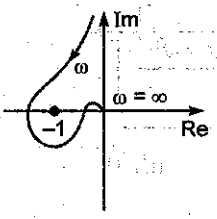
(d)


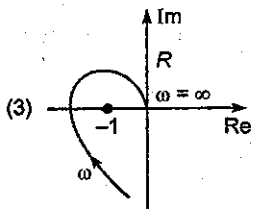
51. $y[n]$ denotes the output and $x[n]$ denotes the input of a discrete-time system given by the difference equation $y[n] - 0.8y[n-1] = x[n] + 1.25\,x[n+1]$. Its right-sided impulse response is

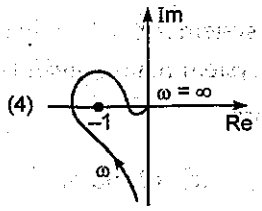
(a) causal (b) unbounded (c) periodic (d) non-negative

52. Consider the following Nyquist plots of loop transfer functions over $\omega = 0$ to $\omega = \infty$. Which of these plots represents a stable closed-loop system?

(1)


(2)






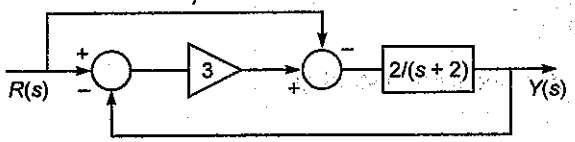
(a) (1) only (b) all, except (1) (c) all, except (3) (d) (1) and (2)

GATE 2005

53. A unity feedback system, having an open-loop gain $G(s)H(s)=\frac{k(1-s)}{(1+s)}$, becomes stable, when

(a) $|k|>1$ (b) $k>1$ (c) $|k|<1$ (d) $k<-1$

54. When subjected to a unit step input, the closed loop control system shown in the figure will have a steady state error of

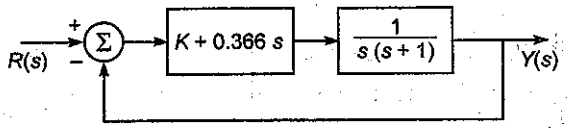


(a) –1.0 (b) –0.5 (c) 0 (d) 0.5

55. If the GH plane, the Nyquist plot of the loop transfer function $G(s)H(s)=\frac{\pi e^{-0.25s}}{s}$ passes through the negative real axis at the point

(a) $(-0.25, j0)$ (b) $(-0.5, j0)$ (c) $(-1, j0)$ (d) $(-2, j0)$

56. If the compensated system shown in the figure has a phase margin of 60° at the cross-over frequency of 1 rad/s, then value of the gain K is



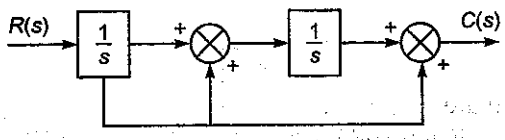
(a) 0.366 (b) 0.732 (c) 1.366 (d) 2.738

GATE 2004

57. For the equation $s^3-4s^2+s+6=0$, the number of roots in the left half of s-plane will be

(a) zero (b) one (c) two (d) three

58. For the block diagram shown in figure, the transfer function $\frac{C(s)}{R(s)}$ is equal to



(a) $\frac{s^2+1}{s^2}$ (b) $\frac{s^2+s+1}{s^2}$

(c) $\frac{s^2+s+1}{s}$ (d) $\frac{1}{s^2+s+1}$

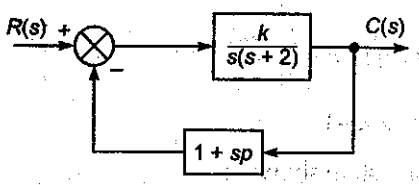
▼ Solution Poir

▼ *Solution Points*

59. The state variable description of a linear autonomous system is $\dot{X} = AX$, where X is the two-dimensional state vector and A is the system matrix given by $A = \begin{bmatrix} 0 & 2 \\ 2 & 0 \end{bmatrix}$. The roots of the characteristic equation are

(a) -2 and $+2$ (b) $-j2$ and $+j2$ (c) -2 and -2 (d) $+2$ and $+2$

60. The block diagram of a closed-loop control system is given by figure. The values of k and p such that the system has a damping ratio of 0.7 and an undamped natural frequency ω_a of 5 rad/s, are respectively equal to



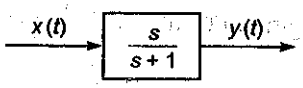
(a) 20 and 0.3 (b) 20 and 0.2
(c) 25 and 0.3 (d) 25 and 0.2

61. The unit impulse response of a second order under damped system starting from rest is given by

$$c(t) = 12.5\, e^{-6t} \sin 8t, t \geq 0$$

The steady state value of the unit step response of the system is equal to
(a) 0 (b) 0.25 (c) 0.5 (d) 1.0

62. In the system shown in figure, the input $x(t) = \sin t$. In the steady state, the response $y(t)$ will be



(a) $\frac{1}{\sqrt{2}} \sin(t - 45°)$ (b) $\frac{1}{\sqrt{2}} \sin(t + 45°)$

(c) $\sin(t - 45°)$ (d) $\sin(t + 45°)$

63. The open-loop transfer function of a unity feedback control system is given as $G(s) = \frac{as + 1}{s^2}$. The value of a to give a phase margin of 45° is equal to

(a) 0.141 (b) 0.441 (c) 0.841 (d) 1.141

GATE 2003

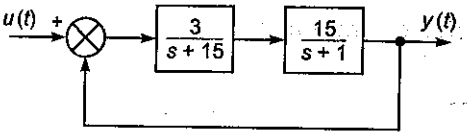
64. A control system with certain excitation is governed by the following mathematical equation

$$\frac{d^2x}{dt^2} + \frac{1}{2}\frac{dx}{dt} + \frac{1}{18}x = 10 + 5e^{-4t} + 2e^{-5t}$$

The natural time constants of the response of the system are
(a) $2s$ and $5s$ (b) $3s$ and $6s$ (c) $4s$ and $5s$ (d) $1/3s$ and $1/6s$

65. The block diagram shown in figure gives a unity feedback closed-loop control system. The steady state error in the response of the above system to unit step input is

▼ *Solution Points*



(a) 25% (b) 0.75% (c) 6% (d) 33%

66. The roots of the closed-loop characteristic equation of the system shown in above figure are

(a) −1 and −15 (b) 6 and 10 (c) −4 and −15 (d) −6 and −10

67. The following equation defines a separately excited DC motor in the form of a differential equation

$$\frac{d^2\omega}{dt^2}+\frac{B}{J}\frac{d\omega}{dt}+\frac{K^2}{LJ}\omega=\frac{K}{LJ}V_a$$

The above equation may be organized in the state space form as follows

$$\begin{bmatrix}\frac{d^2\omega}{dt^2}\\ \frac{d\omega}{dt}\end{bmatrix}=P\begin{bmatrix}\frac{d\omega}{dt}\\ \omega\end{bmatrix}+QV_a$$

where, the P matrix is given by

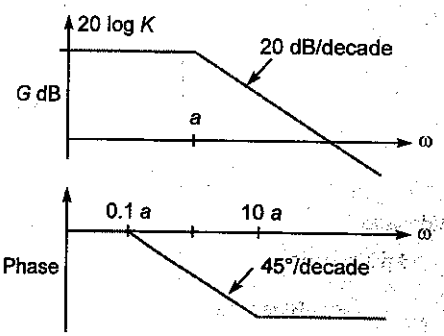
(a) $\begin{bmatrix}-\frac{B}{J} & -\frac{K^2}{LJ}\\ 1 & 0\end{bmatrix}$ (b) $\begin{bmatrix}-\frac{K^2}{LJ} & -\frac{B}{J}\\ 0 & 1\end{bmatrix}$

(c) $\begin{bmatrix}0 & 1\\ -\frac{K^2}{LJ} & -\frac{B}{J}\end{bmatrix}$ (d) $\begin{bmatrix}1 & 0\\ -\frac{B}{J} & -\frac{K^2}{LJ}\end{bmatrix}$

68. The loop gain GH of a closed-loop system is given by the following expression $\frac{K}{s(s+2)(s+4)}$. The value of K for which the system just becomes unstable is

(a) $K=6$ (b) $K=8$ (c) $K=48$ (d) $K=96$

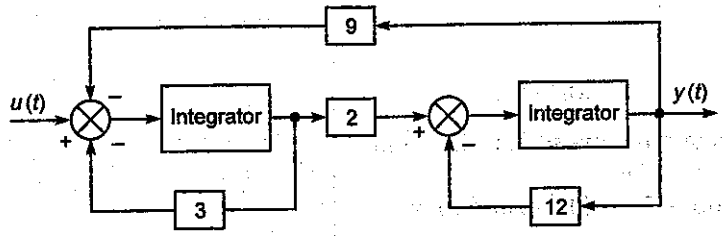
69. The asymptotic Bode plot of the transfer function $\frac{K}{1+\frac{s}{a}}$ is given in figure. The error in phase angle and dB gain at a frequency of $\omega=0.5\,a$ are respectively



(a) 4.9°, 0.97 dB (b) 5.7°, 3 dB (c) 4.9°, 3 dB (d) 5.7°, 0.97 dB

▼ *Solution Poin*

70. The block diagram of a control system is shown in figure. The transfer function $C(s) = Y(s)/U(s)$ of the system is



(a) $\dfrac{1}{18\left(1+\dfrac{s}{12}\right)\left(1+\dfrac{s}{3}\right)}$

(b) $\dfrac{1}{27\left(1+\dfrac{s}{6}\right)\left(1+\dfrac{s}{9}\right)}$

(c) $\dfrac{1}{27\left(1+\dfrac{s}{12}\right)\left(1+\dfrac{s}{9}\right)}$

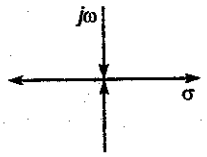
(d) $\dfrac{1}{27\left(1+\dfrac{s}{9}\right)\left(1+\dfrac{s}{3}\right)}$

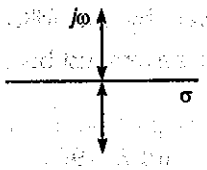
GATE 2002

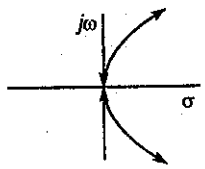
71. For the system $\dot{X} = \begin{bmatrix} 2 & 3 \\ 0 & 5 \end{bmatrix} X + \begin{bmatrix} 1 \\ 0 \end{bmatrix} u$, which of the following statements is true?

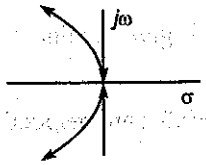
(a) The system is controllable but unstable
(b) The system is uncontrollable and unstable
(c) The system is controllable and stable
(d) The system is uncontrollable and stable

72. A unity feedback system has an open-loop transfer function $G(s) = \dfrac{K}{s^2}$. The root locus plot is

(a) 

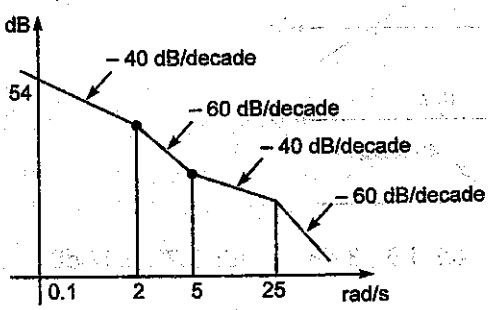
(b) 

(c) 

(d) 

GATE 2001

73. The asymptotic approximation of the log magnitude *versus* frequency plot of a minimum phase system with real poles and one zero is shown in figure. Its transfer function is



(a) $\frac{20(s+5)}{s(s+2)(s+25)}$ (b) $\frac{10(s+5)}{(s+2)^2(s+25)}$

(c) $\frac{20(s+5)}{s^2(s+2)(s+25)}$ (d) $\frac{50(s+5)}{s^2(s+2)(s+25)}$

▼ *Solution Poin*

GATE 2000

74. The characteristic equation of a feedback control system is

$$2s^4 + s^3 + 3s^2 + 5s + 10 = 0$$

The number of roots in the right half of s-plane are

(a) zero (b) 1 (c) 2 (d) 3

75. A unity feedback system has open-loop transfer function $G(s) = 125/[s(s+6)]$. The peak overshoot in the step input response of the system is approximately equal to

(a) 5% (b) 10% (c) 15% (d) 20%

76. Maximum phase lead of the compensator

$$D(s) = \frac{(0.5s+1)}{(0.05s+1)}, \text{ is}$$

(a) 52° at 4 rad/s (b) 52° at 10 rad/s

(c) 55° at 12 rad/s (d) None of these

Answers with Explanations

1. (a) Here, $|z| < 1$

$\therefore \quad |y| > 1$

Since, from the plot, z having positive real and imaginary part.

So, y should have positive real and negative imaginary part.

2. (b) ∵ Stability of open-loop system depends upon the pole location. As the pole is in the left half of s-plane.

Hence, system is stable.

For minimum phase system, there should be no poles and zeros in the right half of s-phase.

But there is one zero on right half of s-plane.

So, system is non-minimum phase.

3. (a) At $\angle G(j\omega) = -180$

Magnitude $X = 0.5$

$\therefore \quad GM = 20 \log\left(\frac{1}{0.5}\right) = 6 \text{ dB}$

At $|G(j\omega)| = 1$

Phase angle $= -150°$

$\therefore \quad PM = 180° + (-150°) = 30°$

4. (a) A steady state error $e_{ss} = 0.1 = \frac{1}{1+k}$ (for step input)

$\Rightarrow \quad k = 9$

$$G(s) = \frac{9}{s+1}$$

Now, input $r(t) = 10\,[u(t) - u(t-1)]$

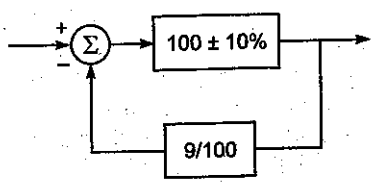
$$r(s) = 10\left[\frac{1-e^{-s}}{s}\right]$$

$$e_{ss} = \lim_{s \to 0} \frac{s \cdot r(s)}{1 + G(s)\,H(s)}$$

$$= \lim_{s \to 0} \frac{s \cdot \dfrac{10\,(1-e^{-s})}{s}}{\dfrac{s+10}{s+1}}$$

$$= \frac{0}{10} = 0$$

5. (a)



Forward gain $= A = 100 \pm 10\%$

Feedback attenuation $= \beta = \frac{9}{100}$

Overall system gain $= A_f = \frac{A}{1 + A\beta}$

For $A = 100 - 10 = 90$,

$$A_f = \frac{90}{1 + 90 \times \frac{9}{100}} = \frac{90}{9.1} \approx 9.89$$

For $A = 100 + 10 = 110$,

$$A_f = \frac{110}{1 + 110 \times \frac{9}{100}} = \frac{110}{10.9} = 10.09$$

$$\text{Actual overall gain} = \frac{100}{1 + 100 \times \frac{9}{100}} = 10$$

Tolerance $= 10 - 10.09 = -0.09 \approx -0.1 = 1\%$

and $10 - 9.89 = +0.11 \approx +0.1 = 1\%$

Hence, overall system gain $= 10 \pm 1\%$.

6. (c) Given, input $U(t) = 1$

$$U(s) = 1/s$$

The output response $= U(s)\,H(s)$

$$= \frac{1}{s} \times \frac{2}{(s+1)} = 2\left(\frac{1}{s} - \frac{1}{s+1}\right)$$

Output $y(t) = 2(1 - e^{-t})\,u(t)$

Final value at $t = \infty$, $y(t = \infty) = 2$

98% of final value $= 2 \times \frac{98}{100} = 1.96$

Let at $t = t_1$, 98% of final value reached.

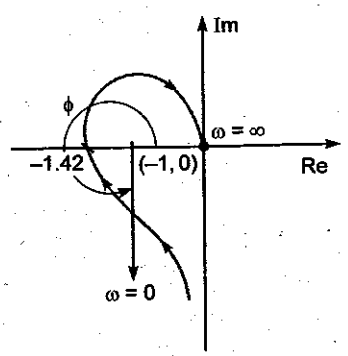
$$1.96 = 2(1 - e^{-t})$$

$$e^{-t} = 0.02$$

$$t = -\ln(0.02) = 3.9 \text{ s} \approx 4 \text{ s}$$

7. (d)

8. (d)



For closed-loop stability, the $(-1, j0)$ point in the $G(j\omega)$-plane must be to the left of the intersect of the Nyquist plot with the real axis, *i.e.*, as the plot includes the $(-1,0)$ point. Hence, the system is unstable. For calculation of number of poles in RHS,

$$\phi = (z-0.5)180°$$

Here $\phi = 270°$ (shown in figure)

$$270° = (z-0.5)180°$$

$$z - 0.5 = 1.5 \Rightarrow z = 2$$

Unstable with two poles in right half of s-plane.

9. (d) The Routh table given is

s^3	2	2
s^2	4	4
⋮		

Hence, the Routh array will be

$$2s^3 + 4s^2 + 2s + 4 = 0$$

$$2s^2(s+2) + 2(s+2) = 0$$

$$(s+2)(2s^2+2) = 0$$

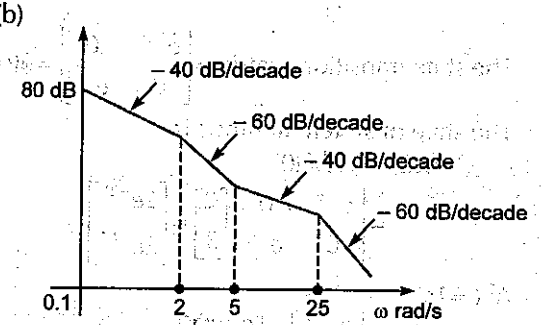
Hence, roots given by

$$s + 2 = 0 \Rightarrow s = -2$$

(one root in left half of s-plane)

$$2s^2 + 2 = 0 \Rightarrow s^2 = -1 \Rightarrow s = \pm j$$

10. (b)



At cross-over frequency $\omega_{c_1} = 2$, the slope change is –20 dB/dec

At cross-over frequency $\omega_{c_2} = 5$, the slope change is +20 dB/dec

At cross-over frequency $\omega_{c_3} = 25$, the slope change is –20 dB/dec

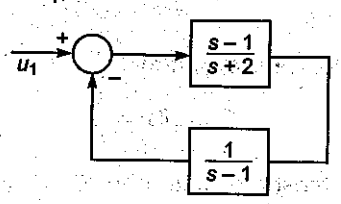
At frequency $\omega = 0.1$, the slope at – 40 dB/dec, hence at origin there are two poles.

$$T(s) = \frac{K(s+5)}{s^2(s+2)(s+25)}$$

Given at $\omega = 0.1$, gain is 80 dB.

$$\therefore\ 80 = 20\log\frac{5k}{(0.1)^2 50} \Rightarrow k = 999.99 \simeq 1000$$

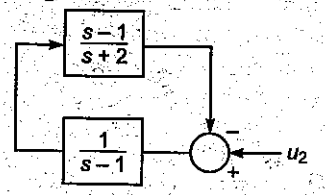
11. (d) For input u_1,



$$\text{The transfer function} = \frac{(s-1)/(s+2)}{1+\left(\frac{1}{s-1}\right)\left(\frac{s-1}{s+2}\right)} = \frac{(s-1)}{(s+3)}$$

For input u_1, the transfer function has pole in le hand side hence stable.

For input u_2,



The transfer function

$$T(s) = \frac{1/s-1}{1+\frac{s-1}{s+2}\times\frac{1}{s-1}} = \frac{s+2}{(s-1)(s+3)}$$

The transfer function has the pole at right ha s-plane. Hence, system is unstable.

12. (b) The characteristics equation of transfer function

$$4s^2 - 2s + 1 = 0$$

Comparing with the general expression

$$a_{n-2}s^2 - a_{n-1}s + a_n = 0$$

Hence, $a_{n-2} = 4$

$a_{n-1} = -2$

$a_n = 1$

The matrix

$$A = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -a_n & -a_{n-1} & -a_{n-2} \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -1 & 2 & -4 \end{bmatrix}$$

13. (c) $T(s) = \dfrac{s^2+4}{(s+1)(s+4)}$

Putting $s = j\omega$,

$$T(j\omega) = \frac{(j\omega)^2+4}{(j\omega+1)(j\omega+4)} = \frac{-\omega^2+4}{(j\omega+1)(j\omega+4)}$$

For system output to be zero,

$$-\omega^2 + 4 = 0$$

$$\omega = +2 \text{ rad/s}$$

14. (a) For given root locus it indicates that the breakaway point is at 0.

Consider for first option,

$$G(s)H(s)=\frac{k}{s^3}$$

Characteristic equation is

$$1+G(s)H(s)=0$$

$$1+\frac{k}{s^3}=0$$

$$s^3+k=0$$

For breakaway point,

$$\frac{dk}{ds}=0$$

$$3s^2=0$$

$$s=0$$

15. (a) $G(s)H(s)=\dfrac{(s+1)}{s^2}$

Putting $s=j\omega$,

$$G(j\omega)H(j\omega)=\frac{(1+j\omega)}{-\omega^2}$$

At phase cross-over frequency ω_{pc},

$$\angle G(j\omega)H(j\omega)|_{\omega=\omega_{pc}}=-180°$$

$$-180°+\tan|\,\omega_{pc}=-180°$$

$$\omega_{pc}=0$$

$$|G(j\omega)H(j\omega)|_{\omega=\omega_{pc}}=\frac{1}{0}=\infty$$

$$GM=\frac{1}{|G(j\omega)H(j\omega)|_{\omega=\omega_{pc}}}$$

$$=\frac{1}{\infty}=0$$

16. (a) The phase margin is the amount of additional phase lag at gain cross-over frequency required to bring the system to refers of instability.

Hence, $PM(\gamma)=180°+\angle G(j\omega)H(j\omega)|_{\omega=\omega_{gc}}$

$=180°+(-180°)$

[As plot passes through $(-1,0)$ point]

$=0°$

17. (b) From intial value theorem,

$$\lim_{t\to 0} f(t)=\lim_{s\to\infty} sF(s)$$

$$=\lim_{s\to\infty} s\times\frac{5}{s(s^2+3s+2)}$$

$=5/2$

18. (c) For a tachometer,

The output voltage $e(t)=K_t\dfrac{d\theta}{dt}$

Taking Laplace transform,

$$E(s)=K_t s\,\theta(s)$$

$$\frac{E(s)}{Q(s)}=K_t s$$

19. (a) For lead compensator,

$$T(s)=\frac{K\left(1+\dfrac{s}{a}\right)}{\left(1+\dfrac{s}{b}\right)}$$

General expression of lead compensator

$$T(s)=K_c\,\alpha\left(\frac{Ts+1}{\alpha Ts+1}\right)\text{ where } 0<\alpha<1$$

Hence, $T=\dfrac{1}{a}$

$$\alpha T=\frac{1}{b}$$

$$\alpha=\frac{a}{b}$$

$$0<\alpha<1 \Rightarrow 0<\frac{a}{b}<1 \Rightarrow a<b$$

20. (a) Given, $X(0)=\begin{bmatrix}2\\3\end{bmatrix}$

The state transition matrix $=\begin{bmatrix}e^{-2t} & 0\\ 0 & e^{-t}\end{bmatrix}=\phi(t)$

The state of system at time t is

$$X(t)=\phi(t)\times(0)$$

$$=\begin{bmatrix}e^{-2t} & 0\\ 0 & e^{-t}\end{bmatrix}\begin{bmatrix}2\\3\end{bmatrix}=\begin{bmatrix}2e^{-2t}\\3e^{-t}\end{bmatrix}$$

At $t=1$s,

$$X(1)=\begin{bmatrix}2e^{-2}\\3e^{-1}\end{bmatrix}=\begin{bmatrix}0.271\\1.100\end{bmatrix}$$

21. (d) For closed-loop stability, the $(-1, j0)$ point in the $G(j\omega)$ plane must be left of the intersect of the Nyquist plot with the real axis. As the plot includes the $(-1,0)$ point. Hence, the system is unstable.

For calculations of number of poles in RHS.

$$\phi=(z-0.5)180°$$

Here, $\phi=270°$

$$270°=(z-0.5)180°$$

$$z=2$$

i.e., system is unstable with two poles in the right half of s-plane.

22. (a) Given, state space equation

$$\dot{x} = \begin{bmatrix} -3 & 1 \\ 0 & -2 \end{bmatrix} x$$

Comparing with general equation

$$\dot{x} = Ax$$

$$A = \begin{bmatrix} -3 & 1 \\ 0 & -2 \end{bmatrix}$$

The solution

$$x(t) = L^{-1}[sI - A]^{-1} \times [0]$$

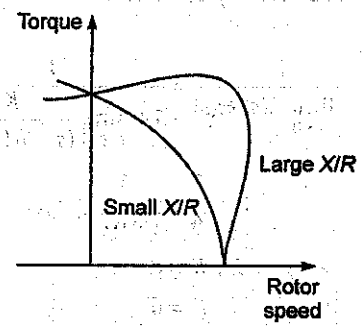
$$= L^{-1}\begin{bmatrix} s+3 & -1 \\ 0 & s+2 \end{bmatrix}^{-1} \times [0]$$

$$= L^{-1}\begin{bmatrix} \frac{1}{s+2} & \frac{-1}{(s+3)(S+2)} \\ 0 & \frac{1}{s+3} \end{bmatrix} \times [0]$$

$$x(t) = \begin{bmatrix} e^{-3t} & e^{-2t} - e^{-3t} \\ 0 & e^{-2t} \end{bmatrix} \begin{bmatrix} 10 \\ -10 \end{bmatrix}$$

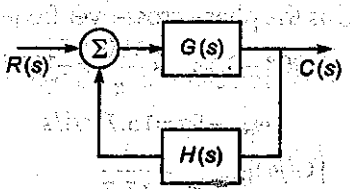
$$x_{st} = \lim_{t \to \infty} x(t) = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

23. (b) The torque-speed characteristics of induction motor



In normal induction motors, X/R ratio is generally kept high so high obtain the maximum torque. The rotor of the servomotor is built to provide linear characteristics by making high resistance so that its X/R ratio is small. Hence, ratio X/R for servomotor is less than that of a normal induction motor.

24. (c)



We can write

$$T(S) = \frac{G}{1+GH}$$

S_G^T = Sensitivity of T with respect to G (forward path)

$$= \frac{1}{1+GH}$$

S_H^T = Sensitivity of T with respect to H (feedback path)

$$= -\frac{GH}{1+GH} \simeq -1 \qquad \text{(directly effect the output)}$$

25. (b) The given table can be used to determine steady state error for different input and type system.

	Type 0	Type 1	Type 2	where
Unit step	$1/(1+K_p)$	0	0	$K_p = \lim_{s \to 0} G(s)$
Unit ramp	∞	∞	$1/K_a$	$K_V = \lim_{s \to 0} sG(s)$
Unit parabolic	∞	∞	$1/K_a$	$K_a = \lim_{s \to 0} s^2G(s)$

26. (c) The transfer function $T(s) = \dfrac{\text{LT of } y(t)}{\text{LT of } x(t)}$

$$= \frac{L[te^{-t}]}{L[u(t)]} = \frac{1/(s+1)^2}{1/s} = \frac{s}{(s+1)^2}$$

27. (a)

$$\frac{Y(s)}{R(s)} = \frac{1}{s^2 + s(k+1) + 1}$$

The characteristic equation is

$$s^2 + s(K+1) + 1 = 0$$

Comparing with general expression

$$s^2 + 2\xi\omega_n s + \omega_n^2 = 0$$

$$\omega_n = 1$$

$$2\xi\omega_n = k+1$$

$$\Rightarrow \qquad \xi = \left(\frac{k+1}{2}\right)$$

and peak overshoot $M_p = e^{-\pi\xi/(1-\xi^2)}$

Therefore, gain k will influence mainly peak overshoot.

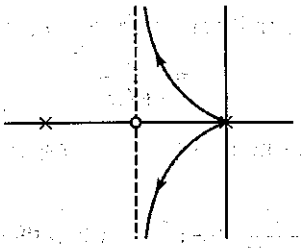
28. (a) Number of poles = 3

Number of zeros = 2

$$\text{Centroid} = \frac{-2 - \left(-\frac{2}{3}\right)}{3-1} = \frac{-2}{3}$$

$$\text{Asymptotes} = \frac{(2q+1)180}{p-2}$$

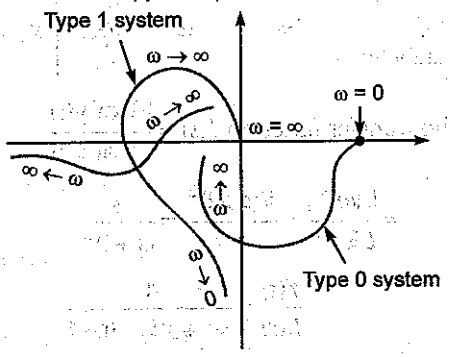
$$= 90° \text{ and } 270°$$



29. (d) The general shape of polar plot for the transfer function

$$G(j\omega)=\frac{K(1+j\omega T_a)(1+j\omega T_b)....}{(j\omega)^{\lambda}(1+j\omega T_1)(1+j\omega T_2)....}$$

For $X=0 \Rightarrow$ Type 0 system
For $X=1 \Rightarrow$ Type 1 system
For $X=2 \Rightarrow$ Type 2 system



The given transfer function is type 0 system.

30. (c) Given $\dot{x}=Ax+Bu$

with $A=\begin{bmatrix}-1 & 2\\ 0 & 2\end{bmatrix}, B=\begin{bmatrix}0\\ 1\end{bmatrix}$

The characteristic equation is

$$[\lambda I-A]=0$$
$$\begin{bmatrix}\lambda & 0\\ 0 & \lambda\end{bmatrix}-\begin{bmatrix}-1 & 2\\ 0 & 2\end{bmatrix}=0$$
$$\begin{bmatrix}\lambda+1 & -2\\ 0 & \lambda-2\end{bmatrix}=0$$
$$(\lambda+1)(\lambda-2)+2=0$$
$$\lambda^2-\lambda-2+2=0$$
$$\lambda(\lambda-1)=0$$
$$\lambda=0,1$$

The root lies in the +ve half of s-plane. Hence, the system is unstable.
For the case,

$$[B\ |\ AB]=\begin{bmatrix}0 & \begin{bmatrix}-1 & 2\\ 2 & 2\end{bmatrix}\begin{bmatrix}0\\ 1\end{bmatrix}\\ 1 & \end{bmatrix}=\begin{bmatrix}0 & 2\\ 1 & 2\end{bmatrix}=\text{non-singular}$$

The system is therefore, controllable.

31. (c) $s(s+1)(s+3)+k(s+2)=0$

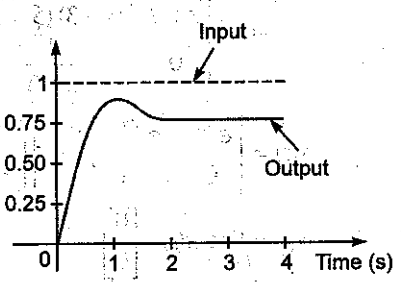
$$1+k\frac{(s+2)}{s(s+1)(s+3)}=0$$

Open loop pole is $0,-1,-3$, *i.e.*, $n=3$
Open loop zero is -2, *i.e.*, $m=1$

The n-m branches of the root locus which tend to infinity along the asymptotes has centriod

$$-\sigma_A=\frac{\text{Sum of real part of pole}-\text{Sum of real part of zero}}{\text{Number of poles}-\text{Number of zeros}}$$
$$=\frac{0-1-3-(-2)}{3-1}=-1$$

32. (d)



From the given response,
We can say steady state error $=1-0.75=1/4$

$$e_{ss}=\frac{1}{1+\lim_{s\to 0}G(s)H(s)}=\frac{1}{1+\lim_{s\to 0}\frac{K}{(s+1)(s+2)}}$$ H(s)= (Give

$$\Rightarrow \quad \frac{1}{4}=\frac{1}{1+K/2}$$
$$\Rightarrow \quad 2+K=8$$
$$\Rightarrow \quad K=6$$

33. (d) Given,

$$G(s)=\frac{e^{-0.1s}}{s}$$
$$G(j\omega)=\frac{e^{-0.1(j\omega)}}{(j\omega)}$$

Let ω_{pc} is the phase cross-over frequency.

$$-90°-0.1\omega_{pc}\times\frac{180}{\pi}=-180°$$
$$\omega_{pc}=5\pi=15.7 \text{ rad/s}$$
$$|G(j\omega)|_{\omega=\omega_{pc}}=\frac{1}{15.7}$$

Gain margin $GM=20\log|G(j\omega)|_{\omega=\omega_{pc}}$

$$=20\log\left(\frac{1}{15.7}\right)=-23.9 \text{ dB}$$

34. (c) Given,

$$\frac{dx_1(t)}{dt} = -3x_1(t) + x_2(t) + 2u(t) \quad \ldots(i)$$

$$\frac{dx_2(t)}{dt} = -2x_2(t) + u(t) \quad \ldots(ii)$$

$$y_1(t) = x_1(t) \quad \ldots(iii)$$

Taking Laplace transform,

$$sX_1(s) - 3X_1(s) + X_2(s) + 2u(s)$$

$$X_1(s)(s+3) = X_2(s) + 2u(s) \quad \ldots(a)$$

$$sX_2(s) = -2X_2(s) + u(s)$$

$$(s+2)X_2(s) = u(s)$$

$$X_2(s) = u(s)/s + 2$$

Putting the value of $X_2(s)$ in, Eq. (a),

$$X_1(s)(s+3) = u(s)\left[\frac{1}{s+2} + 2\right]$$

$$X_1(s) = u(s)\frac{2(s+2)+1}{(s+2)(s+3)}$$

$$Y(s) = X_1(s)$$

$$Y(s) = u(s)\left[\frac{2s+5}{s^2+5s+6}\right]$$

$$\text{Transfer function} = \frac{\text{LT of } y(t)}{\text{LT of } u(t)} = \frac{Y(s)}{u(s)} = \frac{2s+5}{s^2+5s+6}$$

35. (b) The state transition matrix

$$\phi(t) = L^{-1}[(SI - A)^{-1}]$$

$$= L^{-1}\left[\begin{bmatrix} s & 0 \\ 0 & s \end{bmatrix} - \begin{bmatrix} -3 & 1 \\ 0 & -2 \end{bmatrix}\right]^{-1} = L^{-1}\begin{bmatrix} s+3 & -1 \\ 0 & s+2 \end{bmatrix}^{-1}$$

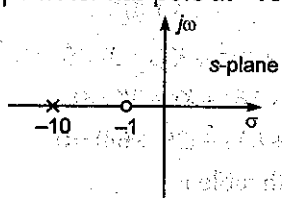
$$= L^{-1}\left\{\frac{1}{(s+3)(s+2)}\begin{bmatrix} s+2 & 1 \\ 0 & s+3 \end{bmatrix}\right\}$$

$$= \left\{L^{-1}\begin{bmatrix} \frac{1}{s+3} & \frac{1}{s+2} - \frac{1}{s+3} \\ 0 & \frac{1}{s+2} \end{bmatrix}\right\}$$

$$= \begin{bmatrix} e^{-3t} & e^{-2t} - e^{-3t} \\ 0 & e^{-2t} \end{bmatrix}$$

36. (a) $C_1 = \dfrac{10(s+1)}{(s+10)}$

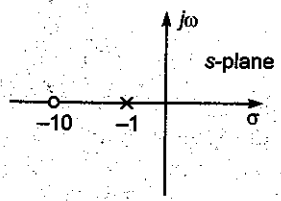
The compensator has pole at −10 and zero at −1.



Zero is near to origin. Hence, zero dominant the characteristics. Hence, it is lead compensator.

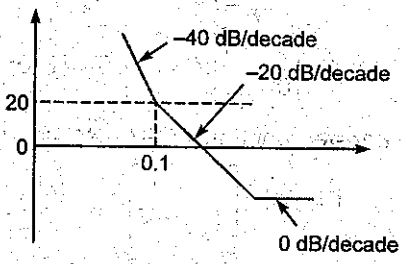
$$\therefore C_2 = \frac{s+10}{10(s+1)}$$

The compensator has pole at −1 and zero at −10.



Pole is near to origin. Hence, pole dominant the characteristics. Hence, it is lag compensator.

37. (c)



There is slope of −40 dB decade. Hence, at origin there is two poles.

At $\omega = 0.1$, the slope changes from $-20 - (-40) = +20$ dB/decade. Hence, there is zero at $\omega = 0.1$.

At $\omega = 10$, the slope change $= 0 - (-20) = 20$ dB/decade. Hence, at $\omega = 10$, there is simple zero.

Hence, there is two poles and two zeros.

38. (c) The characteristic equation is

$$1 + G(s)H(s) = 0$$

$$1 + \frac{K}{s(s+3)(s+10)} = 0$$

$$s(s^2 + 3s + 10s + 30) + K = 0$$

$$s^3 + 13s^2 + 30s + K = 0$$

The Routh table is

s^3	1	30
s^2	13	K
s^1	$\frac{390 - K}{13}$	
1	K	

As $K > 0$; the system to be stable.

$$390 - K < 0$$

$$K < 390$$

$$0 < K < 390$$

39. (c) The characteristic equation is

$$s^2 + 20s + 100 = 0$$

Comparing with the general expression

$$s^2 + 2\xi\omega_n s + \omega_n^2 = 0$$

i.e.,

$$\omega_n = 10$$

$$2\xi\omega_n = 20$$

$$\xi = 1$$

Critically damped system.

40. (a)

41. (d) Given, $\dot{x} = Ax + Bu$

$$y = Cx$$

$$A = \begin{bmatrix} 0 & 1 \\ 0 & -2 \end{bmatrix}; B = \begin{bmatrix} 0 \\ 1 \end{bmatrix}; C = [1 \quad 0]$$

The transfer function $= C[sI - A]^{-1}B + D$

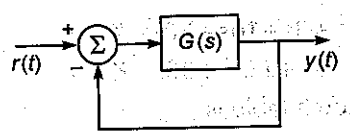
$$[sI - A] = \begin{bmatrix} s & -1 \\ 0 & s+2 \end{bmatrix}$$

$$[sI - A]^{-1} = \frac{\begin{bmatrix} (s+2) & 1 \\ 0 & s \end{bmatrix}}{s(s+2)} = \begin{bmatrix} \frac{1}{s} & \frac{1}{s(s+2)} \\ 0 & \frac{1}{s+2} \end{bmatrix}$$

$$G(s) = [1 \quad 0]\begin{bmatrix} \frac{1}{s} & \frac{1}{s(s+2)} \\ 0 & \frac{1}{s+2} \end{bmatrix}\begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

$$= [1 \quad 0]\begin{bmatrix} \frac{1}{s(s+2)} \\ \frac{1}{s+2} \end{bmatrix} = \frac{1}{s(s+2)}$$

42. (a)



Steady state error

$$e_{ss} = \lim_{s \to 0} \frac{sR(s)}{1 + G(s)H(s)}$$

$$r(t) = u(t)$$

$$R(s) = \frac{1}{s}$$

$$e_{ss} = \lim_{s \to 0} \frac{s \times \frac{1}{s}}{1 + G(s)} = \frac{1}{1 + \lim_{s \to 0} G(s)}$$

$$\lim_{s \to 0} G(s) = \lim_{s \to 0} \frac{1}{s(s+2)} = \infty$$

$$\Rightarrow \quad e_{ss} = \frac{1}{\infty} = 0$$

43. (b)

$$G(s) = \frac{1}{s(s+1)(s+2)}$$

$$= \frac{1}{j\omega(j\omega+1)(j\omega+2)}$$

$$= \frac{-j}{\omega(1+j\omega)(2+j\omega)}$$

Multiplying $(1-j\omega)(2-j\omega)$ to numerator and denominator.

$$G(s) = \frac{-j(1-j\omega)(2-j\omega)}{\omega(1+\omega^2)(4+\omega^2)} = \frac{-j(2-2j\omega-j\omega-\omega^2)}{\omega(1+\omega^2)(4+\omega^2)}$$

$$= \frac{-3-j(2-\omega^2)}{\omega(1+\omega^2)(4+\omega^2)} = \frac{-3\omega}{\omega(1+\omega^2)(4+\omega^2)}$$

$$- \frac{j(2-\omega^2)}{\omega(1+\omega^2)(4+\omega^2)}$$

$$x = \frac{-3}{(1+\omega^2)(4+\omega^2)} \qquad \text{as } \omega \to 0; x = -3/4$$

$$y = \frac{-(2-\omega^2)}{\omega(1+\omega^2)(4+\omega^2)} \qquad \text{as } \omega \to 0; y \to \infty$$

44. (d)

45. (b) $G(z) = z^{-1} + z^{-2}$

The characteristic equation is

$$1 + KG(z) = 0$$

$$1 + K(z^{-1} + z^{-2}) = 0$$

$$1 + \frac{K}{z} + \frac{K}{z^2} = 0$$

$$z^2 + Kz + K = 0$$

For system to be stable

$$|K| < 1$$

$$-1 < K < 1$$

46. (b) The characteristic equation is

$$1 + G(s)H(s) = 0$$

$$1 + \frac{K(s+3)}{(s+8)^2} = 0$$

$$(s+8)^2 + K(s+3) = 0$$

$$s^2 + 64 + 16s + Ks + 3K = 0$$

$$s^2 + (16 + K)s + (3K + 64) = 0$$

The Routh table is

s^2	1	$(3K+64)$
s	$16+K$	0
s	$3K+64$	0

For sustained oscillation, all the elements of row is zero.

$$\therefore \quad 16+K=0$$

$$K=-16$$

Putting the value in characteristic equation,

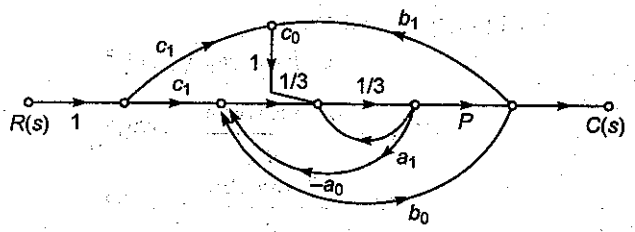
$$s^2+(16-16)s+(3\times-16+64)=0$$

$$s^2=-16$$

$$s=\pm j4=\pm j\omega$$

$$\omega = 4 \text{ rad/s}$$

47. (d) The signal flow graph



Using Mason's gain formula,

$$\text{Transfer function}=\frac{\frac{c_0p}{s^2}+\frac{pc_1}{s}}{1+\frac{a_1}{s}+\frac{a_0}{s^2}-\frac{pb_0}{s^2}-\frac{pb_1}{s}}$$

$$=\frac{(c_0+c_1s)\,p}{(s^2+a_1s+a_0)-p[b_0+sb_1]}$$

$$=\frac{\frac{(c_0+c_1s)\,p}{(s^2+a_1s+a_0)}}{1-\frac{p(b_0+sb_1)}{s^2+a_1s+a_0}}$$

Comparing with reduced form,

$$\frac{XYP}{1-YPZ} \Rightarrow X=(c_0+c_1s)$$

$$Y=\frac{1}{s^2+a_1s+a_0}$$

$$Z=(b_0+sb_1)$$

48. (a) Steady state value of c

$$=\lim_{s\to0} sE(s)\frac{k_i}{s}$$

$$=\lim_{s\to0}\frac{s.\frac{1}{s}.\frac{k_i}{s}}{\left[1+\left(k_p+\frac{k_i}{s}\right)\left(\frac{\omega^2}{s^2+2\xi\omega_ns+\omega_n^2}\right)\right]}$$

$$=1$$

49. (a) $$H(j\omega)=\frac{10^4(1+j\omega)}{(10+j\omega)(100+j\omega)^2}$$

$$H(s)=\frac{10^4(1+s)}{(10+s)(100+s)^2}$$

$$=\frac{10^4(1+s)}{10\left(1+\frac{s}{10}\right)(100)^2\left(1+\frac{s}{100}\right)^2}$$

$$=\frac{0.1(1+s)}{\left(1+\frac{s}{10}\right)\left(1+\frac{s}{100}\right)^2}$$

Gain $=0.1$

Gain in dB $=20\log 0.1=-20$ dB

Bode magnitude plot starts at – 20 dB.

50. (b) The characteristic equation is

$$(s^2-4)(s+1)+k(s-1)=0$$

$$1+\frac{k(s-1)}{(s+1)(s^2-4)}=0$$

$$G(s)H(s)=\frac{(s-1)}{(s+1)(s^2-4)}$$

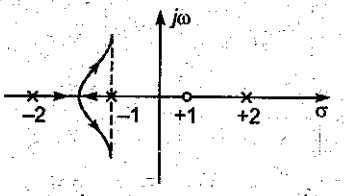
Zero at 1 and poles at -2 and $+2$ and -1.

$$\text{The centriod}=\frac{\text{Real part of pole}-\text{Real part of zero}}{\text{Number of poles}-\text{Number of zeros}}$$

$$=\frac{2-1-2-1}{2}=-1$$

$$\text{Asymptotes} \quad =\pm(n-m)\frac{180^\circ}{2}$$

$$=\pm90^\circ$$



Root locus doesn't exist at right of pole at -1 because number of poles and zeros to right is 0 (even).

51. (a)

52. (a) For stable closed-loop system, the Nyquist plot must not encircle the (−1,0) point. The plot (1) doesn't enclose (−1,0) point whereas other enclosed.

53. (c) The characteristic equation is

$$1+G(s)H(s)=0$$

$$1+\frac{k(1-s)}{(1+s)}=0$$

$$s+1+k-ks=0$$

$$s(1-k)+(1+k)=0$$

The Routh table is

s	$1-k$
s^0	$1+k$

For system to stable,

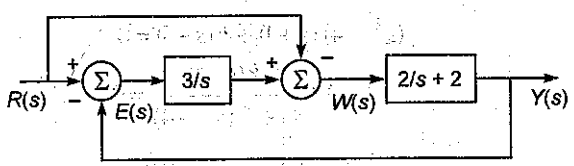
$$1-k>0$$

$$k<1$$

and $1+k>0$ *i.e.*, $-1<k<1$ or $|k|<1$

$$k>-1$$

54. (c) Replacing the integrator by $3/s$, the block diagram



We can write

$$W(s)=\{R(s)-Y(s)\}\,3/s-R(s)$$

and $$Y(s)=\frac{2}{s+2}W(s)$$

Putting the value of $W(s)$,

$$Y(s)=\frac{2}{s+2}\left[\{R(s)-Y(s)\}\frac{3}{s}-R(s)\right]$$

$$Y(s)=\frac{2}{s+2}\left[R(s)\left[\frac{3}{s}-1\right]-\frac{3}{s}Y(s)\right]$$

$$Y(s)\left[1+\frac{2\times 3}{s(s+2)}\right]=R(s)\left[\frac{3-s}{s}\times\frac{2}{s+2}\right]$$

$$Y(s)\left[\frac{s^2+2s+6}{s(s+2)}\right]=R(s)\frac{2(3-s)}{s(s+2)}$$

$$\frac{Y(s)}{R(s)}=\frac{2(3-s)}{s^2+2s+6}$$

Error $$E(s)=R(s)-Y(s)$$

$$=R(s)\left[1-\frac{Y(s)}{R(s)}\right]$$

$$=R(s)\left[1-\frac{2(3-s)}{s^2+2s+6}\right]$$

$$=R(s)\left[\frac{s^2+4s}{s^2+2s+6}\right]$$

Steady state error

$$e_{ss}=\lim_{s\to 0}sE(s)=\lim_{s\to 0}s\times\frac{1}{s}\left[\frac{s^2+4s}{s^2+2s+6}\right]$$

$$=0$$

55. (b) $$G(s)H(s)=\frac{\pi e^{-0.25s}}{s}$$

$$G(j\omega)H(j\omega)=\frac{\pi e^{-0.25(j\omega)}}{j\omega}$$

At negative real axis,

$$\angle G(j\omega)H(j\omega)=-180°$$

$$-190°-\frac{0.25\times 180\omega}{\pi}=-180°$$

$$-90°-\frac{0.25\omega\times 180}{\pi}=-180°$$

$$-\frac{0.25\,\omega}{\pi}=\frac{-90}{180}-\frac{1}{2}$$

$$G(j\omega)H(j\omega)=\frac{\pi[\cos(0.25\omega)-j\sin(0.25\,\omega)]}{j\omega}$$

$$=-\frac{\pi}{\omega}\sin(0.25\,\omega)-j\frac{\pi}{\omega}\cos(0.25\,\omega)$$

$$|G(j\omega)H(j\omega)|_{\omega=2\pi}=-\frac{\pi}{2\pi}\sin(0.25\times 2\pi)-j0$$

$$=-\frac{1}{2}-j0$$

Hence, passes through negative real axis at $(-0.5, j0)$.

56. (c) $$G(s)H(s)=\frac{k+j0.366s}{s(s+1)}$$

$$G(j\omega)H(j\omega)=\frac{k+j0.366\omega}{j\omega(j\omega+1)}$$

$$\angle G(j\omega)H(j\omega)=\tan^{-1}\left(\frac{0.366\omega}{k}\right)-90°-\tan^{-1}(\omega)$$

At ω_{gc}, PM $=60°$

$$\therefore 180°+\tan^{-1}\left(\frac{0.366\,\omega_{gc}}{k}\right)-90°-\tan^{-1}(\omega_{gc})=60°$$

given $\omega_{gc}=1$ rad/s

$$180°+\tan^{-1}\left(\frac{0.366}{k}\right)-90°-\tan^{-1}(1)=60°$$

$$\tan^{-1}\left(\frac{0.366}{k}\right)=90+60+45-180=15$$

$$\Rightarrow \quad \frac{0.366}{k}=\tan(15)$$

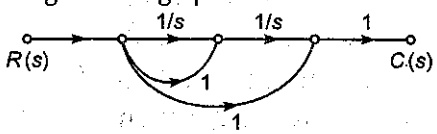
$$k=\frac{0.366}{0.267}=1.366$$

57. (b) The Routh table is

s^3	1	1	0
s^2	−4	6	0
s	2.5	0	0
1	6		

There are two time sign changes in the first column of Routh table. Hence, number of roots in the right half of s-plane is 2. Hence, root in left half s-plane is $3 - 2 = 1$.

58. (b) The signal flow graph is



Using Mason's gain formula,

$$\text{Transfer function}=\frac{p_1\Delta_1+p_2\Delta_2+p_3\Delta_3}{\Delta}$$

Here, $\Delta_1=\Delta_2=\Delta_3=\Delta=1$

$$p_1=\frac{1}{s^2}$$

$$p_2=1$$

$$p_3=\frac{1}{s}$$

$$\text{Transfer function}=\frac{1}{s^2}+1+\frac{1}{s}$$

$$=\frac{1+s^2+2}{s^2}$$

$$=\frac{s^2+s+1}{s^2}$$

59. (a) Given, matrix $A=\begin{bmatrix}0 & 2\\ 2 & 0\end{bmatrix}$

Characteristic equation is

$$\lambda I-A=0$$

$$\begin{bmatrix}\lambda & -2\\ -2 & \lambda\end{bmatrix}=0$$

$$\lambda^2-4=0$$

$$\lambda=\pm 2$$

Hence, roots are − 2 and + 2.

60. (d) $\text{Transfer function}=\dfrac{\dfrac{k}{s(s+2)}}{1+\dfrac{k}{s(s+2)}\times(1+sp)}$

$$=\frac{k}{s^2+2s+ksp+k}$$

$$=\frac{k}{s^2+(2+kp)s+k}$$

The characteristic equation will be

$$s^2+(2+kp)s+k=0$$

Comparing with general expression

$$s^2+2\xi\omega_n s+\omega_n^2=0$$

$$\omega_n=\sqrt{k} \Rightarrow k=25$$

$$2\xi\omega_n=2+kp$$

$$2\times\xi\times\sqrt{k}=2+kp$$

$$2\times 5\times\xi=2+25p$$

$$2+5p=10\times\xi=7$$

$$25p=5$$

$$p=\frac{1}{5}=0.2$$

61. (d) Given, unit impulse response

$$C(t)=12.5\,e^{-6t}\sin 8t$$

Transfer function = LT[impulse response]

Taking Laplace transform,

$$\text{Transfer function}=C(s)=\frac{12.5\times 8}{(s+6)^2+8^2}$$

$$=\frac{100}{s^2+12s+100}$$

The output $Y(s)$ for unit step input

$$Y(s)=C(s)\cdot R(s)$$

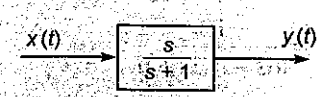
$$Y(s)=\frac{1}{s}\times\frac{100}{s^2+12s+100}$$

$$\text{Steady state output}=\lim_{s\to 0} sY(s)$$

$$=\lim_{s\to 0} s\times\frac{1}{s}\times\frac{100}{s^2+12s+100}$$

$$=\frac{100}{100}=1$$

62. (b)



Given, $x(t)=\sin t$

i.e., $\omega=1$ rad/s

$$H(s)=\frac{s}{s+1}$$

$$H(j\omega)=\frac{j\omega}{1+j\omega}=\frac{j}{1+j}$$
$$=\frac{1}{\sqrt{2}}\angle 90°-45°$$
$$=\frac{1}{\sqrt{2}}\angle 45°$$
$$y(t)=\frac{1}{\sqrt{2}}\sin(t+45°)$$

63. (c) Given that

Phase margin = 45°

$$180°+\angle G(j\omega)H(j\omega)|_{\omega_{gc}}=45°$$
$$\angle G(j\omega)H(j\omega)|_{\omega_{gc}}=-135°$$

Given that

$$G(s)H(s)=\frac{1+as}{s^2}$$
$$G(j\omega)H(j\omega)=\frac{1+aj\omega}{(j\omega)^2}=\frac{1+ja\omega}{-\omega^2}$$
$$\angle G(j\omega)H(j\omega)=-180°+\tan^{-1}(a\omega)$$
$$-180°+\tan^{-1}(a\omega)_{gc}=-135$$
$$\tan^{-1}a\omega_{gc}=45°$$
$$a\omega_{gc}=1$$
$$\omega_{gc}=\frac{1}{a}$$
$$|G(s)H(s)|_{\omega_{gc}}=\frac{\sqrt{1+\omega^2a^2}}{\omega^2}=\frac{\sqrt{1+\frac{1}{a^2}\cdot a^2}}{\frac{1}{a^2}}=1$$
$$\Rightarrow \quad \frac{1}{a^2}=\sqrt{2}$$
$$\Rightarrow \quad a^2=\frac{1}{2^{1/2}}$$
$$\Rightarrow \quad a=\frac{1}{2^{1/4}}=0.841$$

64. (b) The transfer function of the system

$$T(s)=\frac{1}{s^2+\frac{1}{2}s+\frac{1}{18}}=\frac{18}{18s^2+9s+1}$$
$$=\frac{18}{(6s+1)(3s+1)}$$

The general expression $=\frac{A}{(T_1s+1)(T_2s+1)}$

Hence, time constants T_1 and T_2 will be 3s and 6s.

65. (a) $T(s)=\frac{y(s)}{u(s)}=\frac{\left(\frac{3}{s+15}\right)\left(\frac{15}{s+1}\right)}{1+\left(\frac{3}{s+15}\right)\left(\frac{15}{s+1}\right)}=\frac{45}{s^2+16s+60}$

$$c_{ss}=\lim_{s\to 0}sE(s)=\lim_{s\to 0}s[u(s)-y(s)]$$
$$=\lim_{s\to 0}su(s)\left[1-\frac{y(s)}{u(s)}\right]$$
$$=\lim_{s\to 0}s\times\frac{1}{s}\left[1-\frac{45}{s^2+16s+60}\right]$$
$$=\lim_{s\to 0}\frac{s^2+16s+15}{s^2+16s+60}=\frac{15}{60}=\frac{1}{4}=25\%$$

66. (d) The characteristic equation is

$$s^2+16s+60=0$$
$$s^2+10s+6s+60=0$$
$$s(s+10)+6(s+10)=0$$
$$(s+6)(s+10)=0$$

Hence, the roots are (–6, –10).

67. (a) The given differential equation

$$\frac{d^2\omega}{dt^2}+\frac{B}{J}\frac{d\omega}{dt}+\frac{K^2}{LJ}\omega=\frac{K}{LJ}V_a$$

The term $\frac{d\omega}{dt}$ can be written as

$$\frac{d\omega}{dt}=1\cdot\frac{d\omega}{dt}+0\cdot\omega$$

In the matrix form,

$$\begin{bmatrix}\frac{d^2\omega}{dt^2}\\ \frac{d\omega}{dt}\end{bmatrix}=\begin{bmatrix}-B/J & -K^2/LJ\\ 1 & 0\end{bmatrix}\begin{bmatrix}\frac{d\omega}{dt}\\ \omega\end{bmatrix}+\begin{bmatrix}K/LJ\\ 0\end{bmatrix}V_a$$

Comparing the given state-space,

$$\begin{bmatrix}\frac{d^2\omega}{dt^2}\\ \frac{d\omega}{dt}\end{bmatrix}=p\begin{bmatrix}\frac{d\omega}{dt}\\ \omega\end{bmatrix}+QV_a$$

Hence, $[p]=\begin{bmatrix}-B/J & -K^2/LJ\\ 1 & 0\end{bmatrix}$

68. (c) The characteristic equation is

$$1+G(s)H(s)=0$$
$$1+\frac{K}{s(s+2)(s+4)}=0$$

$$s(s^2+6s+8)+K=0$$
$$s^3+6s^2+8s+K=0$$

The Routh table is

s^3	1	8
s^2	6	K
s	$\frac{48-K}{6}$	0
1	K	

For system just becomes stable,

$$48-K=0$$
$$K=48$$

69. (a) Given, $G(s)H(s)=\frac{K}{1+(s/a)}$

$$\angle GH|_{\omega=0.59}=\tan^{-1}(\omega/a)=26.56°$$

From the asymptotic plot,

$$\angle GH=22.56$$

The error = 4.06°

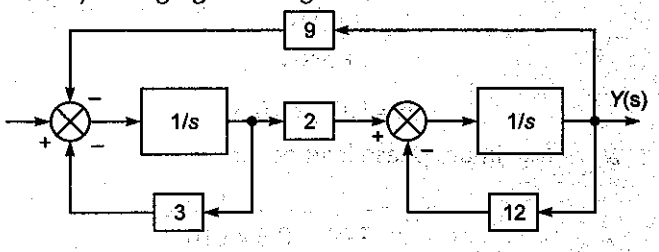
At $\omega=0.59$,

$$|M|=20\log K-20\log\sqrt{1^2+0.5^2}$$
$$=20\log K-0.97$$

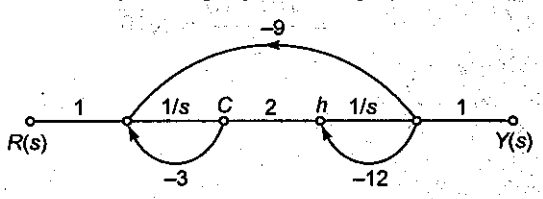
from the plot, $\log|GH|=20\log K$

∴ Error in dB gain = 0.97 dB

70. (b) By changing the integrator,



The corresponding signal flow graph



Using Mason's gain formula,

The overall transfer function

$$=\frac{2/s^2}{1+\frac{3}{s}+\frac{12}{s}+\frac{18}{s^2}+\frac{36}{s^2}}$$
$$=\frac{2}{s^2+15s+54}$$
$$=\frac{2}{(s+9)(s+6)}$$
$$=\frac{2}{9\left(1+\frac{s}{9}\right)\times 6\left(1+\frac{s}{6}\right)}$$
$$=\frac{1}{27\left(1+\frac{s}{9}\right)\left(1+\frac{s}{6}\right)}$$

71. (b) For the given state-space,

$$\dot{X}=\begin{bmatrix}2 & 3\\ 0 & 5\end{bmatrix}X+\begin{bmatrix}1\\ 0\end{bmatrix}u$$

$$A=\begin{bmatrix}2 & 3\\ 0 & 5\end{bmatrix}$$

$$B=\begin{bmatrix}1\\ 0\end{bmatrix}$$

$$[B \;\vdots\; AB]=\begin{bmatrix}1 & 2\\ 0 & 0\end{bmatrix}$$

The rank of matrix is not equal to 2. Hence, uncontrollable.

The characteristic equation is

$$|sI-A|=0$$
$$\begin{bmatrix}s & 0\\ 0 & s\end{bmatrix}-\begin{bmatrix}2 & 3\\ 0 & 5\end{bmatrix}=0$$
$$\begin{bmatrix}s-2 & -3\\ 0 & s-5\end{bmatrix}=0$$
$$(s-2)(s-5)=0$$

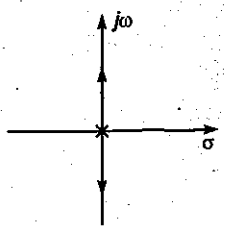
Two roots 2, 5 exist on the positive half of s-plane. Hence, system is unstable.

72. (b) As there is two poles and no zero exist. Hence, Asymptotes 2 goes to infinity.

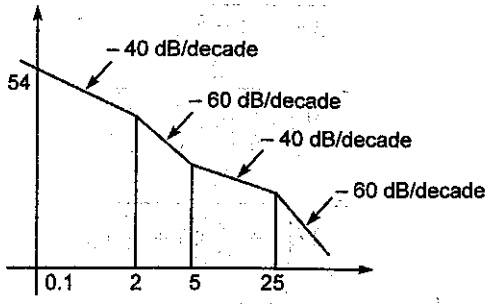
$$\text{Angle of asymptotes}=\frac{\pm(2q+1)180°}{2-0}$$

For $q=0$,

$$\phi_A=\pm 90°$$



73. (d)



Initial slope = – 40 dB/decade; hence two poles at origin.

Phase change at cut-off frequency at $\omega_1 = 2$ is

$-60-(-40) = -20$ dB/decade (pole)

Phase change at cut-off frequency at $\omega_2 = 5$ is

$-40-(-60) = +20$ dB/decade (zero)

Phase change at cut-off frequency at $\omega_3 = 25$ is

$-60-(-40) = -20$ dB/decade (pole)

Hence, Transfer function $= \dfrac{k(s+5)}{s^2(s+2)(s+25)}$

Given at $\omega = 0.1$, gain is 54 dB.

We can write

$$54 = 20\log\frac{5k}{(0.1)^2\,50}$$

Hence, $k = 50$

Transfer function $= \dfrac{50(s+5)}{s^2(s+2)(s+25)}$

74. (c) The characteristic equation is

$$2s^4 + s^3 + 3s^2 + 5s + 10 = 0$$

Routh table is

s^4	2	3	10
s^3	1	5	0
s^2	–7	10	
s^1			
1			

There are two sign changes hence, number of roots in right half s-plane are 2.

75. (b) The characteristic equation is

$$1+\frac{25}{s(s+6)} = 0$$

$$s(s+6)+25 = 0$$

$$s^2 + 6s + 25 = 0$$

Comparing with general expression

$$s^2 + 2\xi\omega_n s + \omega_n^2 = 0$$

$$\omega_n = 5;\ 2\xi\omega_n = 6$$

$$\xi = \frac{6}{2\times 5} = 0.6$$

The peak overshoot

$$= e^{-\pi\xi/\sqrt{1-\xi^2}} = e^{-\pi\times 0.6/\sqrt{1-0.36}}$$

$$= e^{-\pi\times 0.6/0.8} = 0.095 = 10\%$$

76. (d) The given compensator

$$D(s) = \frac{0.5s+1}{0.05s+1}$$

$$= \frac{1+0.5s}{1+0.5\times 10^{-1}\times s}$$

Comparing with the general expression

$$= \frac{1+j\omega\tau}{1+j\omega\alpha\tau}$$

$$\alpha = 10^{-1};\ \tau = 0.5$$

Maximum phase lead occurs at

$$\omega_m = \frac{1}{\tau\sqrt{\alpha}} = \frac{1}{0.5\times\sqrt{10^{-1}}}$$

$$= \frac{\sqrt{10}\times 10}{5} = 2\sqrt{10}$$

7

Measurements and Instruments

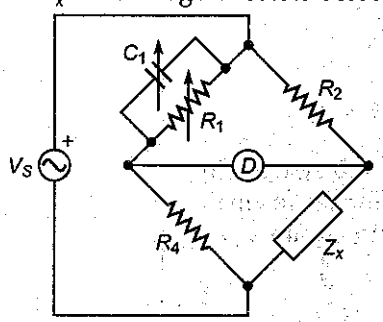
Syllabus

Electrical and Electronic Measurements *Bridges and potentiometers; PMMC, Moving iron, Dynamometer and induction type instruments; measurement of voltage, current, power, energy and power factor. Instrument transformers; digital voltmeters and multimeters; phase, time and frequency measurement; Q-meters; oscilloscopes; potentiometric recorders; error analysis.*

GATE 2011 *(1 Mark Questions)*

▼ *Solution Points*

1. The bridge circuit shown in the figure below is used for the measurement of an unknown element Z_x. The bridge circuit is best suited when Z_x is a



(a) low resistance
(b) high resistance
(c) low Q inductor
(d) lossy capacitor

2. A dual trace oscilloscope is set to operate in the alternate mode. The control input of the multiplexer used in the y-circuit is fed with a signal having a frequency equal to
(a) the highest frequency that the multiplexer can operate properly
(b) twice the frequency of the time base (sweep) oscillator
(c) the frequency of the time base (sweep) oscillator
(d) half the frequency of the time base (sweep) oscillator

3. Consider the following statements :
(i) The compensating coil of a low power factor wattmeter compensates the effect of the impedance of the current coil.

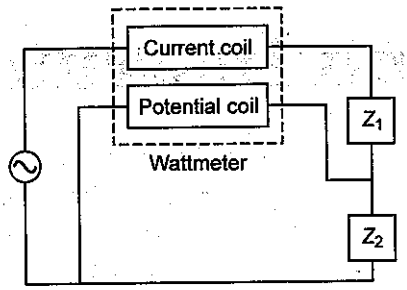
(ii) The compensating coil of a low power factor wattmeter compensates the effect of the impedance of the voltage coil circuit.

Of these statements,

(a) (i) is true but (ii) is false
(b) (i) is false but (ii) is true
(c) both (i) and (ii) are true
(d) both (i) and (ii) are false

GATE 2010

4. A wattmeter is connected as shown in the figure. The wattmeter reads



(a) zero always
(b) total power consumed by Z_1 and Z_2
(c) power consumed by Z_1
(d) power consumed by Z_2

5. An ammeter has a current range of 0-5 A and its internal resistance is 0.2 Ω. In order to change the range of 0-25 A, we need to add a resistance of

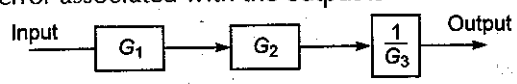
(a) 0.8 Ω in series with the ammeter
(b) 1.0 Ω in series with the ammeter
(c) 0.04 Ω in parallel with the ammeter
(d) 0.05 Ω in parallel with the ammeter

GATE 2009

6. The pressure coil of a dynamometer type wattmeter is

(a) highly inductive
(b) highly resistive
(c) purely resistive
(d) purely inductive

7. The measurement system shown in the figure uses three sub-systems in cascade whose gains are specified as G_1, G_2 and $1/G_3$. The relative small errors associated with each respective sub-system G_1, G_2 and G_3 are $\varepsilon_1, \varepsilon_2$ and ε_3. The error associated with the output is



(a) $\varepsilon_1 + \varepsilon_2 + \frac{1}{\varepsilon_3}$
(b) $\frac{\varepsilon_1 \varepsilon_2}{\varepsilon_3}$
(c) $\varepsilon_1 + \varepsilon_2 - \varepsilon_3$
(d) $\varepsilon_1 + \varepsilon_2 + \varepsilon_3$

8. The two inputs of a CRO are fed with two stationary periodic signals. In the *X-Y* mode, the screen shows a figure which changes from ellipse to circle and back to ellipse with its major axis changing orientation slowly and repeatedly. Which of the following inference can be made from this?

(a) The signals are not sinusoidal
(b) The amplitudes of the signals are very close but not equal
(c) The signals are sinusoidal with their frequencies very close but not equal
(d) There is a constant but small phase difference between the signals

▼ *Solution Points*

▼ *Solution Points*

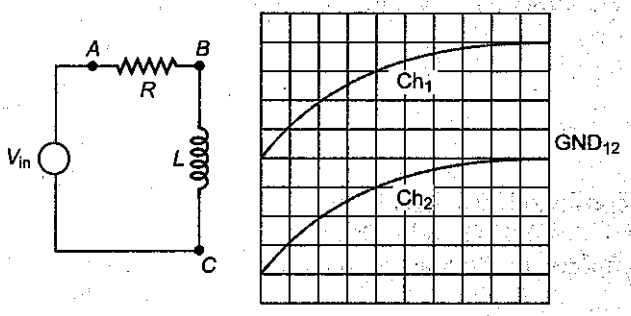
GATE 2008

9. Two 8-bit ADCs, one of single slope integrating type and other of successive approximate type, take T_A and T_B times to convert 5 V analog input signal to equivalent digital output. If the input analog signal is reduced to 2.5 V, the approximate time taken two ADCs will respectively, be

(a) T_A, T_B (b) $T_{A/2}, T_B$ (c) $T_A, T_{B/2}$ (d) $T_{A/2}, T_{B/2}$

GATE 2007

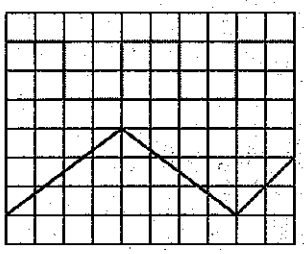
10. The probes of a non-isolated, two-channel oscilloscope are clipped to points A, B and C in the circuit of the adjacent figure. V_{in} is a square wave of a suitable low frequency. The display on Ch_1 and Ch_2 are as shown on the right. Then, the Signal and Ground probes S_1, G_1 and S_2, G_2 of Ch_1 and Ch_2 respectively are connected to points



(a) A, B, C, A (b) A, B, C, B (c) C, B, A, B (d) B, A, B, C

GATE 2006

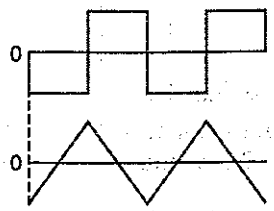
11. The time/div and voltage/div axes of an oscilloscope have been erased. A student connects a 1 kHz, 5 V p-p square wave calibration pulse to channel 1 of the scope and observes the screen to be as shown in the upper trace of the figure. An unknown signal is connected to channel 2 (lower trace) of the scope. If the time/div and voltage/div on both channels are the same, the amplitude (p-p) and period of the unknown signal are respectively



(a) 5 V, 1 ms (b) 5 V, 2 ms (c) 7.5 V, 2 ms (d) 10 V, 1 ms

12. A sampling wattmeter (that computes power from simultaneously sampled values of voltage and current) is used to measure the average power of a load. The peak to peak voltage of the square wave is 10 V and the current is a triangular wave of 5 A (p-p) as shown in the figure. The period is 25 ms. The reading in watt will be

▼ *Solution Points*



(a) zero (b) 25 W (c) 50 W (d) 100 W

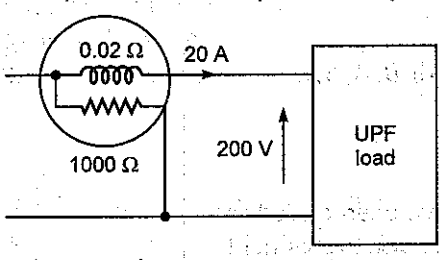
GATE 2005

13. The Q-meter works on the principle of
(a) mutual inductance (b) self inductance
(c) series resonance (d) parallel resonance

14. A PMMC voltmeter is connected across a series combination of a DC voltage source $V_1 = 2\,V$ and an AC voltage source $V_2(t) = 3\sin(4t)$. The voltmeter reads
(a) 2 V (b) 5 V (c) $(2+\sqrt{3}/2)$ V (d) $(\sqrt{17}/2)$ V

GATE 2004

15. The circuit in figure is used to measure the power consumed by the load. The current coil and the voltage coil of the wattmeter have 0.02 Ω and 1000 Ω resistances respectively. The measured power compared to the load power will be



(a) 0.4% less (b) 0.2% less (c) 0.2% more (d) 0.4% more

16. A galvanometer with a full scale current of 10 mA has a resistance of 1000 Ω. The multiplying power (the ratio of measured current to galvanometer current) of a 100 Ω shunt with this galvanometer is
(a) 110 (b) 100 (c) 11 (d) 10

GATE 2003

17. A Manganin swap resistance is connected in series with a moving-coil ammeter consisting of a milli ammeter and a suitable shunt in order to
(a) minimise the effect of temperature variation
(b) obtain large deflecting torque
(c) reduce the size of the meter
(d) minimise the effect of stray magnetic fields

18. The effect of stray magnetic fields on the actuating torque of a portable instrument is maximum when the operating field of the instrument and the stray fields are
(a) perpendicular (b) parallel
(c) inclined at 60° (d) inclined at 30°

▼ *Solution Points*

19. A reading of 120 is obtained when a standard inductor was connected in the circuit of a Q-meter and the variable capacitor is adjusted to a value of 300 pF. A lossless capacitor of unknown value C_x is then connected in parallel with the variable capacitor and the same reading was obtained when the variable capacitor is readjusted to a value of 200 pF. The value of C_x in pF is
(a) 100 (b) 200 (c) 300 (d) 500

GATE 2002

20. Two in-phase, 50 Hz sinusoidal waveforms of unit amplitude are fed into channel 1 and channel 2 respectively of an oscilloscope. Assuming that the voltage scale, time scale and other settings are exactly the same for both the channels, what would be observed, if the oscilloscope is operated in *X-Y* mode?
(a) A circle of unit radius
(b) An ellipse
(c) A parabola
(d) A straight line inclined at 45° with respect to the *x*-axis

GATE 2001

21. If an energymeter disc makes 10 revolutions in 100 s when a load of 450 W is connected to it, the meter constant (in rev/kWh) is
(a) 1000 (b) 500 (c) 1600 (d) 800

22. The minimum number of wattmeters required to measure 3-phase, 3-wire balanced or unbalanced power is
(a) 1 (b) 2 (c) 3 (d) 4

GATE 2000

23. Triangular PWM control, when applied to a three-phase BJT based voltage source, inverter introduces
(a) low order harmonic voltages on the DC side
(b) very high order harmonic voltages on the DC side
(c) low order harmonic voltages on the AC side
(d) very high order harmonic voltages on the DC side

24. Instrument transformers are known to introduce magnitude and phase errors in measurements. These are primarily due to
(a) improper connections on the primary side
(b) measurement errors inherent in the meter connected to the transformer secondary
(c) open and short circuit parameters of the instrument transformers
(d) None of the above

25. An analog electronic circuit that measures rms value of the input voltage by averaging the square of the instantaneous voltage level, responds slowly to changes in the input signal due to
(a) the square function built into the circuit
(b) the square-root function built into the circuit
(c) the averaging function built into the circuit
(d) None of the above

▼ *Solution Points*

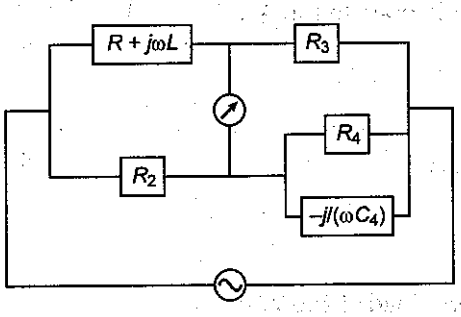
GATE 2011 *(2 Marks Questions)*

26. A $4\frac{1}{2}$ digit DMM has the error specification as 0.2% of reading + 10 counts. If a DC voltage of 100 V is read on its 200 V full scale, the maximum error that can be expected in the reading is

(a) ±0.1% (b) ±0.2% (c) ±0.3% (d) ±0.4%

GATE 2010

27. The Maxwell's bridge shown in the figure is at balance. The parameters of the inductive coil are



(a) $R = R_2R_3/R_4, L = C_4R_2R_3$ (b) $L = R_2R_3/R_4, R = C_4R_2R_3$

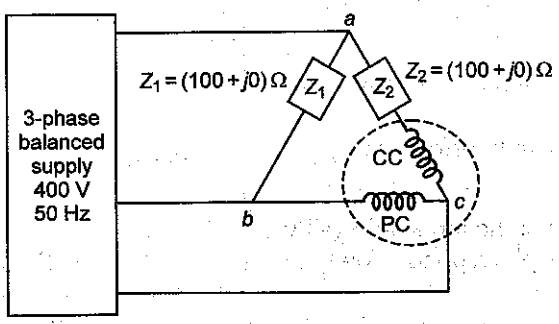
(c) $R = R_4/R_2R_3, L = 1(C_4R_2R_3)$ (d) $L = R_4/R_2R_3, R = 1/(C_4R_2R_3)$

GATE 2009

28. An average-reading digital multimeter reads 10 V when fed with a triangular wave, symmetric about the time axis. For the same input, an rms reading meter will read

(a) $\frac{20}{\sqrt{3}}$ (b) $\frac{10}{\sqrt{3}}$ (c) $20\sqrt{3}$ (d) $10\sqrt{3}$

29. The figure shows a three-phase delta connected load supplied from a 400 V, 50 Hz, 3-phase balanced source. The Pressure Coil (PC) and Current Coil (CC) of a wattmeter are connected to the load as shown, with the coil polarities suitably selected to ensure a positive deflection. The wattmeter reading will be



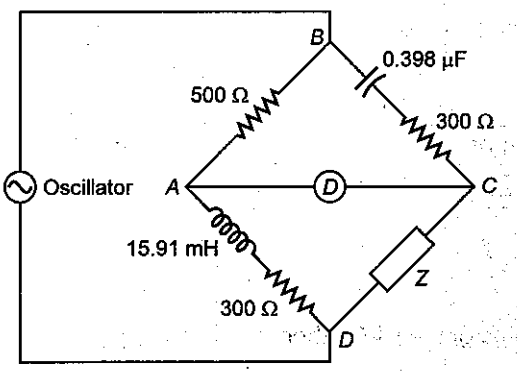
(a) zero (b) 1600 W

(c) 800 W (d) 400 W

▼ *Solution Points*

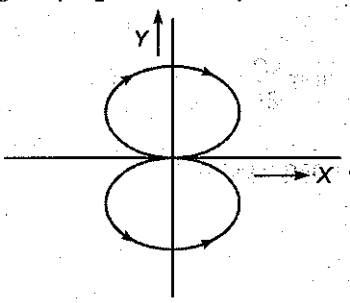
GATE 2008

30. The AC bridge shown in the figure is used to measure the impedance Z. If the bridge is balanced for oscillator frequency $f = 2\,\text{kHz}$, then the impedance Z will be



(a) $(260 + j0)\,\Omega$ (b) $(0 + j200)\,\Omega$
(c) $(260 - j200)\,\Omega$ (d) $(260 + j200)\,\Omega$

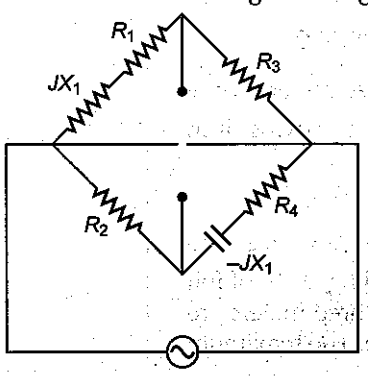
31. Two sinusoidal signals $p(\omega_1 t) = A \sin \omega_1 t$ and $q(\omega_2 t)$ are applied to X and Y inputs of a dual channel CRO. The Lissajous figure displayed on the screen is shown below. The signal $q(\omega_2 t)$ will be represented as



(a) $q(\omega_2 t) = A \sin \omega_2 t, \omega_2 = 2\omega_1$ (b) $q(\omega_2 t) = A \sin \omega_2 t, \omega_2 = \omega_1/2$
(c) $q(\omega_2 t) = A \cos \omega_2 t, \omega_2 = 2\omega_1$ (d) $q(\omega_2 t) = A \cos \omega_2 t, \omega_2 = \omega_1/2$

GATE 2007

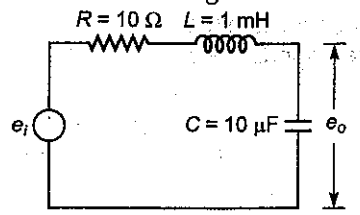
32. A bridge circuit is shown in the figure below. Which one of the sequences given below is most suitable for balancing the bridge?



(a) First adjust R_4 and then adjust R_1 (b) First adjust R_2 and then adjust R_3
(c) First adjust R_2 and then adjust R_4 (d) First adjust R_4 and then adjust R_2

▼ *Solution Points*

Statement for Linked Answer Questions 33 and 34
Consider the *R-L-C* circuit shown in figure.



33. For a step-input e_i, the overshoot in the output e_o will be
(a) 0, since the system is not under-damped
(b) 5%
(c) 16%
(d) 48%

34. If the above step response is to be observed on a non-storage CRO, then it would be best to have the e_i as a
(a) step function
(b) square wave of frequency 50 Hz
(c) square wave of frequency 300 Hz
(d) square wave of frequency 2.0 kHz

GATE 2006

35. Consider the following statements with reference to the equation $\frac{\delta p}{\delta t}$:
(1) This is a point form of the continuity equation.
(2) Divergence of current density is equal to the decrease of charge per unit volume per unit at every point.
(3) This is Maxwell's divergence equation.
(4) This represents the conservation of charge.
Of these statements, which one is correct?
(a) 2 and 4 (b) 1, 2 and 3
(c) 2, 3 and 4 (d) 1, 2 and 4

36. An energymeter connected to an immersion heater (resistive) operating on an AC 230 V, 50 Hz, AC single-phase source reads 2.3 unit (kWh) in 1h. The heater is removed from the supply and now connected to a 400 V peak to peak square wave source of 150 Hz. The power in kW dissipated by the heater will be
(a) 3.478 (b) 1.739 (c) 1.540 (d) 0.870

37. A current of $-8 + 6\sqrt{2}(\sin \omega t + 30°)$ A is passed through three meters. They are a centre zero PMMC meter, a true rms meter and a moving iron instrument. The respective readings (in ampere) will be
(a) 8, 6, 10 (b) 8, 6, 8 (c) –8, 10, 10 (d) –8, 2, 2

38. A variable w is related to three other variables x, y, z as $w = xy/z$. The variables are measured with meters of accuracy ±0.5% reading, ±1% of full scale value and ±1.5% reading. The actual readings of the three meters are 80, 20 and 50 with 100 being the full scale value for all three. The maximum uncertainty in the measurement of w will be
(a) ±0.5% reading (b) ±5.5% reading
(c) ±6.7% reading (d) ±7.0% reading

▼ *Solution Points*

39. A 200/1 Current Transformer (CT) is wound with 200 turns on the secondary on a toroidal core. When it carries a current of 160 A on the primary, the ratio and phase errors of the CT are found to be – 0.5% and 30 min respectively. If the number of secondary turns is reduced by 1, the new ratio error (%) and phase error (min) will be respectively
(a) 0.0, 30 (b) – 0.5, 35 (c) –1.0, 30 (d) – 1.0, 25

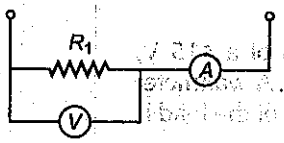
40. R_1 and R_4 are the opposite arms of a Wheatstone bridge as are R_3 and R_2. The source voltage is applied across R_1 and R_3. Under balanced conditions which one of the following is true?
(a) $R_1 = R_3R_4/R_2$ (b) $R_1 = R_2R_3/R_4$
(c) $R_1 = R_2R_4/R_3$ (d) $R_1 = R_2 + R_3 + R_4$

GATE 2005

41. The simultaneous application of signals $x(t)$ and $y(t)$ to horizontal and vertical plates, respectively, of an oscilloscope, produce a vertical figure of 8 display. If P and Q are constants and $x(t) = P\sin(4t + 30)$, then $y(t)$ is equal to
(a) $Q\sin(4t - 30)$ (b) $Q\sin(2t + 15)$ (c) $Q\sin(8t + 60)$ (d) $Q\sin(4t + 30)$

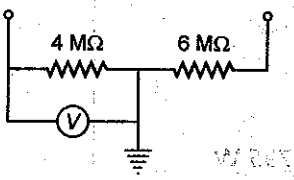
42. A DC ammeter has a resistance of 0.1 Ω and its current rating is 0-100 A. If the range is to be extended to 0-500 A, then the ammeter requires the shunt resistance of
(a) 0.010 Ω (b) 0.011Ω (c) 0.025 Ω (d) 1.0 Ω

43. The set-up in the figure is used to measure resistance R. The ammeter and voltmeter resistances are 0.01 Ω and 2000 Ω, respectively. The readings are 2 A and 180 V, respectively giving measure resistance of 90 Ω. The percentage error in the measurement is



(a) 2.25 % (b) 2.35 % (c) 4.5% (d) 4.71%

44. A 1000 V DC supply has two one-core cables as its positive and negative leads, their insulation resistances to earth are 4 MΩ and 6 MΩ respectively as shown in figure. A voltmeter with resistance 50 kΩ is used to measure the insulation of the cable. When connected between the positive core and earth, then the voltmeter reads



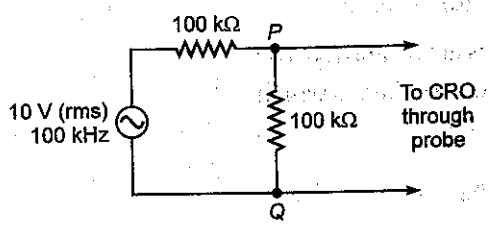
(a) 8 V (b) 16 V
(c) 24 V (d) 40 V

45. Two wattmeters, which are connected to measure the total power on a three-phase system supplying a balanced load read 10.5 kW and – 2.5 kW respectively. The total power and the power factor, respectively, are
(a) 13.0 kW, 0.334 (b) 130.0 kW, 0.684
(c) 8.0 kW, 0.52 (d) 8.0 kW, 0.334

▼ *Solution Points*

GATE 2004

46. A CRO probe has an impedance of 500 kΩ in parallel with a capacitance of 10 pF. The probe is used to measure the voltage between *P* and *Q* as shown in figure. The measured voltage will be



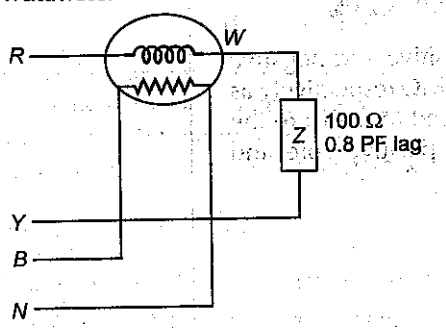
(a) 3.53 V (b) 4.37 V (c) 4.54 V (d) 5.00 V

47. A moving coil of a meter has 100 turns, and a length and depth of 10 mm and 20 mm respectivley. It is positioned in a uniform radial flux density of 200 mT. The coil carries a current of 50 mA. The torque on the coil is
(a) 200 μN-m (b) 100 μN-m (c) 2μN-m (d) 1μN-m

48. A DCA-h meter is rated for 15 A, 250 V. The meter constant is 14.4 A-s/rev. The meter constant at rated voltage may be expressed as
(a) 3750 rev/kWh (b) 3600 rev/kWh
(c) 1000 rev/kWh (d) 960 rev/kWh

49. A moving iron ammeter produces a full scale torque of 240 μN-m with a deflection of 120° at a current of 10 A. The rate of change of self inductance (μH/rad) of the instrument at full scale is
(a) 2.0 μH/rad (b) 4.8 μH/rad
(c) 12.0 μH/rad (d) 114.6 μH/rad

50. A single-phase load is connected between *R* and *Y* terminals of a 415 V, symmetrical, 3-phase, 4 wire system with phase sequence *RYB*. A wattmeter is connected in the system as shown in figure. The power factor of the load is 0.8 lagging. The wattmeter will read



(a) –795 W (b) –597 W (c) +597 W (d) +795 W

51. A 50 Hz, bar primary CT has a secondary with 500 turns. The secondary supplies 5 A current into a purely resistive burden of 1 Ω. The magnetizing ampere turns are 200. The phase angle between the primary and secondary current is
(a) 4.6° (b) 85.4° (c) 94.6° (d) 175.4°

52. The core flux in the CT of Q. 51, under the given operating condition is
(a) 0 (b) 45.0 μWb (c) 22.5 mWb (d) 100.0 mWb

GATE 2003

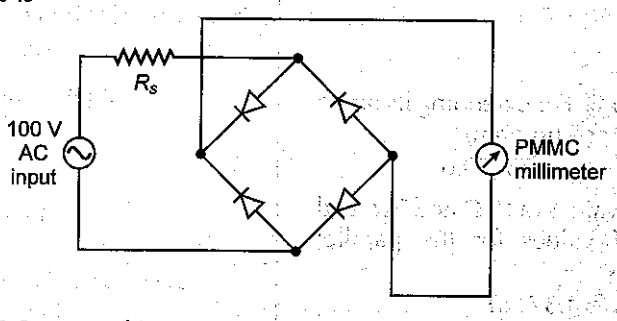
▼ *Solution Points*

53. The items in Group I represent the various types of measurements to be made with a reasonable accuracy using a suitable bridge. The items in Group II represent the various bridges available for this purpose. Select the correct choice of the item in Group II for the corresponding item in Group I from the following.

Group I	Group II
P. Resistance in the milliohm range	1. Wheatstone bridge
Q. Low values of capacitance	2. Kelvin double bridge
R. Comparison of resistances which are nearly equal	3. Schering bridge
S. Inductance of a coil with a large time constant	4. Wien's bridge
	5. Hay's bridge
	6. Carey-Foster bridge

(a) P-2, Q-3, R-6, S-5 (b) P-2, Q-6, R-4, S-5
(c) P-2, Q-3, R-5, S-4 (d) P-1, Q-3, R-2, S-6

54. A rectifier type AC voltmeter consists of a series resistance R_s an ideal full-wave rectifier bridge and a PMMC instrument as shown in figure. The internal resistance of the instrument is 100 Ω and a full scale deflection is produced by a DC current of 1 mA. The value of R_s required to obtain full scale deflection with an AC voltage of 100 V (rms) applied to the input terminals is



(a) 63.56 Ω (b) 86.93 Ω (c) 89.93 Ω (d) 141.3 kΩ

55. A wattmeter reads 400 W when its current coil is connected in the *R* phase and its pressure coil is connected between this phase and the neutral of a symmetrical 3-phase system supplying a balanced star connected 0.8 PF inductive load. The phase sequence is *RYB*. What will be the reading of this wattmeter, if its pressure coil alone is reconnected between the *B* and *Y* phases, all other connections remaining as before?
(a) 400.0 (b) 519.6 (c) 300.0 (d) 692.8

56. The inductance of a certain moving-iron ammeter is expressed as $L = 10 + 3\theta - \theta^2/4\ \mu H$, where θ is the deflection in radian from the zero position. The control spring torque is 25×10^{-6} N-m/rad. The deflection of the pointer in radian when the meter carries a current of 5 A, is
(a) 2.4 (b) 2.0 (c) 1.2 (d) 1.0

57. A 500 A/5 A, 50 Hz current transformer has a bar primary. The secondary burden is a pure resistance of 1Ω and it draws a current of 5 A. If the magnetic core requires 250 AT for magnetization, the percentage ratio of error is
(a) 10.56 (b) –10.56 (c) 11.80 (d) –11.80

▼ *Solution Points*

58. The voltage-flux adjustment of a certain 1-phase 220 V induction watt-hour meter is altered so that the phase angle between the applied voltage and the flux due to it is 85° (instead of 90°). The errors introduced in the reading of this meter when the current is 5 A at power factor of unity and 0.5 lagging are respectively

(a) 3.8 mW, 77.4 mW
(b) –3.8 mW, –77.4 mW
(c) – 4.2 W, –85.1 W
(d) 4.2 W, 85.1 W

59. Group II represents the figure obtained on a CRO screen when the voltage signals $V_1 = V_{in} \sin \omega t$ and $V_2 = V_{in} \sin(\omega t + \phi)$ are given to its X and Y plates respectively and ϕ is changed. Choose the correct value of ϕ from Group I to match with the corresponding figure of Group II.

Group I

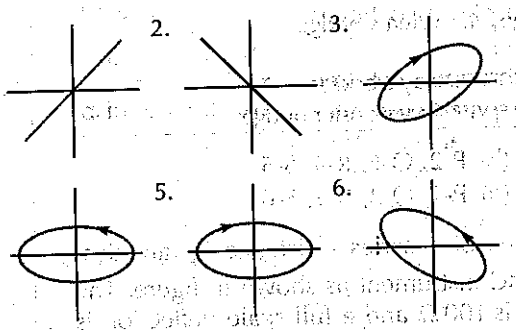
P. $\phi = 0$

Q. $\phi = \pi/2$

R. $\pi < \phi < 3\pi/2$

S. $\phi = 3\pi/2$

Group II



(a) P-1, Q-3, R-6, S-5
(b) P-2, Q-6, R-4, S-5
(c) P-2, Q-3, R-5, S-4
(d) P-1, Q-5, R-6, S-6

GATE 2001

60. A 100 μA ammeter has an internal resistance of 100 Ω. For extending its range to measure 500 μA, the shunt required is of resistance (in ohm)

(a) 20.0 (b) 22.22 (c) 25.0 (d) 50.0

61. Resistances R_1 and R_2 have, respectively, nominal values of 10 Ω and 5 Ω, and tolerances of ±5% and ±10%. The range of values for the parallel combination of R_1 and R_2 is

(a) 3.077 Ω to 3.636 Ω
(b) 2.805 Ω to 3.371 Ω
(c) 3.237 Ω to 3.678 Ω
(d) 3.192 Ω to 3.435 Ω

GATE 2000

62. The two-wattmeter method is used to measure active power on a three-phase, three-wire system. If the phase voltage is unbalanced, then the power reading is

(a) affected by both negative sequence and zero sequence voltages
(b) affected by negative sequence voltages but not by zero sequence voltages
(c) affected by zero sequence voltages but not by negative sequence voltages
(d) not affected by negative or zero sequence voltages

Answers with Explanations

1. (c) The circuit shown in the figure is the Maxwell's inductance-capacitance bridge. In this bridge, an inductance is measured by comparison with a standard capacitance. This bridge is limited to measurement of low Q coil.

2. (c) Oscilloscope with two vertical inputs, referred to as dual trace oscilloscope. Using a single-beam CRT, they multiplex the inputs, usually switching between them fast enough to display two traces apparently at once.
Switching channels can be asynchronous, *i.e.*, free running with trace blanking while switching or after each horizontal sweep is complete. Asynchronous switching is usually designated chopped while sweep-synchronised is designated att [esnate].

3. (b)

4. (c) The fixed coil or field coil is connected in series with the load and so carries the current in the circuit. The fixed coil therefore forms the current coil. The moving coil is connected across the voltage and therefore carries a current proportional to voltage.

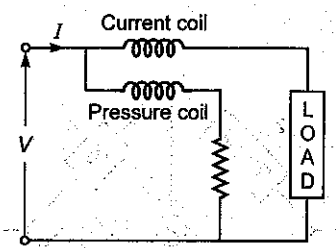
5. (d) Multiplying power

$$m = \frac{I}{I_m} = \frac{25}{5} = 5$$

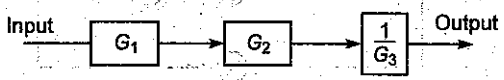
Resistance of shunt

$$R_{sh} = \frac{R_m}{m-1} = \frac{0.2}{5-1} = \frac{0.2}{4} = 0.05\,\Omega$$

6. (b) The moving coil is connected across the voltage. A high non-inductive resistance is connected in series with moving coil to limit the current to a small value.



7. (d) The error associated with product or quotient of two or more than two quantities is equal to the sum of individual errors.



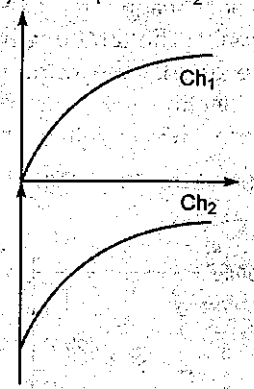
If $x = x_1 x_2 x_3$

$$\frac{\delta x}{x} = \pm\left(\frac{\delta x_1}{x_1} + \frac{\delta x_2}{x_2} + \frac{\delta x_3}{x_3}\right)$$

8. (b) In X-Y mode, if figure shows ellipse that indicates the amplitude is not equal but it changes to circle means amplitudes are equal. The inference can be made from the amplitudes of the signal are very close but not equal.

9. (a) The time taken by both ADCs will remain T_A, T_B.

10. (b) The display on Ch_1 and Ch_2 are



If the input is square wave then, the channel 1 indicates the voltage across resistor and channel 2 indicates the voltage across inductor.
Hence, the probe is connected to A-B for channel 1 and C-B for channel 2.

11. (c) Peak-to-peak voltage of upper traces = 5 V
Peak-to-peak division of upper traces = 2

$$\frac{\text{Voltage}}{\text{Division}} = \frac{5\text{ V}}{2} = 2.5\text{ V} \qquad \ldots(i)$$

Peak-to-peak voltage of lower traces = x volt
Peak-to-peak division of lower traces = 3

$$\frac{\text{Voltage}}{\text{Division}} = \frac{x}{3}$$

From Eq. (i),

$$\frac{x}{3} = 2.5$$

$$x = 7.5\text{ V}$$

Time period of upper traces $= \frac{1}{1\text{ kHz}} = 1\text{ ms}$

Time division of upper traces = 4

$$\frac{\text{Time period}}{\text{Division}} = \frac{1\text{ ms}}{4} \qquad \ldots(ii)$$

Time period of lower traces = x ms

Time division of lower traces = 8

$$\frac{\text{Time period}}{\text{Division}} = \frac{x}{8}$$

From Eq. (ii), $\frac{x}{8} = \frac{1}{4}$ ms

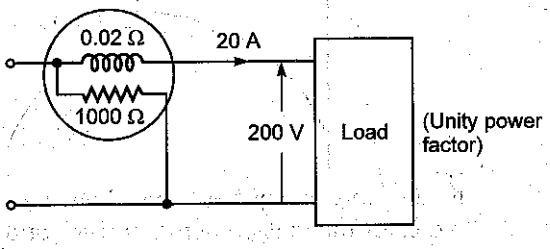
$\Rightarrow$ $x = 2$ ms

12. (a) From waveform it is clear that for half of time period the power is of opposite sign of equal amplitude to another half of the time period. The total power in one period is zero. Hence, the reading of wattmeter is zero.

13. (c)

14. (a) The permanent magnet moving coil measures the DC value only. The torque will be reversed if the current is reversed. If the instrument is connected to AC, the deflection responds to the mean torque which is zero. Hence, voltmeter reads 2 V.

15. (c) Load power = $20 \times 200 = 4000$ W



Power consumed in coil

$$= (20)^2 \times 0.02$$

$$= 400 \times 0.02 = 8 \text{ W}$$

Wattmeter reading = 4008

$$\% \text{ increase} = \frac{4008 - 4000}{4000}$$

$$= \frac{8}{4000} = 0.2\% \text{ more}$$

16. (c) Shunt resistance $R_{sh} = \frac{R_m}{m-1}$

where, m is the multiplying factor.

$$100 = \frac{1000}{m-1}$$

$$m - 1 = 10$$

$$m = 11$$

17. (a) A manganin swap resistance is connected in series with a moving coil ammeter consisting of a multimeter to minimise the effect of temperature variation.

18. (b) Actuating torque $\propto$ cos (angle between operating field and magnetic field)

For maximum actuating torque both are parallel.

19. (a) Reading of Q-meter = $\frac{1}{\omega RC}$

Given, $120 = \frac{1}{\omega RC} = \frac{1}{\omega R(C' + C_x)}$

i.e., $C = C' + C_x$

$$300 = 200 + C_x$$

$$C_x = 100 \text{ pF}$$

20. (d) As the two signals have the same frequency with no phase difference the straight line observed in oscilloscope operated in X-Y mode. As the signal are of same amplitude, the slope is 45°.

21. (d) Meter constant is defined as the number of revolution made per kilowatt hour (kWh).

$$K_m = \frac{100}{(0.450) \times \frac{100}{60 \times 60}} = 800$$

22. (a) If the common point is located one of the lines, then the power is measured using two wattmeters either the load is balanced or unbalanced. But if load is balanced one wattmeter can measure the power.

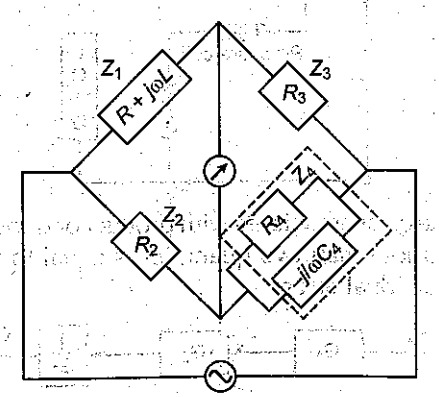
23. (d) When triangular PWM control applied to a three-phase BJT based voltage source, inverter introduces very high order harmonic voltage on the DC side.

24. (c) The magnitude and phase errors in measurement of instrument transformer are due to open and short circuit parameters of the instrument transformer.

25. (c) Because of averaging function built in into the circuit.

26. (c) ±0.3%

27. (a)



At balance,

$$\frac{Z_1}{Z_2}=\frac{Z_3}{Z_4}=Z_3\cdot Y_4$$

$$\frac{R+j\omega L}{R_2}=R_3\left(\frac{1}{R_4}+j\omega C_4\right)$$

$$\frac{R}{R_2}+j\frac{\omega L}{R_2}=\frac{R_3}{R_4}+j\omega R_3C_4$$

Equating real and imaginary parts,

$$\frac{R}{R_2}=\frac{R_3}{R_4}$$

$$R=\frac{R_2R_3}{R_4}$$

$$\frac{\omega L}{R_2}=\omega R_3C_4$$

$$L=R_2R_3C_4$$

28. (d) For triangular wave,

$$\text{Average value}=V_m/3$$

$$\text{rms}=V_m/\sqrt{3}$$

$$V_m/3=10\text{ V}\Rightarrow V_m=30$$

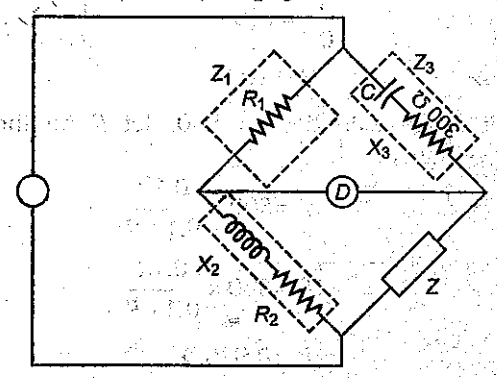
$$\text{rms}=\frac{30}{\sqrt{3}}=10\sqrt{3}$$

29. (c) Wattmeter reading = |Current through current coil| × |Voltage across pressure coil| × cos (angle between them)

$$=\left|\frac{400\angle 120}{100}\right|\times|400\angle -120|\times\cos 240$$

$$=-4\times 400\times\frac{1}{2}=-800\text{ W}$$

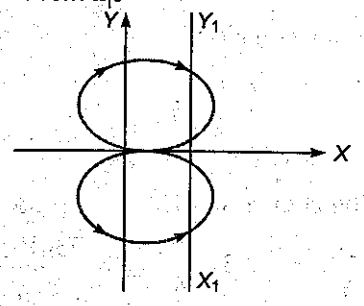
30. (a) For balance, $ZZ_1=Z_2Z_3$



$$\Rightarrow\quad Z=\frac{Z_2Z_3}{Z_1}$$

$$Z=(260+j0)\Omega$$

31. (d) $p(\omega_1t)=A\sin\omega_1t$



Y_1-line cuts 4 times the Lissajous pattern.

X_1-line cuts 2 times the Lissajous pattern.

Maximum number of intersection of a horizontal line $\frac{\omega_y}{\omega_x}$

$$=\frac{\text{With Lissajous pattern}}{\text{Maximum number of intersection of a vertical line with Lissajous pattern}}$$

$$\therefore\quad\frac{\omega_y}{\omega_x}=\frac{2}{4}=\frac{1}{2}$$

$$\therefore\quad(\omega\times 2\omega_1)=(2\omega_y=2\omega_2)$$

$$\omega_2=\omega_1/\omega_2$$

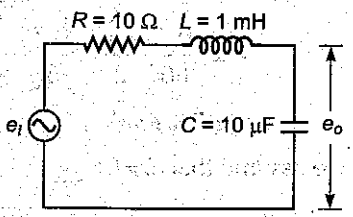
and $q(\omega_2,t)$ will lead $p(\omega_1,t)$ by 90°.

$$q(\omega_2,t)<A\sin(\omega_2t+90°)$$

$$q(\omega_2t)<A\cos\omega_2t$$

32. (a) For balancing bridge first adjust R_4 and then R_1.

33. (c)



The characteristic equation

$$s^2+\frac{R}{L}s+\frac{1}{LC}=0$$

$$s^2+2\xi\omega_ns+\omega_n^2=0$$

$$2\xi\omega_n=\frac{R}{L}$$

$$\omega_n=\frac{1}{\sqrt{LC}}$$

$$\xi=\frac{R}{L}\times\frac{\sqrt{LC}}{2}=0.5$$

$$\text{Peak overshoot}=\frac{e^{-\pi\xi}}{\sqrt{1-\xi^2}}\times 100\%$$

$$=16\%$$

34. (a)

35. (d) Continuity equation

$$\nabla \cdot J = \frac{de}{dt}$$

is the conservation of charge.

36. (b) Reading of energymeter

$$2.3 = \frac{V^2}{R} \times e = \frac{(230)^2}{R} \times 1$$

$$R = 23\,\Omega$$

$$\text{Power dissipated} = \frac{V^2}{R} = \frac{(200)^2}{23}$$

$$= 1.739 \text{ kW}$$

37. (c) PMMC instrument reads only DC value and since, it is centre zero type so, it will give –8 value. RMS meter will read rms value of the current.

$$\text{So, RMS} = \sqrt{8^2 + \left(\frac{6\sqrt{2}}{\sqrt{2}}\right)^2} = 10 \text{ A}$$

Moving iron also reads rms value, so its reading will also be 10 A, hence ultimately it will be (–8, 10, 10).

38. (d) $w = \frac{xy}{z}$

Relative limiting error of w,

$$\frac{\delta w}{w} = \left(\frac{\delta x}{x} + \frac{\delta y}{y} + \frac{\delta z}{z}\right)$$

$$= \pm\left(\frac{0.4}{80} + \frac{1}{20} + \frac{0.75}{50}\right) \times 100$$

$$= \pm 7\% \text{ reading}$$

39. (a) Here, we assume that ($\delta = 0$)

$$nI_s = I_p$$

$$R = n + \frac{I_0 \cos(\delta + \alpha)}{I_g} = 200 + I_0 \cos\alpha$$

$$\text{But \% error ratio} = -0.5 = \frac{k_n - R}{R}$$

$$-0.5 R_1 = 100 k_n - 100 R_1 \qquad (k_n = 200)$$

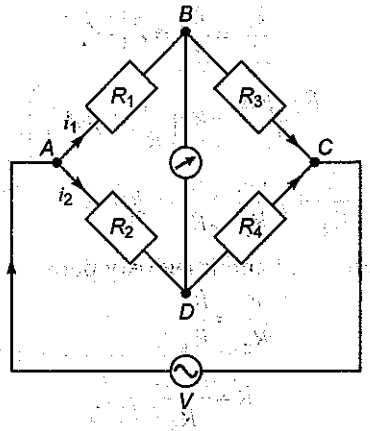
$$R_1 = 201$$

$$\frac{I_0 \cos\alpha}{I_g} = 1$$

$\therefore$ n becomes 199.

% error in ratio = 0

40. (b) At balance condition,



$$V_B = V_D$$

$$V - i_1 R_1 = V - i_2 R_2$$

and $$V - i_1 R_3 = V - i_2 R_4$$

$$\frac{i_1}{i_2} = \frac{R_2}{R_1} = \frac{R_4}{R_3}$$

$$\Rightarrow \quad R_1 = \frac{R_2 R_3}{R_4}$$

41. (b) Given, $x(t) = p \sin(4t + 30)$

As display at CRO, giving a figure 8

$$\frac{f_y}{f_x} = \frac{1}{2}$$

$$f_y = \frac{f_x}{2}$$

Hence, $y(t) = Q \sin\left(\frac{4t + 30}{2}\right) = Q \sin(2t + 15)$

42. (c) We can write

$$\frac{V}{0.1} = 100$$

$$\Rightarrow \quad V = 10 \text{ V}$$

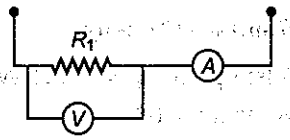
If range extended to 500, let R be the shunt resistance.

$$\Rightarrow \quad V = 500 \times \frac{0.1R}{0.1 + R}$$

$$\Rightarrow \quad 10 = 500 \times \frac{0.1R}{0.1 + R}$$

$$\Rightarrow \quad R = 0.025\,\Omega$$

43. (d) The current through voltmeter $= I_V = \frac{180}{2000} = \frac{9}{100}$ A



The current through resistance

$$I_R = 2 - \frac{9}{100} = 1.91 \text{ A}$$

Since,

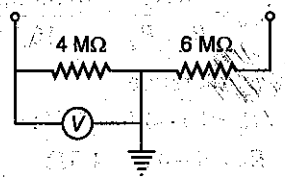
$$1.91R = 180$$

$$R = 94.24\ \Omega$$

$$\text{Error} = \frac{94.24 - 90}{90} \times 100$$

$$= 4.71\%$$

44. (a) Since, $R_T = \frac{4 \times 0.05}{4.05} + 6 = 6.05\,\Omega$



$$I_T = \frac{1000}{6.05} \text{ A}$$

$$V_T = \frac{1000}{6.05} \times 0.05 = 8 \text{ V}$$

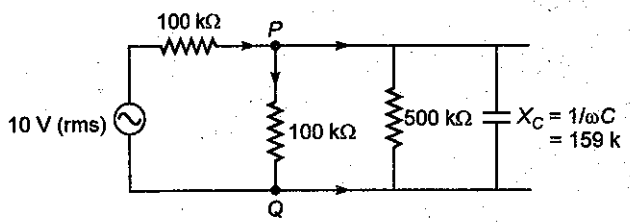
45. (d) The total power = 10.5 – 2.5

= 8 kW

and $\tan\theta = \sqrt{3} \times \frac{13}{8}$

$\Rightarrow$ $\cos\theta = 0.334$

46. (a) Let potential at P is V.



Applying KCL at P,

$$\frac{10 - V}{100} = \frac{V}{100} + \frac{V}{159} + \frac{V}{500}$$

$$\Rightarrow \quad \frac{10 - V}{100} = 0.0182 \text{ V}$$

$$\Rightarrow \quad V = 3.53 \text{ V}$$

47. (a) The deflecting torque $T_d = NBI\,dl$

$= (100)(200 \times 10^{-3})(10 \times 10^{-3})(20 \times 10^{-3})(50 \times 10^{-3})$

= 200 μN-m

48. (b) The given meter constant = 14.4 A-s/rev

The meter constant at rated voltage = 14.4 × 250 = 3600 rev/kWh

49. (b) At full scale position,

$$T_c = \frac{1}{2} I^2 \frac{dL}{d\theta}$$

$$\frac{dL}{d\theta} = \frac{2T_c}{I^2} = 4.8\ \mu\text{H/rad}$$

50. (b) We can write

$$V_{RY} = 415 \angle 30°$$

$$V_{BN} = \frac{415}{\sqrt{3}} \angle 120°$$

$$I_c = \frac{V_{RY}}{Z} = \frac{415 \angle 30°}{100 \angle 36.87°} = 4.15 \angle -6.87°$$

$$S = VI^* = \frac{415}{\sqrt{3}} \times 4.15 \angle (-120° - 6.87°)$$

$$= 994.34 \angle -126.87$$

$$= -597 \text{ W}$$

51. (a) The phase angle between the primary and secondary currents is

$$\theta° = \left(\frac{180}{\pi}\right)\left(\frac{200 \times 1}{500 \times 5}\right) \simeq 4.6°$$

52. (b) Voltage induced in secondary

$$E_s = I_s \times 2_s = 5 \text{ V}$$

$$\Rightarrow \quad 4.44 f \phi_m N = 5$$

$$\Rightarrow \quad \phi_m = \frac{5}{4.44 \times 50 \times 500} = 45\ \mu\text{Wb}$$

53. (a) Resistance in milliohm — Kelvin double bridge range

Low value of capacitance — Schering bridge

Comparison of resistance which are normally equal — Carey-Foster bridge

Inductance of coil with a large time constant — Hay's bridge

54. (c) DC sensitivity

$$= \frac{1}{\text{Full scale current}} = \frac{1}{1\text{mA}} = 10^3\ \Omega/\text{V}$$

For full wave rectifier, AC sensitivity

$$= 0.9\, S_{DC} = 0.9 \times 10^3 = 900\ \Omega/\text{V}$$

Resistance $R_S = S_{AC} V - R_m$

$$= 900 \times 100 - 100$$

$$= 89.90\ \text{k}\Omega$$

55. (b)

56. (c) At equilibrium,

$$K_\theta = \frac{1}{2} I^2 \frac{dL}{d\theta}$$

Given, $L = 10 + 3\theta - \frac{\theta^2}{4}$

$$\frac{dL}{d\theta} = 3 - \frac{\theta}{2}$$

$$K_\theta = \frac{1}{2} \times 25 \times \left(3 - \frac{\theta}{2}\right)$$

$$25 \times 10^{-6} = \frac{1}{2} \times 25 \times \left(3 - \frac{\theta}{2}\right)$$

$$\theta = 1.2 \text{ rad}$$

57. (b) $I_m = 250$ A

$$I_p = \sqrt{(nI_s)^2 + (I_m)^2}$$

$$n = \frac{500}{5} = 100$$

$$I_p = \sqrt{(100 \times 5)^2 + 250^2}$$

$$R = \frac{I_p}{I_s} = \frac{559.0169}{5} = 111.80$$

$$\text{Ratio error} = \frac{100 - 111.80}{111.80} = -10.56$$

58. (c) Error $= VI \sin(\phi) - VI \cos(\phi)$

$$= VI [\sin\phi - \cos\phi] = -85.1 \text{ W}$$

For power factor $= 0°$

$$= VI \sin(85° - \cos 0°)$$

$$= -4.2 \text{ W}$$

59. (d)

60. (c) Magnifying power $= \frac{500}{100} = 5$

$$R_{sh} = \frac{R_m}{m-1} = \frac{100}{5-1} = \frac{100}{4} = 25\Omega$$

61. (a) For positive tolerance,

$$R_1 = 10 + 0.5 = 10.5\Omega$$

$$R_2 = 5 + 0.5 = 5.5\Omega$$

$$R_{eq} = \frac{R_1 R_2}{R_1 + R_2} = \frac{10.5 \times 5.5}{16} = 3.6$$

For negative tolerance,

$$R_1 = 10 - 0.5 = 9.5\Omega$$

$$R_2 = 5 - 0.5 = 4.5\Omega$$

$$R_{eq} = \frac{R_1 R_2}{R_1 + R_2} = \frac{9.5 \times 4.5}{14} = 3.06$$

Hence, range is 3.06 to 3.6.

62. (d) In two-wattmeter reading method, the power reading is independent of negative and zero sequence voltages.

❐

8

Analog and Digital Electronics

Syllabus

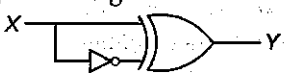
Analog and Digital Electronics *Characteristics of diodes, BJT, FET. Amplifiers - biasing, equivalent circuit and frequency response. Oscillators and feedback amplifiers. Operational amplifiers - characteristics and applications. Simple active filters. VCOs and timers. Combinational and sequential logic circuits. Multiplexer. Schmitt trigger. Multi-vibrators. Sample and hold circuits. A/D and D/A converters. 8-bit microprocessor basics, architecture, programming and interfacing.*

GATE 2011

(1 Mark Questions)

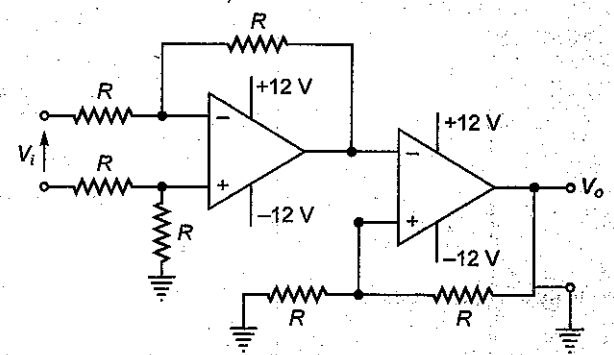
▼ *Solution Points*

1. The output Y of the logic circuit given below is



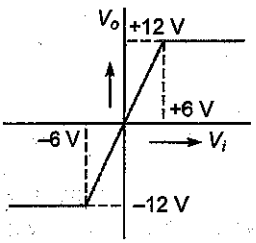
(a) 1 (b) zero (c) X (d) $\overline{X}$

2. For the circuit shown below,

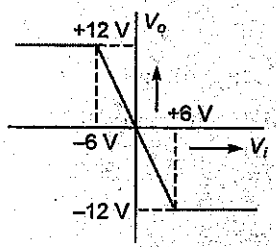


The correct transfer characteristic is

(a)

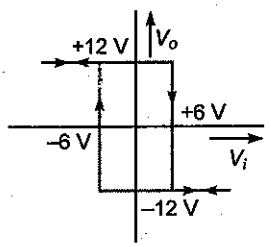


(b)

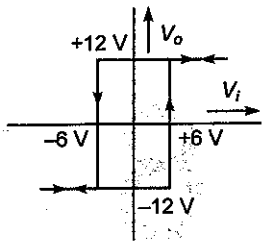


▼ *Solution Points*

(c)

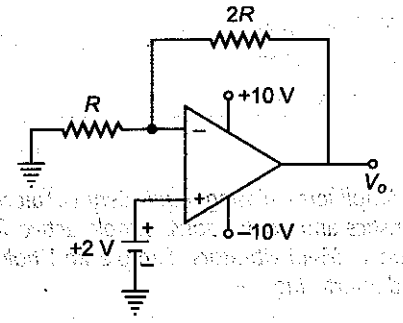


(d)



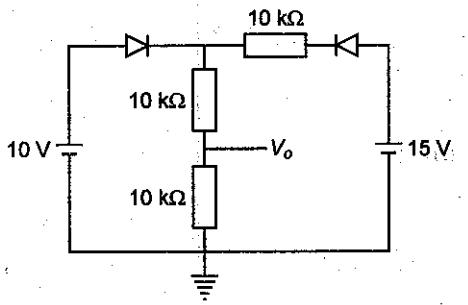
GATE 2010

3. Given that the op-amp is ideal, this output voltage V_o is



(a) 4 V
(b) 6 V
(c) 7.5 V
(d) 12.12 V

4. Assuming that the diodes in the given circuit are ideal, the voltage V_o is



(a) 4 V
(b) 5 V
(c) 7.5 V
(d) 12.12 V

GATE 2009

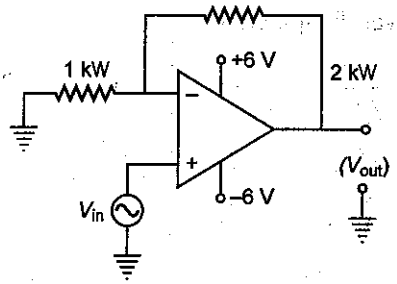
5. The increasing order of speed of data access for the following devices is

(i) Cache Memory
(ii) CDROM
(iii) Dynamic RAM
(iv) Processor Registers
(v) Magnetic Tape

(a) (v), (ii), (iii), (iv), (i)
(b) (v), (ii), (iii), (i), (iv)
(c) (ii), (i), (iii), (iv), (v)
(d) (v), (ii), (i), (iii), (iv)

▼ *Solution Points*

6. The nature of feedback in the op-amp circuit shown is



(a) current-current feedback
(b) voltage-voltage feedback
(c) current-voltage feedback
(d) voltage-current feedback

7. The complete set of only those Logic gates designated as Universal gates is
(a) NOT, OR and AND gates
(b) XNOR, NOR and NAND gates
(c) NOR and NAND gates
(d) XOR, NOR and NAND gates

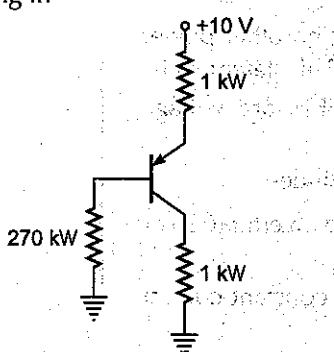
GATE 2008

8. Two 8-bit ADCs, one of single slope integrating type and other of successive approximation type, take T_A and T_B times to convert 6 V analog input signal to equivalent digital output. If the input analog signal is reduced to 2.5 V, the approximate time taken by the two ADCs will respectively be
(a) T_A, T_B
(b) $\frac{T_A}{2}, T_B$
(c) $T_A, \frac{T_B}{2}$
(d) $\frac{T_A}{2}, \frac{T_B}{2}$

9. An input device is interfaced with intel 8085 A microprocessor as memory mapped in/out. The address of the device is 2500 H. In order to input data from the device to accumulator, the sequence of instructions will be
(a) LXI H,2500H
MOV A, M
(b) LXI H,2500H
MOV M, A
(c) LHLD 2500H
MOV A, M
(d) LHLD 2500H
MOV M, A

GATE 2007

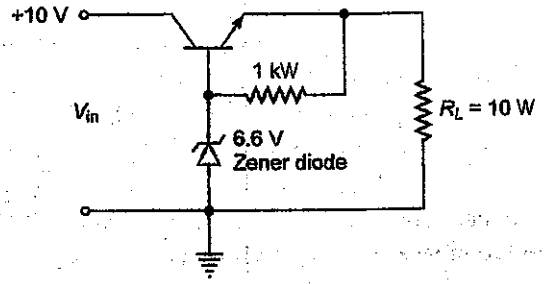
10. The common emitter forward current gain of the transistor shown in $\beta_f = 100$. The transistor is operating in



(a) saturation region
(b) cut–off region
(c) reverse active region
(d) forward active region

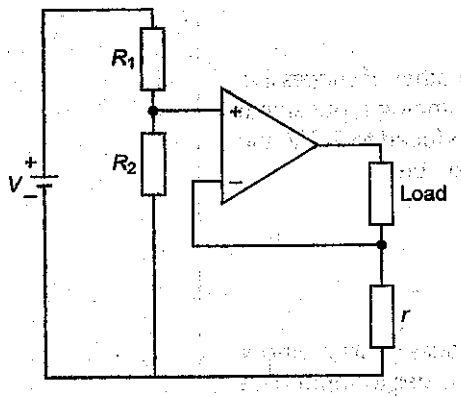
▼ *Solution Points*

11. The three-terminal linear voltage regulator is connected to a 10 Ω load resistor as shown in the figure. If V_{in} is 10 V, what is the power dissipated in the transistor?



(a) 0.6 W (b) 2.4 W
(c) 4.2 W (d) 5.4 W

12. The circuit shown in the figure is



(a) a voltage source with voltage $\frac{rV}{R_1 \| R_2}$

(b) a voltage source with voltage $\frac{r \| R_2}{R_1} V$

(c) a current source with current $\frac{r \| R_2}{R_1 + R_2} \frac{V}{r}$

(d) a current source with current $\frac{R_2}{R_1 + R_2} \frac{V}{r}$

13. Six MOSFETs connected in a bridge configuration (having no other power device) must be operated as a Voltage Source Inverter (VSI). This statement is

(a) true, because being majority carrier devices, MOSFETs are voltage driven

(b) true, because MOSFETs have inherently anti–parallel diodes

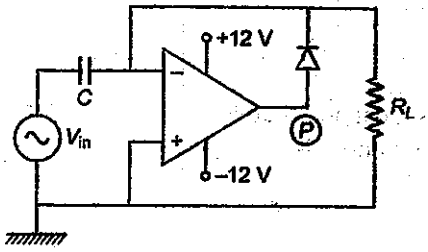
(c) false, because it can be operated both as Current Source Inverter (CSI) or a VSI

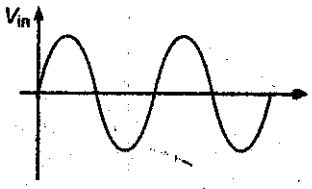
(d) false, because MOSFETs can be operated as excellent constant current sources in the saturation region

GATE 2006

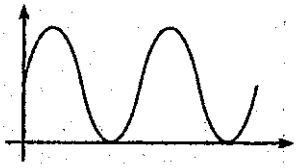
▼ *Solution Points*

14. For a given sinusoidal input voltage, the voltage waveform at point *P* of the clamper circuit shown in figure will be

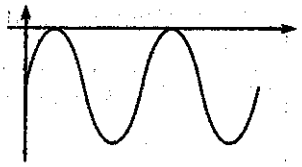




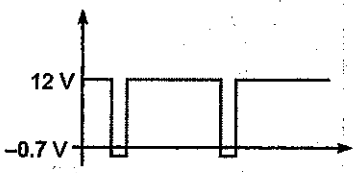
(a)



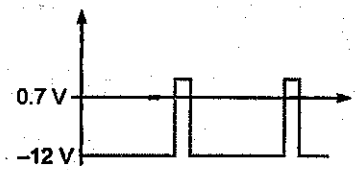
(b)



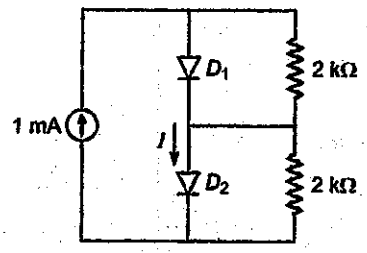
(c)



(d)



15. Assume that D_1 and D_2 in figure are ideal diodes. The value of current I is



(a) zero
(b) 0.5 mA
(c) 1 mA
(d) 2 mA

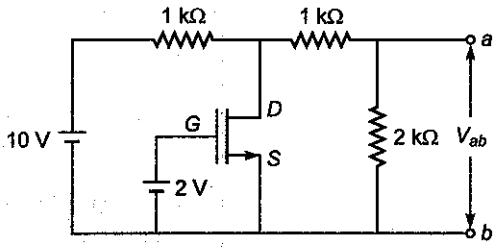
16. The 8085 assembly language instruction that stores the contents of H and L registers into the memory locations 2050 H and 2051 H respectively, is

(a) SPHL 2050 H (b) SPHL 2051 H
(c) SHLD 2050 H (d) STAX 2050 H

▼ *Solution Points*

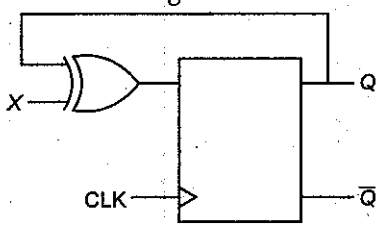
GATE 2005

17. Assume that the *N*-channel MOSFET shown in the figure is ideal and that its threshold voltage is +1.0 V, the voltage V_{ab} between nodes *a* and *b* is



(a) 5 V (b) 2 V (c) 1 V (d) zero

18. The digital circuit shown in the figure works as



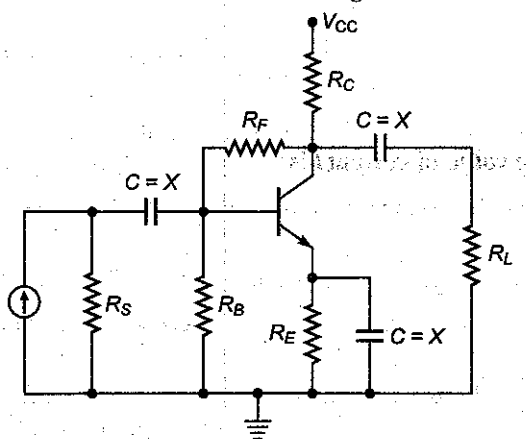
(a) *J-K* flip-flop (b) clocked *R-S* flip-flop
(c) *T* flip-flop (d) ring counter

19. A digital-to-analog converter with a full-scale output voltage of 3.5 V has a resolution close to 14 mV. Its bit size is

(a) 4 (b) 8 (c) 16 (d) 32

GATE 2004

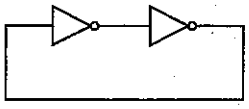
20. The feedback used in the circuit shown in figure can be classified as



(a) shunt-series feedback (b) shunt-shunt feedback
(c) series-shunt feedback (d) series-series feedback

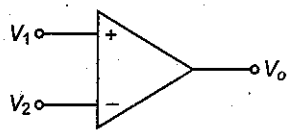
▼ *Solution Points*

21. The digital circuit using two inverters shown in figure will act as



(a) a bistable multi-vibrator (b) an astable multi-vibrator
(c) a monostable multi-vibrator (d) an oscillator

22. The voltage comparator shown in figure can be used in the analog-to-digital conversion as

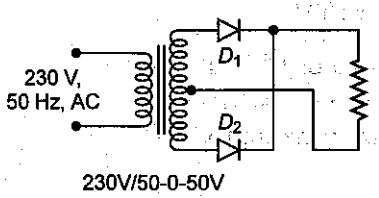


(a) a 1-bit quantizer (b) a 2-bit quantizer
(c) a 4-bit quantizer (d) a 8-bit quantizer

23. A Bipolar Junction Transistor (BJT) is used as a power control switch by biasing it in the cut-off region (OFF state) or in the saturation region (ON state). In the ON state, for the BJT

(a) both the base-emitter and base-collector junctions are reverse biased
(b) the base-emitter junction is reverse biased and the base-collector junction is forward biased
(c) the base-emitter junction is forward biased and the base-collector junction is reverse biased
(d) both the base-emitter and base-collector junctions are forward biased

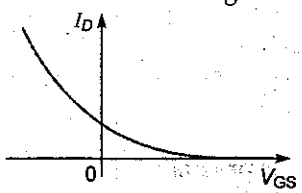
24. The circuit in figure shows a full-wave rectifier. The input voltage is 230 V (rms) single-phase AC. The peak reverse voltage across the diodes D_1 and D_2 is



(a) $100\sqrt{2}$ V (b) 100 V (c) $50\sqrt{2}$ V (d) 50 V

GATE 2003

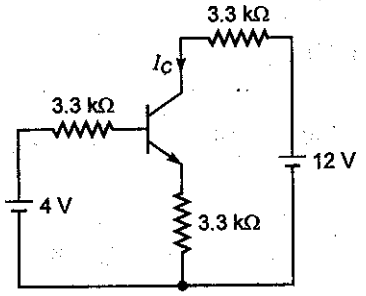
25. The variation of drain current with gate-to-source voltage (I_D-V_{GS} characteristic) of a MOSFET is shown in figure. The MOSFET is



(a) an *n*-channel depletion mode device
(b) an *n*-channel enhancement mode device
(c) a *p*-channel depletion mode device
(d) a *p*-channel enhancement mode device

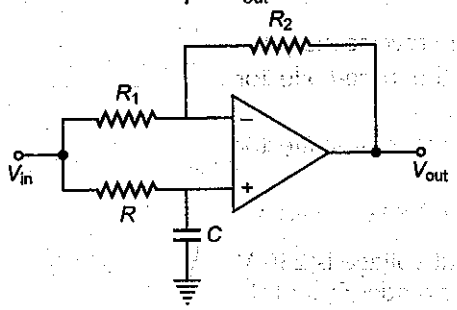
▼ *Solution Points*

26. In the circuit shown in figure, assume that the transistor has $h_{FE} = 99$ and $V_{BE} = -0.7\,V$. The value of collector current I_C of the transistor is approximately



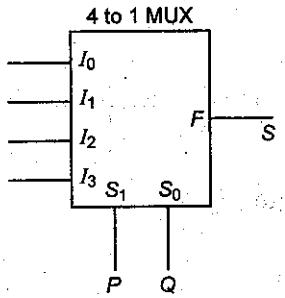
(a) [3.3/3.3] mA
(b) [3.3/(3.3 + 0.33)] mA
(c) [3.3/33] mA
(d) [3.3/(33 + 3.3)] mA

27. For the circuit shown in figure with an ideal operational amplifier, the maximum phase shift of the output V_{out} with reference to the input V_{in} is



(a) 0°
(b) ≈90°
(c) +90°
(d) ±180°

28. Figure shows a 4 to 1 MUX to be used to implement the sum S of 1-bit full added with input bits P and Q and the carry input C_{in}. Which of the following combinations of inputs to I_0, I_1, I_2 and I_3 of the MUX will realize the sum S?



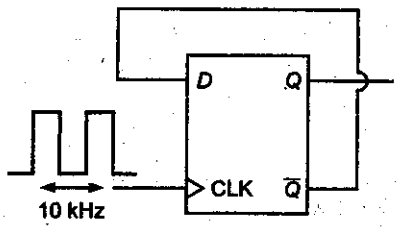
(a) $I_0 = I_1 = C_{in}; I_2 = I_3 = \overline{C}_{in}$
(b) $I_0 = I_1 = \overline{C}_{in}; I_2 = I_3 = C_{in}$
(c) $I_0 = I_3 = C_{in}; I_1 = I_2 = \overline{C}_{in}$
(d) $I_0 = I_3 = \overline{C}_{in}; I_1 = I_2 = C_{in}$

29. When a program is being executed in an 8085 microprocessor, its program counter contains
(a) the number of instructions in the current program that have already been executed
(b) the total number of instructions in the program being executed
(c) the memory address of the instruction that is being currently executed
(d) the memory address of the instruction that is to be executed next

▼ *Solution Points*

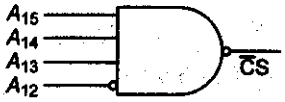
GATE 2002

30. The frequency of the clock signal applied to the rising edge triggered D flip-flop shown in figure is 10 kHz. The frequency of the signal available at Q is



(a) 10 kHz (b) 2.5 kHz (c) 20 kHz (d) 5 kHz

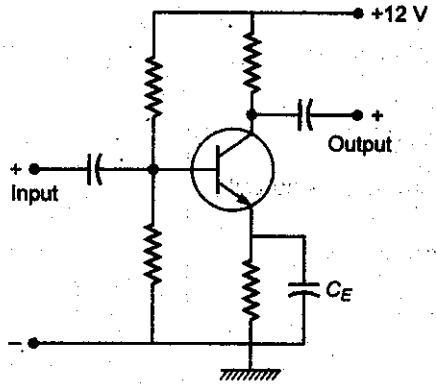
31. The logic circuit used to generate the active low chip select ($\overline{CS}$) by an 8085 microprocessor to address a peripheral is shown in figure. The peripheral will respond to addresses in the range



(a) E000-EFFF (b) 000E-FFFE
(c) 1000-FFFF (d) 0001-FFF1

GATE 2001

32. In the single-stage transistor amplifier circuit shown in figure, the capacitor C_E is removed. Then, the AC small-signal midband voltage gain of the amplifier



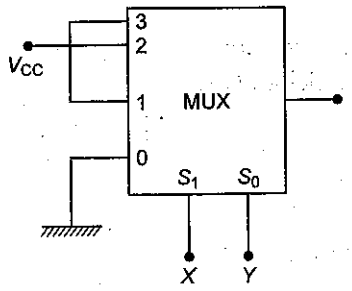
(a) increases (b) decreases (c) is unaffected (d) drops to zero

33. Among the following four, the slowest ADC (analog-to-digital converter) is
(a) parallel-comparator (*i.e.*, flash) type
(b) successive approximation type
(c) integrating type
(d) counting type

34. The output of a logic gate is 1 when all its inputs are at logic 0. The gate is either
(a) a NAND or an Ex-OR gate (b) a NOR or an Ex-OR gate
(c) an AND or an Ex-NOR gate (d) a NOR or an Ex-NOR gate

▼ *Solution Points*

35. The output f of the 4-to-1 MUX shown in figure is



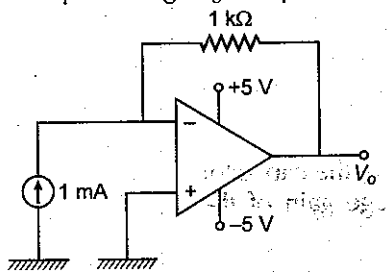
(a) $\overline{XY} + X$ (b) $X + Y$ (c) $\overline{X} + \overline{Y}$ (d) $XY + \overline{X}$

36. An op-amp has an open-loop gain of 10^5 and an open-loop upper cut-off frequency of 10 Hz. If this op-amp is connected as an amplifier with a closed-loop gain of 100, then the new upper cut-off frequency is

(a) 10 Hz (b) 100 Hz (c) 10 kHz (d) 100 kHz

GATE 2000

37. The circuit shown in figure uses an ideal op-amp working with +5 V and –5 V power supplies. The output voltage V_o is equal to



(a) +5 V (b) –5 V (c) +1 V (d) –1 V

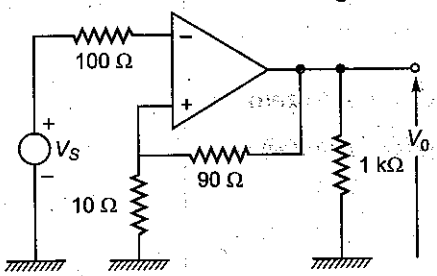
38. Which one of the following is not a vectored interrupt?

(a) TRAP (b) INTR (c) RST7.5 (d) RST3

39. The type of power amplifier which exhibits cross-over distortion in its output is

(a) Class A (b) Class B (c) Class AB (d) Class C

40. The feedback factor for the circuit shown in figure is

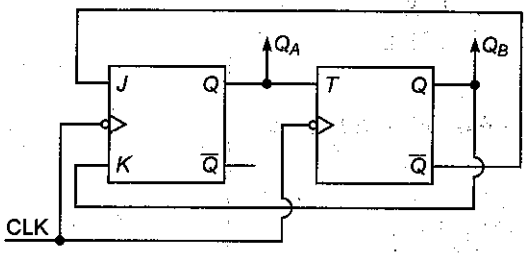


(a) 9/100 (b) 9/10 (c) 1/9 (d) 1/10

GATE 2011 *(2 Marks Questions)*

▼ *Solution Points*

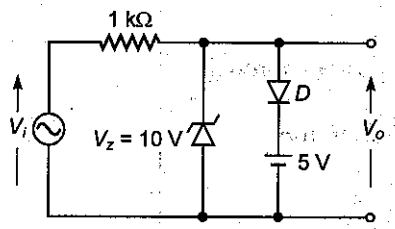
41. A two-bit counter circuit is shown below.



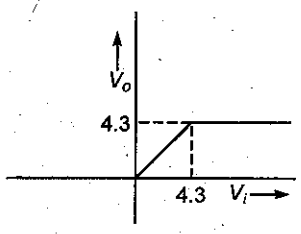
If the state Q_AQ_B of the counter at the clock time t_n is 10 then, the state Q_AQ_B of the counter at $t_n + 3$ (after three clock cycles) will be

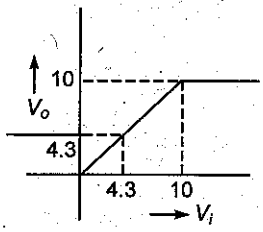
(a) 00 (b) 01 (c) 10 (d) 11

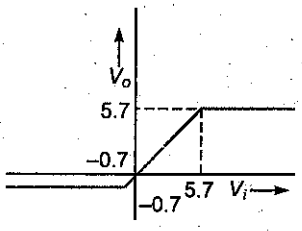
42. A clipper circuit is shown below.

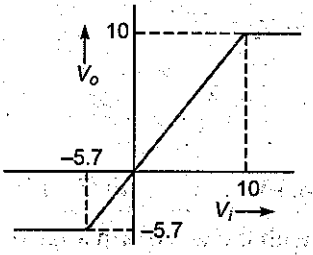


Assuming forward voltage drops of the diodes to be 0.7V, the input-output transfer characteristics of the circuit is

(a)


(b)


(c)


(d)


43. A portion of the main program to call a subroutine SUB in an 8085 environment is given below.

```
        :
        :
        LXI   D, DISP
LP :    CALL  SUB
        :
        :
```

It is desired that control be returned to LP + DISP + 3 when the RET instruction is executed in the subroutine. The set of instructions that precede the RET instruction in the subroutine area

▼ *Solution Points*

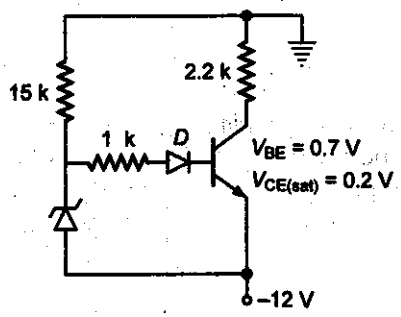
(a) POP D
DAD H
PUSH D

(b) POP H
DAD D
INX H
INX H
INX H
PUSH H

(c) POP H
DAD D
PUSH H

(d) XTHL
INX D
INX D
INX D
XTHL

44. The transistor used in the circuit shown below has a β of 30 and I_{CBO} is negligible.

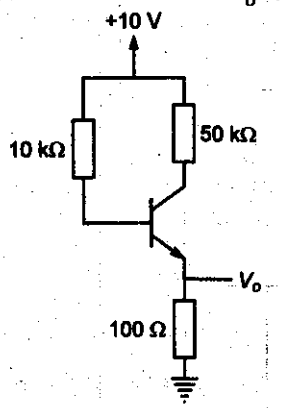


If the forward voltage drop of diode is 0.7 V, then the current through collector will be

(a) 168 mA (b) 108 mA (c) 20.54 mA (d) 5.36 mA

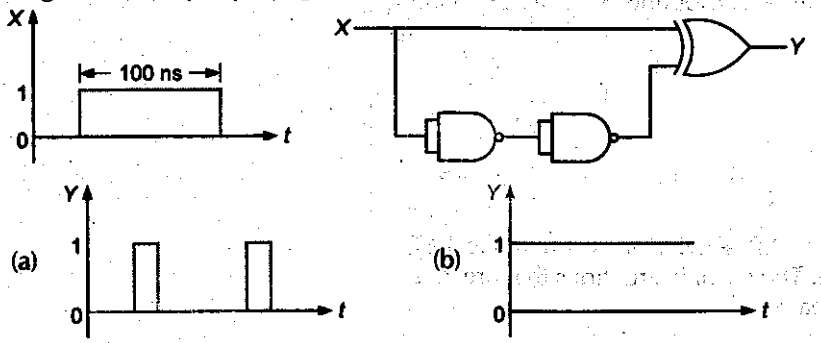
GATE 2010

45. The transistor circuit shown uses a silicon transistor with V_{BE} =0.7 V, $I_C \approx I_E$ and a DC current gain of 100. The value of V_o is



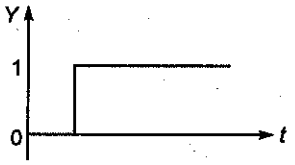
(a) 4.65 V (b) 5 V (c) 6.3 V (d) 7.23 V

46. The TTL circuit shown in the figure is fed with the waveform X (also shown). All gates have equal propagation delay of 10 ns. The output Y of the circuit is

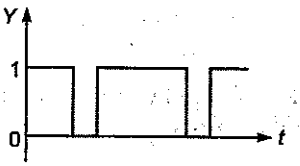


▼ *Solution Points*

(c)



(d)



47. When a CALL Addr instruction is executed, the CPU carries out the following sequential operations internally.

Note : (R) means content of register R

((R)) means content of memory location pointed to by R

PC means program counter

SP means stack pointer

(a) (SP) incremented
(PC) ← Addr
((SP)) ← (PC)

(b) (PC) ← Addr
((SP)) ← (PC)
(SP) incremented

(c) (PC) ← Addr
(SP) incremented
((SP)) ← (PC)

(d) ((SP)) ← (PC)
(SP) incremented
(PC) ← Addr

Statement for Linked Answer Questions 48 and 49

The following Karnaugh map represents a function *F*.

F	00	01	11	10
0	1	1	1	0
1	0	0	1	0

(X)

48. A minimized form of the function *F* is

(a) $F = \overline{X}Y + YZ$

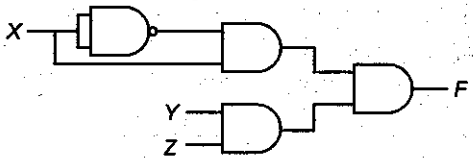
(b) $F = \overline{X}\,\overline{Y} + YZ$

(c) $F = \overline{X}\,\overline{Y} + Y\overline{Z}$

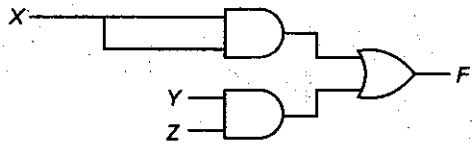
(d) $F = \overline{X}\,\overline{Y} + \overline{Y}Z$

49. Which of the following circuits is a realization of the above function *F*?

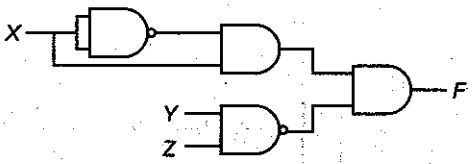
(a)



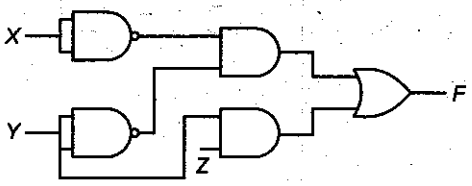
(b)



(c)



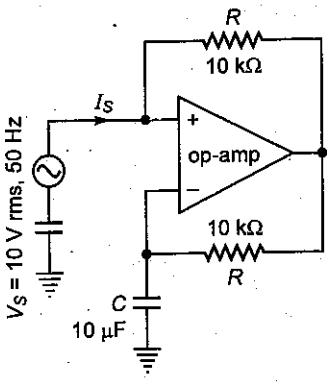
(d)



▼ *Solution Points*

GATE 2009

50. The following circuit has $R = 10\ k\Omega, C = 10\ \mu F$. The input voltage is a sinusoid at 50 Hz with an rms value of 10 V. Under ideal conditions, the current I_S from the source is



(a) 10π mA leading by 90°
(b) 20π mA leading by 90°
(c) 10 mA leading by 90°
(d) 10π mA lagging by 90°

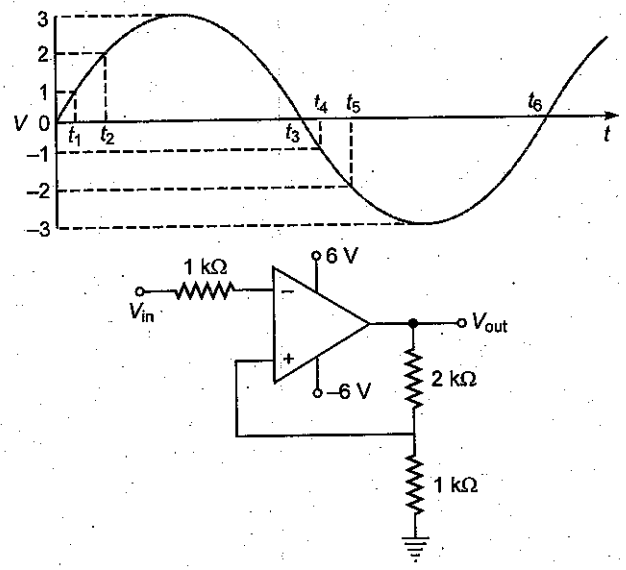
51. Transformer and emitter follower can both be used for impedance matching at the output of an audio amplifier. The basic relationship between the input power P_{in} and output power P_{out} in both the cases is
(a) $P_{in} = P_{out}$ for both transformer and emitter follower
(b) $P_{in} > P_{out}$ for both transformer and emitter follower
(c) $P_{in} < P_{out}$ for transformer and $P_{in} = P_{out}$ for emitter follower
(d) $P_{in} = P_{out}$ for transformer and $P_{in} < P_{out}$ for emitter follower

52. In an 8085 microprocessor, the contents of the accumulator, after the following instructions are executed will become

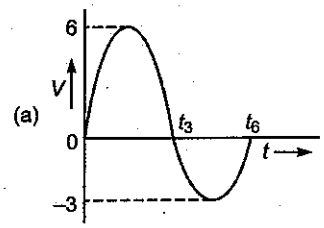
```
XRA  A
MVI  B, F0H
SUB  B
```

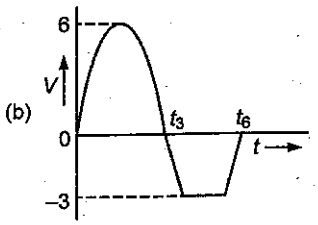
(a) 01 H (b) 0F H (c) F0 H (d) 10 H

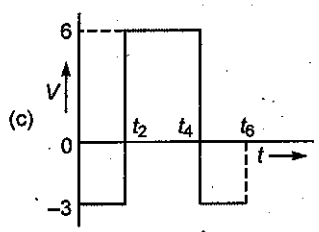
53. An ideal op-amp circuit and its input waveform are shown in the figures. The output waveform of this circuit will be

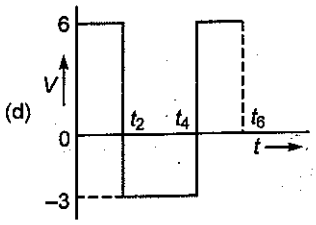


▼ *Solution Points*



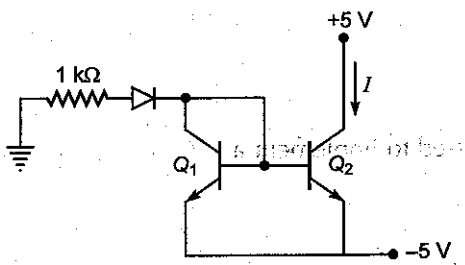






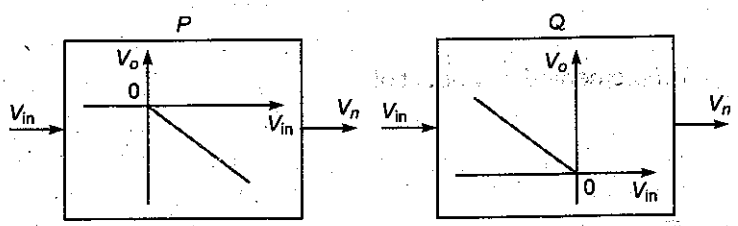
GATE 2008

54. Two perfectly matched silicon transistors are connected as shown in the figure. Assuming the β of the transistors to be very high and the forward voltage drop in diodes to be 0.7 V, the value of current *I* is

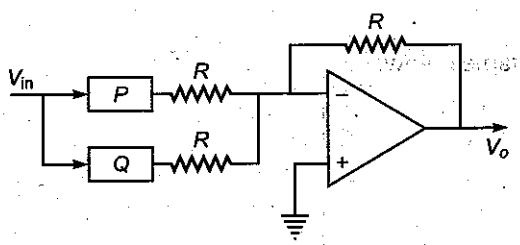


(a) zero (b) 3.6 mA (c) 4.3 mA (d) 5.7 mA

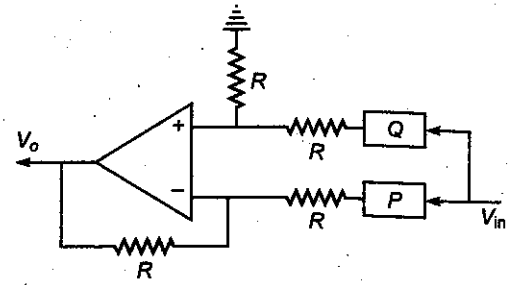
55. The block diagrams of two types of half-wave rectifiers are shown in the figure. The transfer characteristics of the rectifiers are also shown within the block. It is desired to make full-wave rectifier using above two half-wave rectifiers. The resultant circuit will be



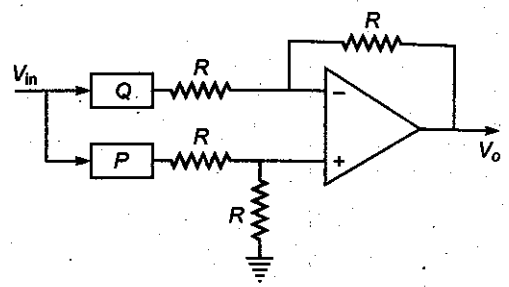
(a)



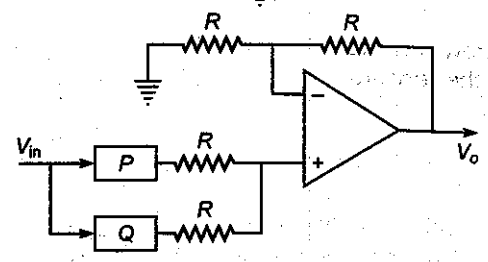
(b)



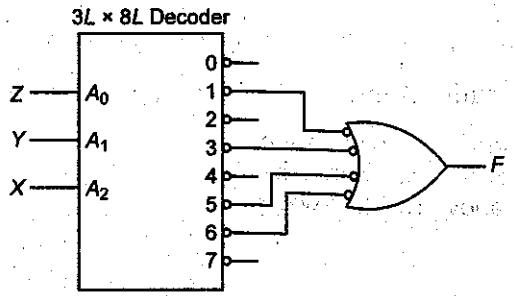
(c)



(d)



56. A 3 line to 8 line decoder, with active low outputs, is used to implement a 3-variable Boolean function as shown in the figure.



The simplified form of Boolean function $F(A, B, C)$ implemented in product of sums form will be

(a) $(X + Z) \cdot (\overline{X} + \overline{Y} + \overline{Z}) \cdot (Y + Z)$

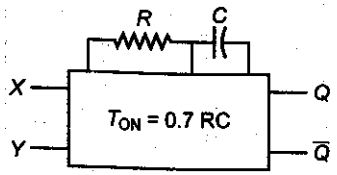
(b) $(\overline{X} + \overline{Z}) \cdot (X + Y + Z) \cdot (\overline{Y} + \overline{Z})$

(c) $(\overline{X} + \overline{Y} + Z) \cdot (\overline{X} + Y + Z) \cdot (X + \overline{Y} + Z) \cdot (X + Y + \overline{Z})$

(d) $(\overline{X} + \overline{Y} + \overline{Z}) \cdot (\overline{X} + Y + \overline{Z}) \cdot (X + Y + Z) \cdot (X + \overline{Y} + \overline{Z})$

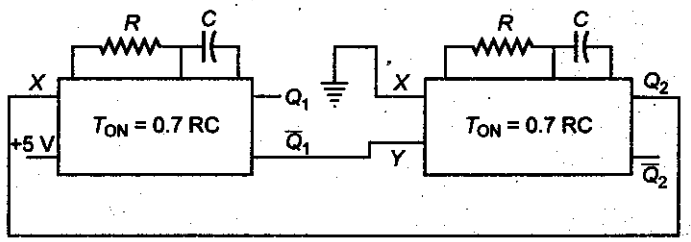
57. The truth table of a monoshot shown in the figure is given in the table below.

X	Y	Q	$\overline{Q}$
0	↑	⎍	⎍
↓	1	⎍	⎍



▼ *Solution Points*

Two monoshots, one positive edge triggered and other negative edge triggered, are connected as shown in the figure. The pulse widths of the two monoshot outputs Q_1 and Q_2 are T_{ON_1} and T_{ON_2} respectively.



The frequency and the duty cycle of the signal at Q_1 will respectively be

(a) $f = \dfrac{1}{T_{ON_1} + T_{ON_2}}, D = \dfrac{1}{5}$

(b) $f = \dfrac{1}{T_{ON_1} + T_{ON_2}}, D = \dfrac{T_{ON_2}}{T_{ON_1} + T_{ON_2}}$

(c) $f = \dfrac{1}{T_{ON_1}}, D = \dfrac{T_{ON_1}}{T_{ON_1} + T_{ON_2}}$

(d) $f = \dfrac{1}{T_{ON_2}}, D = \dfrac{T_{ON_1}}{T_{ON_1} + T_{ON_2}}$

58. The contents (in hexadecimal) of some of the memory locations in an 8085 A based system are given below.

Address	Contents
...	...
26FE	00
26FF	01
2700	02
2701	03
2702	04
...	...

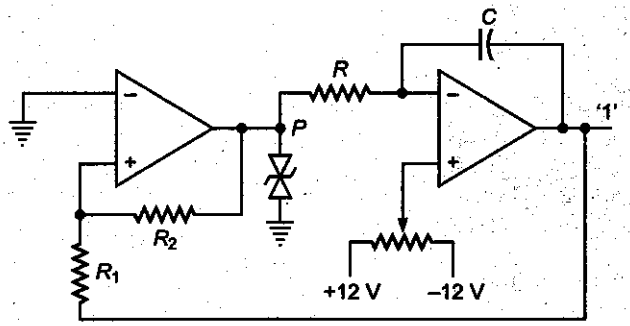
The contents of Stack Pointer (SP), Program Counter (PC) and (HL) are 2700H 2100H and 0000H respectively. When the following sequence of instructions are executed,

2100 H: DAD SP
2101 H: PCHL

the contents of (SP) and (PC) at the end of execution will be

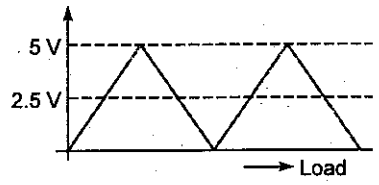
(a) (PC) = 2102H, (SP) = 2700H
(b) (PC) = 2700H, (SP) = 2700H
(c) (PC) = 2800H, (SP) = 26 FEH
(d) (PC) = 2A02H, (SP) = 2702 H

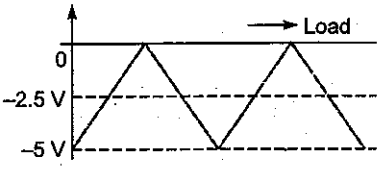
59. A waveform generator circuit using op-amps is shown in the figure. It produces a triangular wave at point P with a peak to peak voltage of 5 V for $V_1 = 0$.

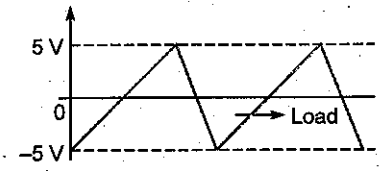


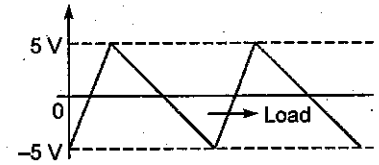
▼ *Solution Points*

If the voltage V_1 is made + 2.5 V, the voltage waveform at point P will become

(a)


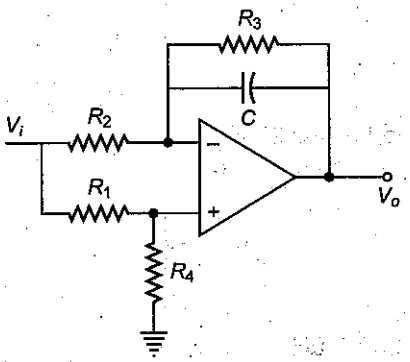
(b)


(c)


(d)


Statement for Linked Answer Questions 60 and 61

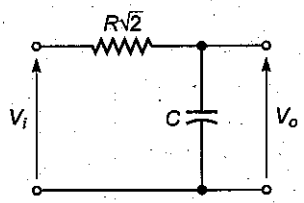
A general filter circuit is shown in the figure.



60. If $R_1 = R_2 = R_A$ and $R_1 = R_4 = R_B$, the circuit acts as a

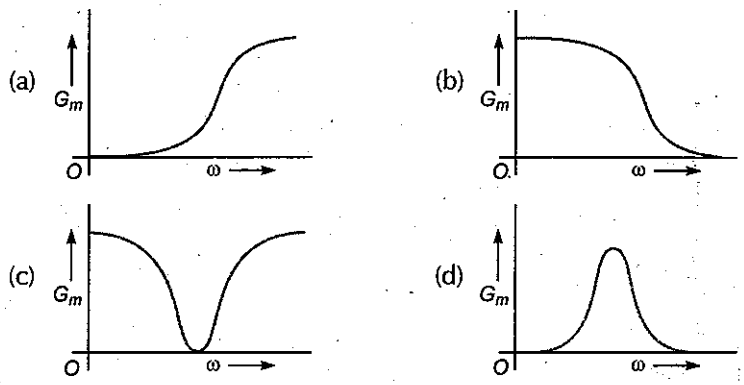
(a) all-pass filter (b) band-pass filter
(c) high-pass filter (d) low-pass filter

61. The output of the filter is given to the circuit shown in figure.



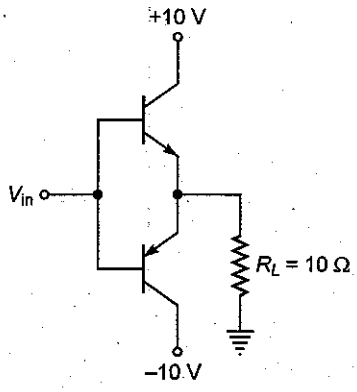
The gain *vs* frequency characteristics of the output (V_o) will be

▼ *Solution Points*



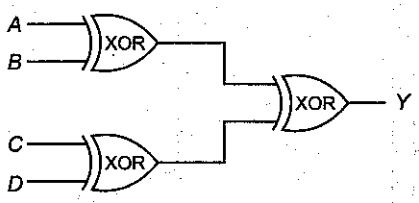
GATE 2007

62. The input signal V_{in} shown in the figure is a 1kHz square wave voltage that alternates between +7 V and −7 V with a 50% duty cycle. Both transistors have the same current gain, which is large. The circuit delivers power to the load resistor R_L. What is the efficiency of this circuit for the given input? Choose the closest answer.



(a) 46% (b) 55% (c) 63% (d) 92%

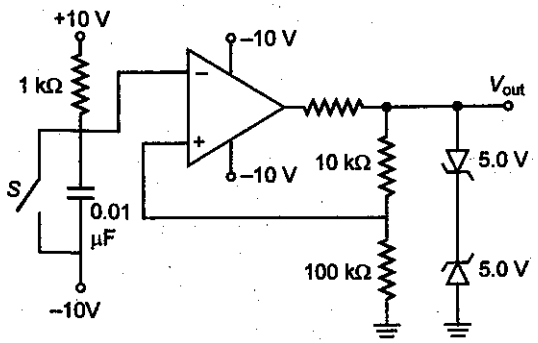
63. A, B, C and D are input bits, and Y is the output bit in the XOR gate circuit of the figure below. Which of the following statements about the sum S of A, B, C, D and Y is correct?



(a) S is always either zero or odd
(b) S is always either zero or even
(c) $S = 1$ only, if the sum of A, B, C and D is even
(d) $S = 1$ only, if the sum of A, B, C and D is odd

64. The switch S is the circuit of the figure is initially closed. It is opened at time $t = 0$. You may neglect the Zener diode forward voltage drops. What is the behaviour of V_{out} for $t > 0$?

▼ *Solution Points*



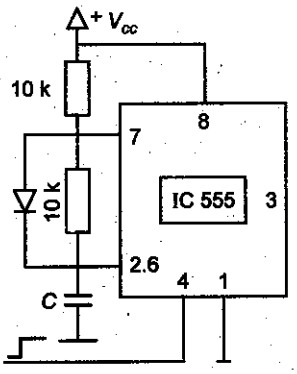
(a) It makes a transition from −5 V to +5 V at $t = 12.98\ \mu s$

(b) It makes a transition from −5 V to +5 V at $t = 2.57\ \mu s$

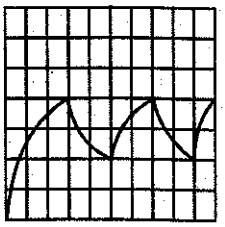
(c) It makes a transition from +5 V to −5 V at $t = 12.98\ \mu s$

(d) It makes a transition from +5 V to −5 V at $t = 2.57\ \mu s$

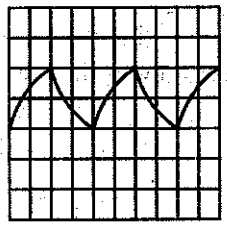
65. IC 555 in the adjacent figure is configured as an astable multi-vibrator. It is enabled to oscillate at $t = 0$ by applying a high input to pin 4. The pin description is 1 and 8-supply; 2-trigger; 4-reset; 6-threshold; 7-discharge. The waveform appearing across the capacitor starting from $t = 0$, as observed on a storage CRO is



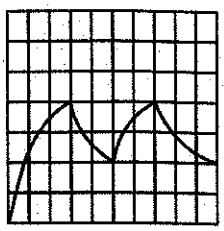
(a)



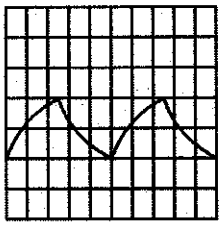
(b)



(c)



(d)



66. Which one of the following statements regarding the INT (interrupt) and the BRQ (bit request) pins in a CPU is true?

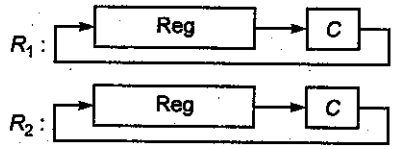
(a) The BRQ pin is sampled after every instruction cycle, but the INT is sampled after every machine cycle

▼ *Solution Points*

(b) Both INT and BRQ are sampled after every machine cycle
(c) The INT pin is sampled after every instruction cycle, but the BRQ is sampled after every machine cycle
(d) Both INT and BRQ are sampled after every instruction cycle

Statements for Linked Answer Questions 67 and 68

The associated figure shows the two types of rotate right instructions R_1 and R_2 available in a microprocessor where, Reg is a 8-bit register and C is the carry bit. The rotate left instructions L_1 and L_2 are similar except that C now links the most significant bit of Reg instead of the least significant one.

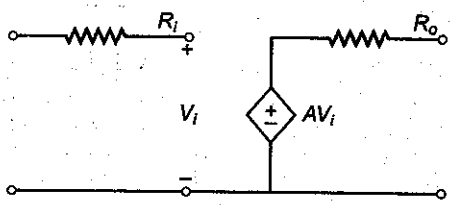


67. Suppose Reg contains the 2's complement number 11010110. If this number is divided by 2, the answer should be
(a) 01101011 (b) 10010101 (c) 11101001 (d) 11101011

68. Such a division can be correctly performed by which of the following sets of operations?
(a) L_2, R_2, R_1 (b) L_2, R_1, R_2 (c) R_2, L_1, R_1 (d) R_1, L_2, R_2

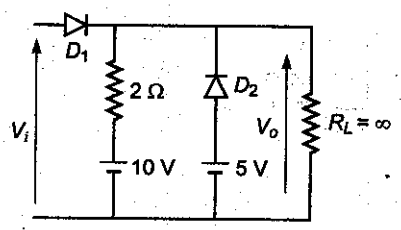
GATE 2006

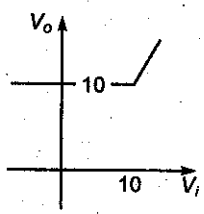
69. The parameters of the circuit shown in the figure are $R_i = 1\text{M}\Omega, R_o = 10\,\Omega$, $A = 10^4$ V/V. If $V_i = 1\mu$V, then output voltage, input impedance and output impedance respectively are

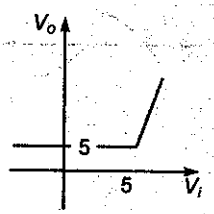


(a) 1 V, ∞, 10 Ω (b) 1V, 0, 10 Ω (c) 1V, 0, ∞ (d) 10 V, ∞, 10 Ω

70. Assuming the diodes D_1 and D_2 of the circuit shown in figure to be ideal ones, the transfer characteristics of the circuit will be

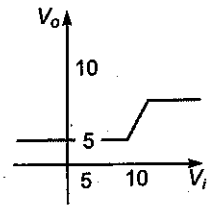


(a)


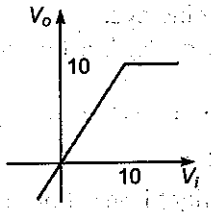
(b)


▼ *Solution Points*

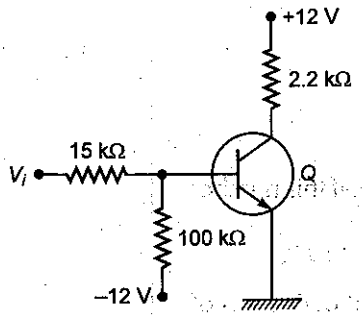
(c)



(d)

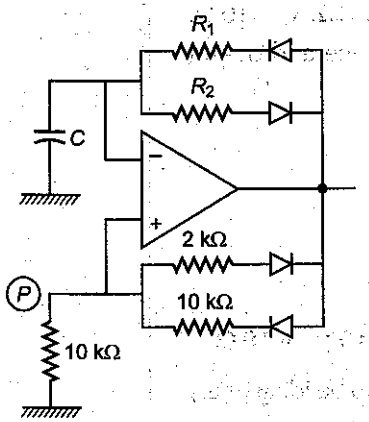


71. Consider the circuit shown in figure. If the β of the transistor is 30 and I_{CBO} is 20 nA and the input voltage is +5 V, then transistor would be operating in

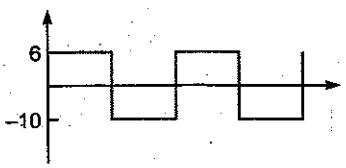


(a) saturation region
(b) active region
(c) breakdown region
(d) cut-off region

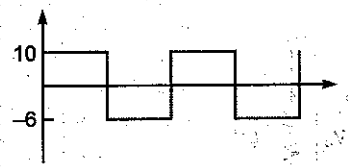
72. A relaxation oscillator is made using op-amp as shown in figure. The supply voltages of the op-amp are ±12 V. The voltage waveform at point *P* will be



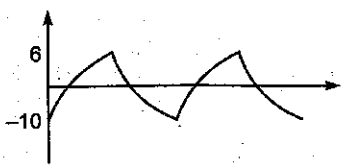
(a)



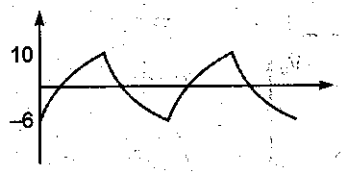
(b)



(c)

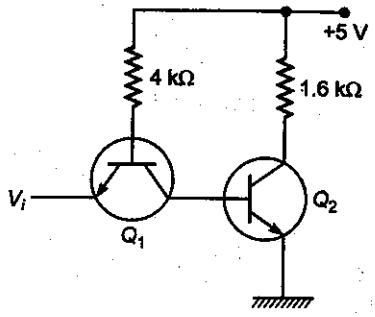


(d)



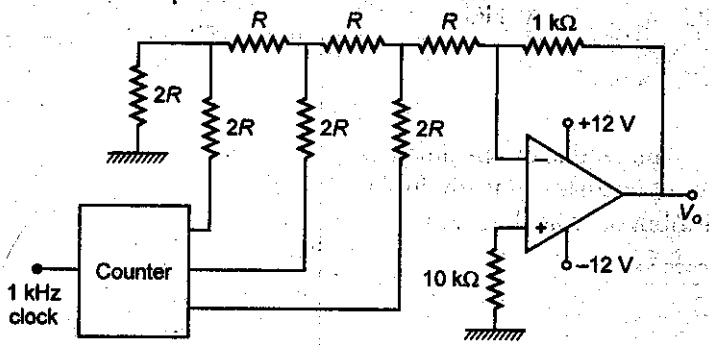
▼ *Solution Points*

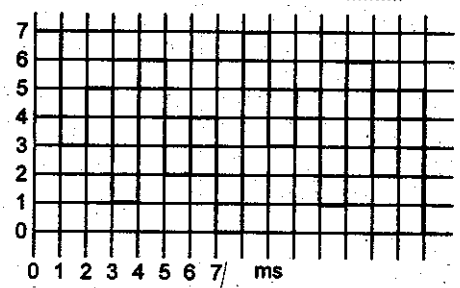
73. A TTL NOT gate circuit is shown in figure. Assuming $V_{BE} = 0.7$ V of both the transistors, if $V_i = 3.0$ V, then the states of the two transistors will be



(a) Q_1 ON and Q_2 OFF
(b) Q_1 reverse ON and Q_2 OFF
(c) Q_1 reverse ON and Q_2 ON
(d) Q_1 OFF and Q_2 reverse ON

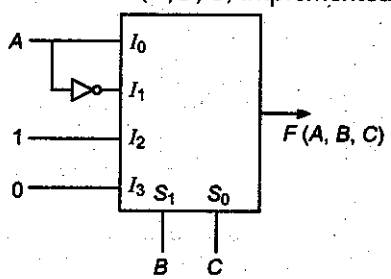
74. A student has made a 3-bit binary down counter and connected to the R–$2R$ ladder type DAC [Gain $= (-1\ k\Omega/2R)$ as shown in figure to generate a staircase waveform. The output achieved is different as shown in figure. What could be the possible cause of this error?





(a) The resistance values are incorrect
(b) The counter is not working properly
(c) The connection from the counter to DAC is not proper
(d) The R and $2R$ resistances are interchanged

75. A 4×1 MUX is used to implement a 3-input Boolean function as shown in figure. The Boolean function $F(A, B, C)$ implemented is



▼ *Solution Points*

(a) $F(A,B,C)=\Sigma(1,2,4,6)$ (b) $F(A,B,C)=\Sigma(1,2,6)$
(c) $F(A,B,C)=\Sigma(2,4,5,6)$ (d) $F(A,B,C)=\Sigma(1,5,6)$

76. A software delay subroutine is written as given below.

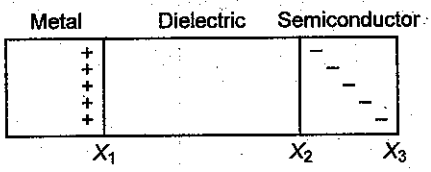
```
DELAY : MVI   H   255D
  MVI   L   255 D
LOOP :  DCR   L
  JNZ   LOOP
  DCR   H
  JNZ   LOOP
```

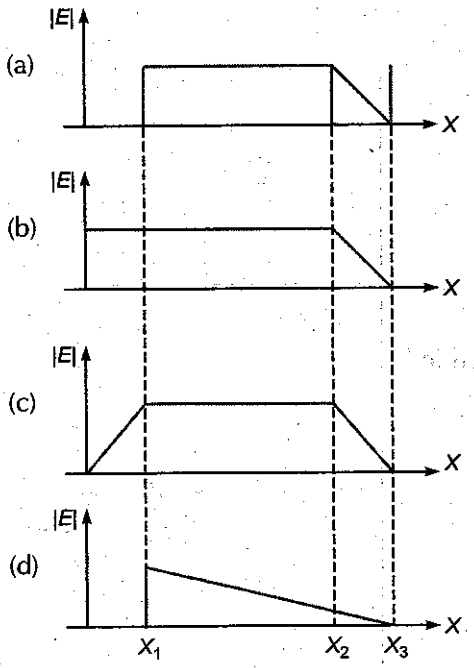
How many times DCR L instruction will be executed?
(a) 255 (b) 510 (c) 65025 (d) 65279

77. In an 8085 A microprocessor based system, it is desired to increment the contents of memory location whose address is available in (D, E) register pair and store the result in same location. The sequence of instructions is
(a) XCHG
INR M
(b) XCHG
INX H
(c) INX D
XCHG
(d) INR M
XCHG

GATE 2005

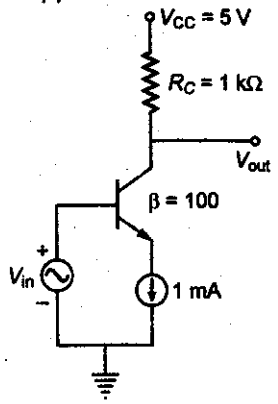
78. The charge distribution in a metal-dielectric semiconductor specimen is shown in the figure. The negative charge density decreases linearly in the semiconductor as shown. The electric field distribution is as shown in





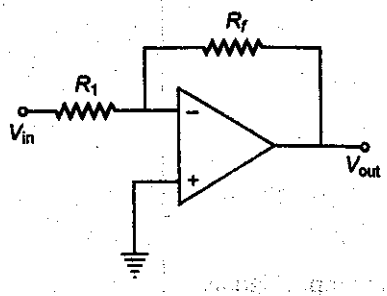
▼ *Solution Points*

79. The common emitter amplifier shown in the figure is biased using a 1mA ideal current source. The approximate base current value is



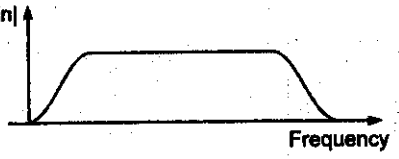
(a) zero (b) 10 μA (c) 100 μA (d) 1000 μA

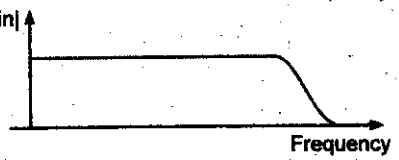
80. Consider the inverting amplifier, using an ideal operational amplifier shown in figure. The designer wishes to realise the input resistance shown by the small signal source to be as large as possible, while keeping the voltage gain between –10 and –25. The upper limit on R_f is 1MΩ. The value of R_1 should be

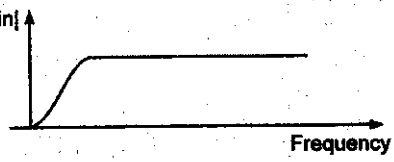


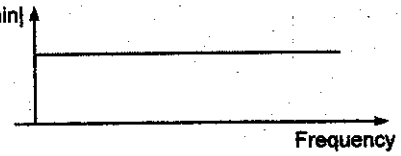
(a) infinite (b) 1MΩ (c) 100 kΩ (d) 40 kΩ

81. The typical frequency response of a two-stage direct coupled voltage amplifier is as shown in

(a)


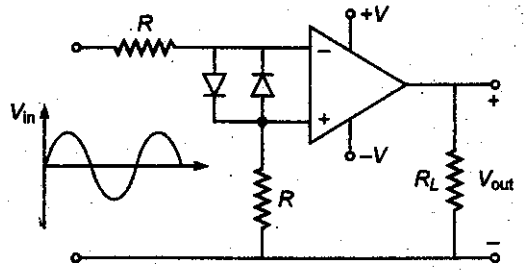
(b)


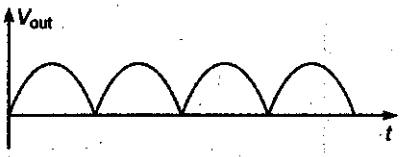
(c)


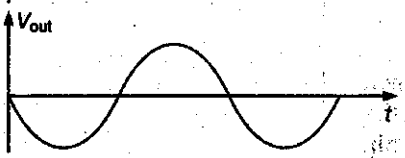
(d)


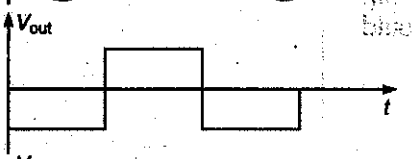
▼ *Solution Points*

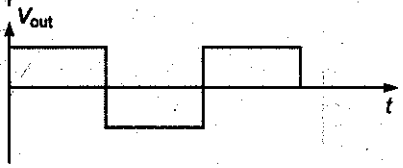
82. In the given figure, if the input is a sinusoidal signal, the output will appear as shown in



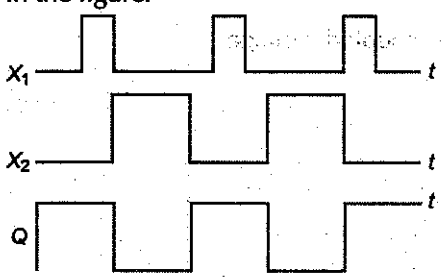
(a)


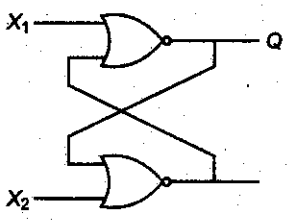
(b)


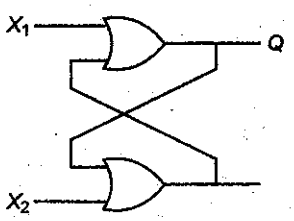
(c)


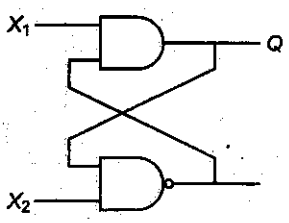
(d)


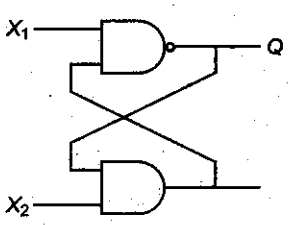
83. Select the circuit which will produce the given output Q for the input signals X_1 and X_2 given in the figure.



(a)


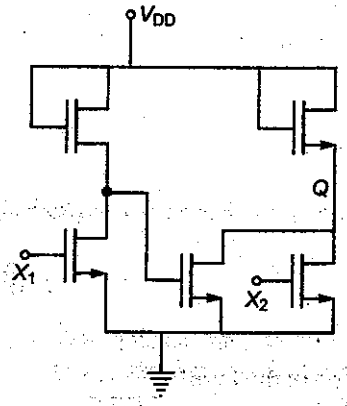
(b)


(c)


(d)


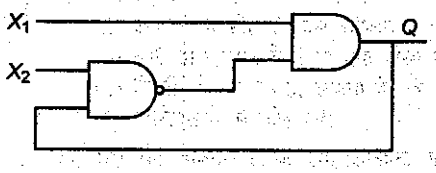
▼ *Solution Points*

84. If X_1 and X_2 are the inputs to the circuit shown in figure, the output Q is



(a) $\overline{X_1 + X_2}$
(b) $\overline{X_1 \cdot X_2}$
(c) $\overline{X}_1 \cdot X_2$
(d) $X_1 \cdot \overline{X}_2$

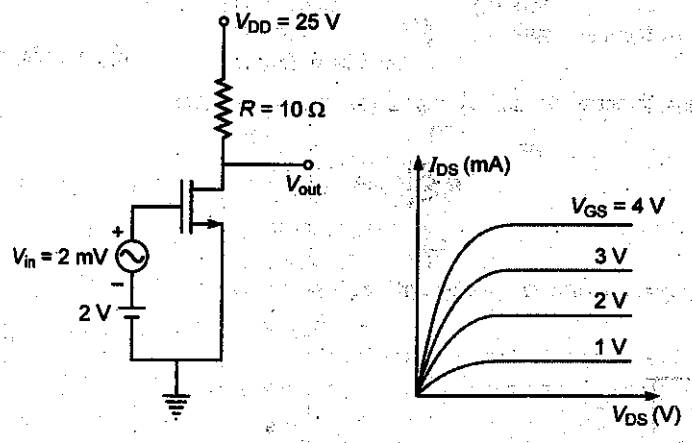
85. In the figure, as long as $X_1 = 1$ and $X_2 = 1$, the output Q remains



(a) at 1
(b) at 0
(c) at its initial value
(d) unstable

Statement for Linked Answer Questions 86 and 87

Assume that the threshold voltage of the N-channel MOSFET shown in figure below.



86. The transconductance of MOSFET is

(a) 0.75 ms
(b) 1 ms
(c) 2 ms
(d) 10 ms

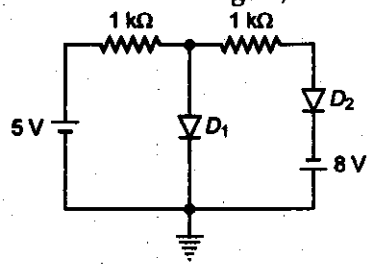
87. The voltage gain of the amplifier is

(a) +5
(b) −7.5
(c) +10
(d) −10

GATE 2004

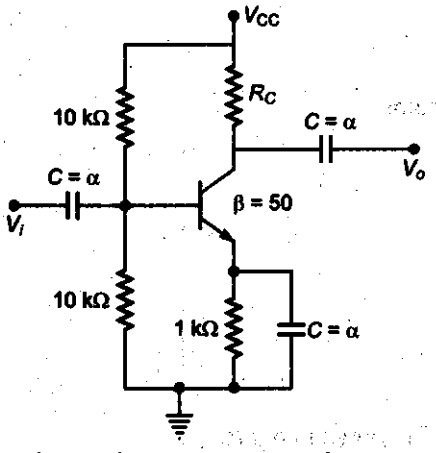
▼ *Solution Points*

88. Assuming that the diodes are ideal in figure, the current in diode D_1 is



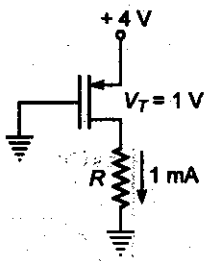
(a) 8 mA (b) 5 mA (c) zero (d) –3 mA

89. The transconductance of the transistor shown in figure. The value of the input resistance R_{in} is



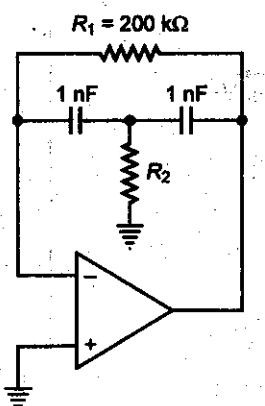
(a) 10.0 kΩ (b) 8.3 kΩ (c) 5.0 kΩ (d) 2.5 kΩ

90. The value of R for which the PMOS transistor in figure will be biased in linear region is



(a) 220 Ω
(b) 170 Ω
(c) 680 Ω
(d) 1200 Ω

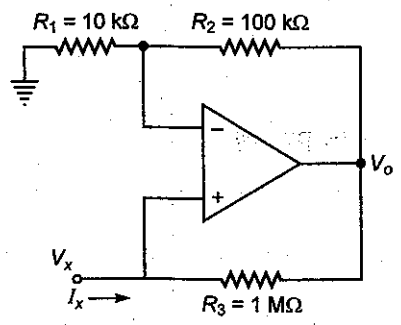
91. In the active filter circuit shown in figure. If $Q = 1$, a pair of poles will be realized with ω_0 equal to



(a) 1000 rad/s (b) 100 rad/s (c) 10 rad/s (d) 1 rad/s

▼ *Solution Points*

92. The input resistance $R_{in}(=V_x/I_x)$ of the circuit in figure is



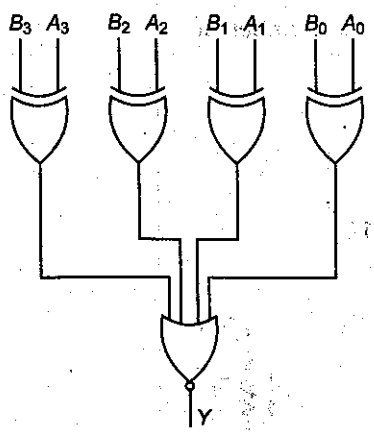
(a) +100 kΩ (b) −100 kΩ (c) +1MΩ (d) −1MΩ

93. The simplified form of the Boolean expression
$Y=(\overline{A}\cdot BC+D)(\overline{A}\cdot D+\overline{B}\cdot\overline{C})$ can be written as

(a) $\overline{A}\cdot D+\overline{B}\cdot\overline{C}\cdot D$ (b) $AD+B\cdot\overline{C}\cdot D$

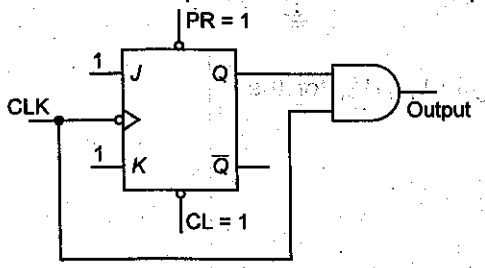
(c) $(\overline{A}+D)(\overline{B}\cdot C+\overline{D})$ (d) $A\cdot\overline{D}+BC\cdot\overline{D}$

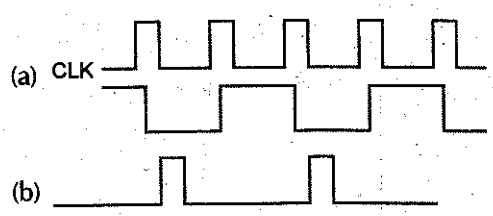
94. A digital circuit which compares two numbers $A_3A_2A_1A_0, B_3B_2B_1B_0$ is shown in figure. To get output $Y=0$, choose one pair of correct input numbers.



(a) 1010, 1010 (b) 0101, 0101 (c) 0010, 0010 (d) 0010, 1011

95. The digital circuit shown in figure generates a modified clock pulse at the output. Choose the correct output waveform from the options given below.



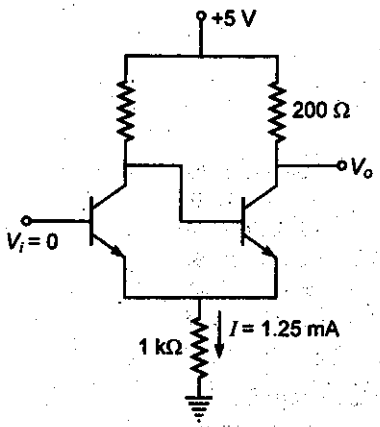


▼ *Solution Points*

(c)

(d)

96. In the Schmitt trigger circuit shown in figure, if $V_{CE,sat} = 0.1V$, the output logic low level (V_{OL}) is



(a) 1.25 V (b) 1.35 V (c) 2.50 V (d) 5.00 V

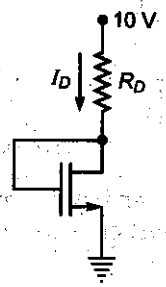
97. If the following program is executed in a microprocessor, the number of instruction cycles it will take from START to HALT is

```
START MVIA, 14 H; Move 14H to register A
SHIFT RLC ; Rotate left without carry
JNZ SHIFT; Jump on non-zero to SHIFT
HALT
```

(a) 4 (b) 8 (c) 13 (d) 16

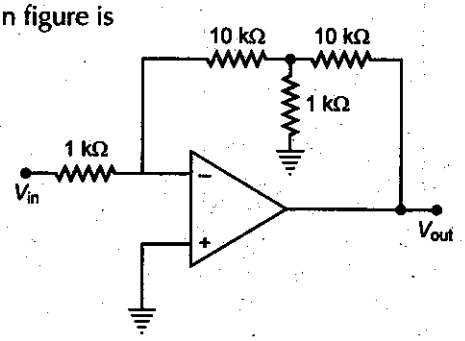
GATE 2003

98. For the n-channel enhancement MOSFET shown in figure, the threshold voltage $V_{th} = 2$ V. The drain current I_D of the MOSFET is 4 mA when the drain resistance R_D is 1kΩ. If the value of R_D is increased to 4 kΩ, drain current I_D will become



(a) 2.8 mA (b) 2.0 mA
(c) 1.4 mA (d) 1.0 mA

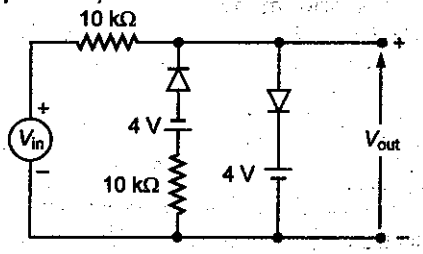
99. Assuming the operational amplifier to be ideal, the gain V_{out}/V_{in} for the circuit shown in figure is



(a) −1 (b) −20 (c) −100 (d) −120

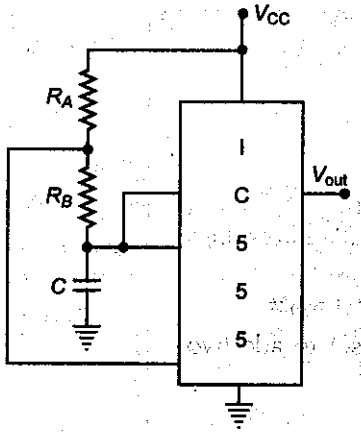
▼ *Solution Points*

100. A voltage signal 10 sin ωt is applied to the circuit with ideal diodes, as shown in figure. The maximum and minimum values of the output waveform V_{out} of the circuit are respectively



(a) +10 V and −10 V (b) +4 V and −4 V
(c) +7 V and −4 V (d) +4 V and −7 V

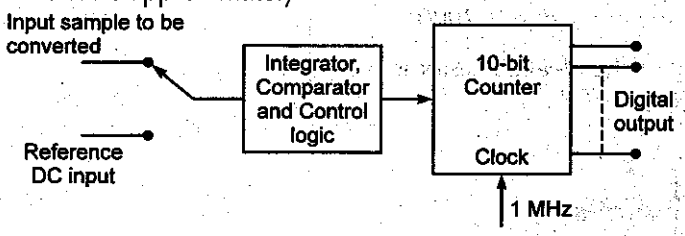
101. The circuit of figure shows a 555 Timer IC connected as an astable multi-vibrator. The value of the capacitor C is 10 nF.



The values of the resistors R_A and R_B for a frequency of 10 kHz and a duty cycle of 0.75 for the output voltage waveform are

(a) $R_A = 3.62\ k\Omega, R_B = 3.62\ k\Omega$ (b) $R_A = 3.62\ k\Omega, R_B = 7.25\ k\Omega$
(c) $R_A = 7.25\ k\Omega, R_B = 3.62\ k\Omega$ (d) $R_A = 7.25\ k\Omega, R_B = 7.25\ k\Omega$

102. The simplified block diagram of a 10-bit A/D converter of dual slope integrator type is shown in figure. The 10-bit counter at the output is clocked by a 1 MHz clock. Assuming negligible timing overhead for the control logic, the maximum frequency of the analog signal that can be converted using this A/D converter is approximately



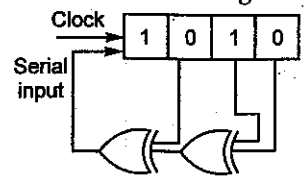
(a) 2 kHz (b) 1 kHz (c) 500 Hz (d) 250 Hz

103. The Boolean expression $\overline{X}Y\overline{Z} + \overline{X}\overline{Y}Z + X\overline{Y}\overline{Z} + X\overline{Y}Z + XYZ$ can be simplified to

(a) $X\overline{Z} + \overline{X}Z + YZ$ (b) $XZ + \overline{Y}Z + Y\overline{Z}$
(c) $\overline{X}Y + YZ + XZ$ (d) $\overline{XY} + Y\overline{Z} + \overline{X}Z$

▼ *Solution Points*

104. The shift register shown in figure is initially loaded with the bit pattern 1010. Subsequently the shift register is clocked, and with each clock pulse the pattern gets shifted by one bit position to the right. With each shift, the bit at the serial input is pushed to the left most position (msb). After how many clock pulses will the content of the shift register become 1010 again?



(a) 3 (b) 7 (c) 11 (d) 15

105. An *X-Y* filp-flop, whose characteristic table is given below is to be implemented by using a *J-K* flip-flop.

X	Y	Q_{n+1}
0	0	1
0	1	Q_n
1	0	Q_n
1	1	0

This can be done by making

(a) $J=X, K=\overline{Y}$ (b) $J=\overline{X}, K=Y$ (c) $J=Y, K=\overline{X}$ (d) $J=\overline{Y}, K=X$

106. A memory system has a total of 8 memory chips each with 12 address lines and 4 data lines. The total size of the memory system is

(a) 6 kbyte (b) 32 kbyte (c) 48 kbyte (d) 64 kbyte

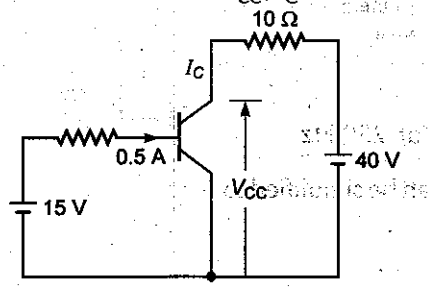
107. The following program is written for an 8085 microprocessor to add two bytes located at memory addresses 1FFE and 1FFF

```
LXI    H, 1FFE
MOV    B, M
INR    L
MOV    A, M
ADD    B
INR    L
MOV    M, A
XOR    A
```

On completion of the execution of the program, the result of addition is found

(a) in the register *A* (b) at the memory address 1000
(c) at the memory address 1F00 (d) at the memory address 2000

108. In the circuit shown in figure, the current gain (β) of the ideal transistor is 10. The operating point of the transistor (V_{CC}, I_C) is

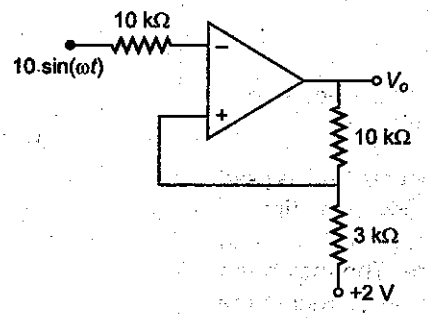


(a) (40 V, 4 A) (b) (40 V, 5 A) (c) (0, 4 A) (d) (15 V, 4 A)

▼ *Solution Points*

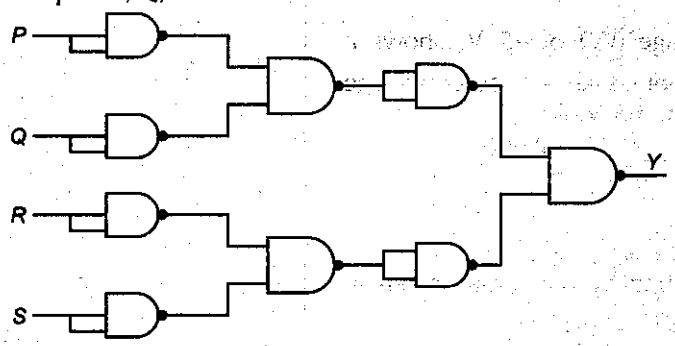
GATE 2002

109. The output voltage (V_o) of the Schmitt trigger as shown in figure swings between +15 V and −15 V. Assume that the operational amplifier is ideal. The output will change from +15 V to −15 V when the instantaneous value of the input sine wave is



(a) 5 V in the positive slope only
(b) 5 V in the negative slope only
(c) 5 V in the positive and negative slopes
(d) 3 V in the positive and negative slopes

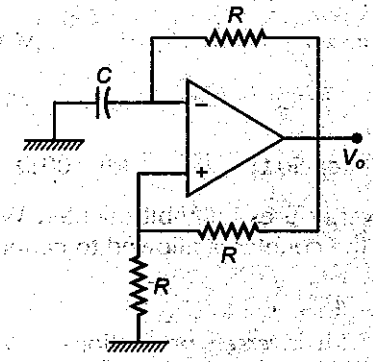
110. For the circuit shown in figure, the Boolean expression for the output Y in terms of inputs P, Q, R and S is



(a) $\overline{P}+\overline{Q}+\overline{R}+\overline{S}$
(b) $P+Q+R+S$
(c) $(\overline{P}+\overline{Q})(\overline{R}+\overline{S})$
(d) $(P+Q)(R+S)$

GATE 2001

111. For the oscillator circuit shown in figure, the expression for the time period of oscillation can be given by (where, $\tau = RC$)



(a) $\tau \ln 3$
(b) $2\tau \ln 3$
(c) $\tau \ln 2$
(d) $2\tau \ln 2$

▼ *Solution Points*

112. An Intel 8085 processor is executing the program given below.

```
        MVIA, 10H
        MVIB, 10H
BACK    NOP
        ADD B
        RLC
        INC BACK
        HLT
```

The number of times that the operation NOP will be executed is equal to
(a) 1 (b) 2 (c) 3 (d) 4

113. A sample-and-hold (S/H) circuit, having a holding capacitor of 0.1 nF, is used at the input of an ADC (analog-to-digital converter). The conversion time of the ADC is 1μs, and during this time the capacitor should not lose more than 0.5% of the charge put across it during the sampling time. The maximum value of the input signal to the S/H circuit is 5 V. The leakage current of the S/H circuit should be less than
(a) 2.5 mA (b) 0.25 mA (c) 25.0 μA (d) 2.5 μA

114. An op-amp, having a slew rate of 62.8 V/μs, is connected in a voltage follower configuration. If the maximum amplitude of the input sinusoidal is 10 V, then the minimum frequency at which the slew rate limited distortion would set in at the output is
(a) 1.0 MHz (b) 6.28 MHz (c) 10.0 MHz (d) 62.8 MHz

115. An *n*-channel JFET, having a pinch-off voltage (V_p) of –5 V, shows a transconductance (g_m) of 1mA/V when the applied gate-to-source voltage (V_{GS}) is –3 V. Its maximum transconductance (in mA/V) is
(a) 1.5 (b) 2.0 (c) 2.5 (d) 3.0

GATE 2000

116. The minimal product-of-sums function described by the K-map given in figure is

C \ AB	00	01	11	10
0	1	1	ϕ	0
1	0	0	ϕ	0

(a) $A'C'$ (b) $A'+C'$ (c) $A+C$ (d) AC

117. A diode whose terminal characteristics are related as $I_D = I_S\left(\frac{V}{V_T}\right)$ where I_S is the reverse saturation current and V_T is the thermal voltage (= 25 mV), is biased at $I_D = 2$mA. Its dynamic resistance is
(a) 25 Ω (b) 12.5 Ω (c) 50 Ω (d) 100 Ω

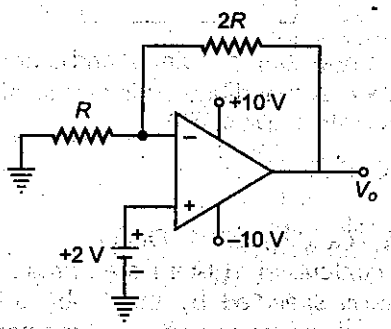
118. A dual-slope analog-to-digital converter uses an *N*-bit counter. When the input signal V_a is being integrated, the counter is allowed to count up to a value
(a) equal to $2^N - 2$ (b) equal to $2^N - 1$
(c) proportional to V_a (d) inversely proportional to V_a

Answers with Explanations

1. (a) $Y = X \cdot \overline{\overline{X}} + \overline{X} \cdot \overline{X} = X + \overline{X} = 1$

2. (d) The given circuit will work as Schmitt trigger and there is zero phase shift.

3. (b)

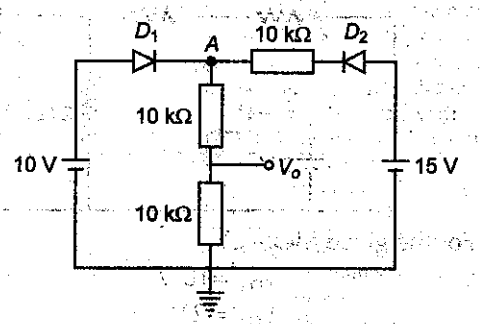


This is a non-inverting amplifier.

The output voltage $V_o = (1 + 2R/R)\, V_{in}$

$= (1 + 2) \times 2 = 6$ V

4. (b)



From figure, it is clear that the diode D_1 is forward biased, hence conduct and diode D_2 is reverse biased, hence cut-off.

As the diode D_1 is ideal.

$$V_A = 10 \text{ V}$$

Hence, $V_o = 10 \times \dfrac{10}{10 + 10} = 5$ V

(From voltage divider rule)

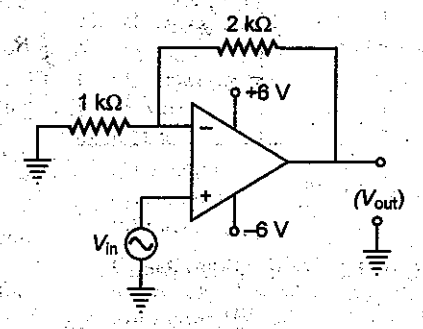
5. (b) Processor registers are located inside the processor. Registers are technically among the fastest of all forms of data storage. Cache memory is intermediate stage of data process slower to processor register.

RAM is the primary storage having higher speed than secondary storage. (Such that magnetic tape or CDROM).

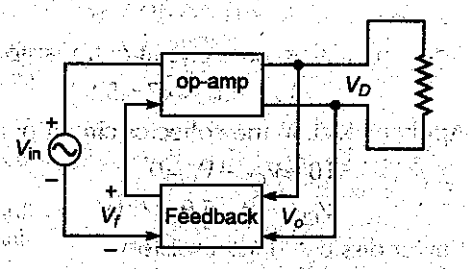
Hence, increasing order sequence is

Magnetic tape < CDROM < Dynamic RAM < Cache Memory < Processor register

6. (b)



The circuit can be realised as



Hence, it is voltage-voltage feedback circuit.

7. (c) With the NOR and NAND gates we are able to release all the logic circuit. Hence, NOR and NAND gates are designated as universal gate.

8. (d)

9. (c) The given condition is the content of the device having address 2500 is to be load to memory and after that the memory content is moved to accumulator.

LHLD 2500 H

The instruction copies the content of memory location pointed out by the 16-bit address register L and copies the content of next memory location in H.

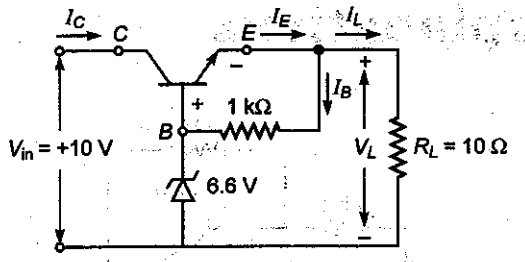
MOV A, M

Transfer the content of memory to accumulator.

10. (d) The base of the *p-n-p* transistor is grounded, while the emitter is connected to positive through R_E. It follows that the emitter-base junction will be forward biased with $V_E = V_{EB} = 0.7$ V.

Since, the collector is connected to a ground through R_C, it is possible that this transistor is operating in forward active mode.

11. (b)



We can write

$$I_C = I_E = I_L - I_B$$

As transistor is in active region

$$V_{BE} = 0.7$$

$$I_B = \frac{0.7}{1} = 0.7 \times 10^{-3} \text{A}$$

Applying KVL in base circuit of transistor,

$$V_L = 6.6 - 0.7 = 5.9 \text{ V}$$

Applying KVL in the collector circuit of transistor,

$$10 - V_{CE} - V_L = 0$$

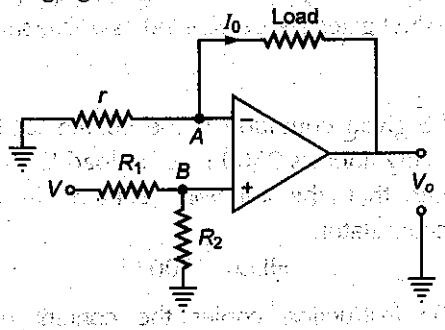
$$V_{CE} = 10 - 5.9 = 4.1 \text{V}$$

Power dissipated in transistor

$$P_{t2} = V_{CE} \times I_C = 4.1 \times 0.7 \times 10^{-3}$$

$$= 2.87 \text{ mW}$$

12. (d) Arranging the circuit,



By virtual ground concept,

$$V_A = V_B$$

$$V_A = \frac{R_2}{R_1 + R_2} V$$

Applying KCL at node A,

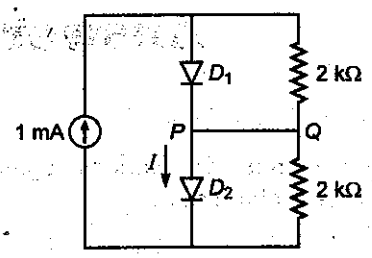
$$\frac{V_A}{r} = I_0$$

$$I_0 = \frac{R_2}{R_1 + R_2} \cdot \frac{V}{r}$$

Hence, the circuit is a current source with current $\frac{R_2}{R_1 + R_2} \frac{V}{r}$.

13. (a) **14.** (d)

15. (a)



As the potentials of points P and Q are same, hence we can say that diode D_2 is reverse biased. Hence, the current $I = 0$.

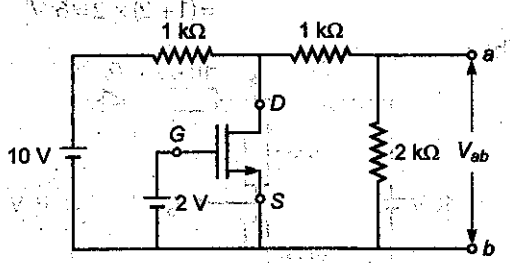
16. (c) SHLD 2050 H

↓

Store H and L registers Direct

The contents of register L are stored in the memory location specified by the 16-bit address in the operand and the contents of H register are stored in the next memory location by incrementing the operand.

17. (d)



For the given MOSFET,

$$V_{DS} = 10 \text{ V}$$

$$V_{GS} = 2 \text{V}$$

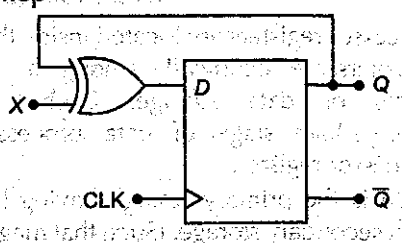
$$V_t = 1 \text{V}$$

Hence, $V_{DS} > V_{GS} - V_t$

The N-MOSFET is in saturation region. The voltage V_{ab} between nodes A and B is zero.

18. (c) The output $Q(t+1) = X^* \times Q(t)$

Hence, the digital circuit shown in figure works as T flip-flop



19. (a) Let n be the bit size.

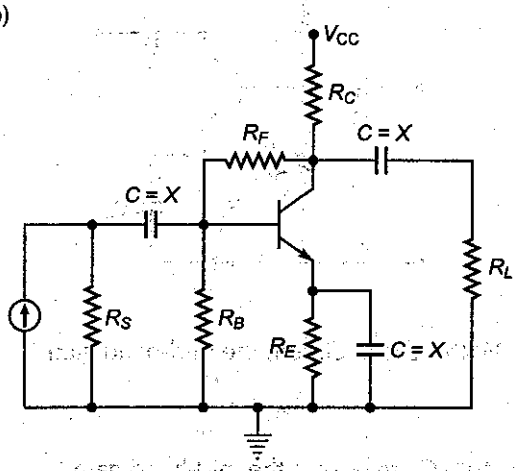
$$\text{Resolution} = \frac{1}{2^n} \times \text{FSV}$$

$$14 \times 10^{-3} = \frac{1}{2^n} \times 3.5$$

$$2^n = \frac{3.5}{14\times10^{-3}} = \frac{3.5\times10^3}{14} = 15.83 \simeq 16$$

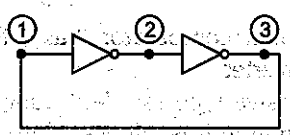
Hence, $n = 4$

20. (b)

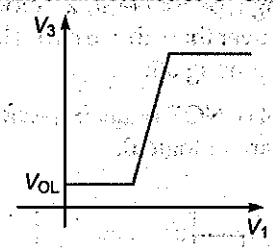


The feedback is provided by R_F which samples the output voltage V_o and feedback a current that is mixed with the source current.
Hence, the given configuration is shunt-shunt feedback.

21. (a)

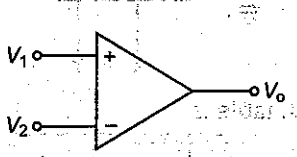


The transfer characteristics of circuit



shows that it has two stable state. Hence, the digital circuit is bistable circuit. The stable state in which the circuit operates depend upon the external excitation that force is to the particular state.

22. (b)



As the op-amp is in open-loop configuration.
Hence, if $V_1 > V_2 \Rightarrow V_o = +V_{sat}$
$V_1 < V_2 \Rightarrow V_o = -V_{sat}$
Hence, comparator used in A/D converter is a 2-bit quantizer.

23. (d) To operate transistor in saturation region (ON state), both the base-emitter and base-collector junctions should be forward biased.

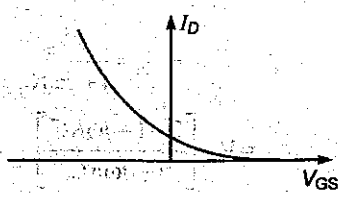
24. (a) As the step-down transformer is used. Hence, the input supply voltage will be 50 V (rms).

$$V_{in} = \sqrt{2}\,V_{rms} = 50\sqrt{2}$$

For full-wave rectifier shown in figure

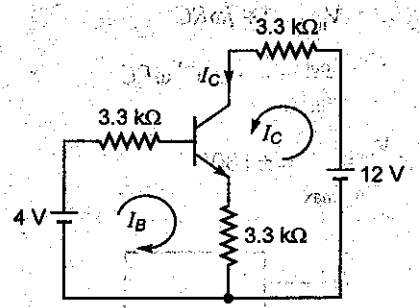
$$PIV = 2V_{in} = 2\times50\sqrt{2} = 100\sqrt{2}\ V$$

25. (d)



From the transfer curve when V_{GS} is made negative, the drain current increases as channel conductivity increases. Hence, it is a p-channel enhancement device.

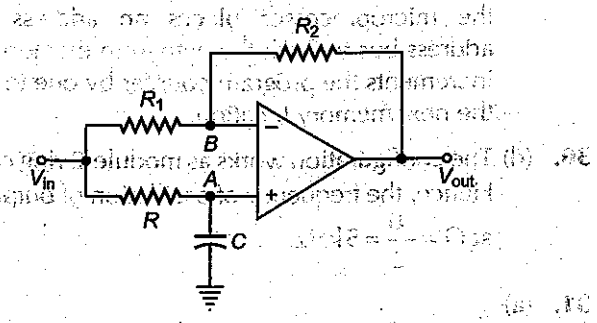
26. (b)



Applying KVL in base-emitter loop,

$$A - 33k I_B - V_{BE} - 3.3 I_E = 0$$
$$I_E = (\beta+1) I_B$$
$$4 - 33k I_B - 0.7 - 3.3k(99+1) I_B = 0$$
$$3.3 = (33k + 330k) I_B$$
$$I_B = \left(\frac{3.3}{33+330}\right) \text{mA}$$
$$I_C = \beta I_B = \frac{(99)(3.3)}{(33+330)} \text{mA}$$
$$I_C \simeq \left(\frac{3.3}{3.3+0.33}\right) \text{mA}$$

27. (d)



From voltage divider rule A,

$$V_A = \frac{V_{in} \times 1/sC}{R + 1/sC} = \frac{V_{in}}{1 + sRC} = \frac{V_{in}}{1 + j\omega RC}$$

From virtual ground concept,

$$V_A = V_B$$

Applying KCL at node B,

$$\frac{V_{in} - V_B}{R_1} = \frac{V_B - V_{out}}{R_1}$$

$$V_{out} = 2V_B - V_{in} = \frac{2V_{in}}{1 + j\omega RC} - V_{in}$$

$$= V_{in}\left[\frac{2 - 1 - j\omega RC}{1 + j\omega RC}\right]$$

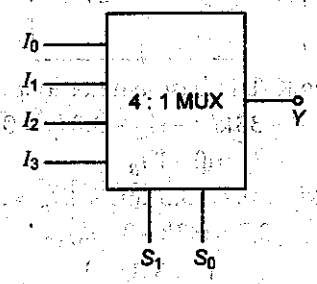
$$V_{out} = V_{in}\left[\frac{1 - j\omega RC}{1 + j\omega RC}\right]$$

$$\frac{V_{out}}{V_{in}} = \frac{1 - j\omega RC}{1 + j\omega RC}$$

$$\angle\frac{V_{out}}{V_{in}} = -2\tan^{-1}\omega RC$$

$$\angle\frac{V_{out}}{V_{in}}\bigg|_{max} = \pm 180°$$

28. (c)



$$Y = \bar{S}_1\bar{S}_0I_0 + \bar{S}_1S_0I_1 + S_1\bar{S}_0I_2 + S_1S_0I_3$$

For full adder circuit,

$$Y = A \oplus B \oplus C_{in}$$

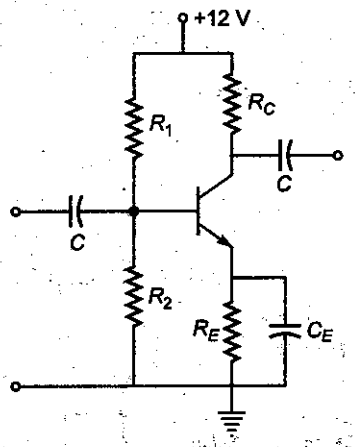
$$I_0 = C_{in}, I_1 = \bar{C}_{in}, I_2 = \bar{C}_{in}, I_3 = C_{in}$$

29. (d) The program counter points to the memory address from which the next byte is to be fetched and when the microprocessor places an address on the address bus to fetch the byte from memory, it then increments the program counter by one to point to the next memory location.

30. (d) The configuration works as module-2 ring counter. Hence, the frequency of oscillation of output signal at $Q = \frac{10}{2} = 5\,\text{kHz}$.

31. (a)

32. (b)



With C_E is in circuit, the mid-band gain

$$A_{V_1} = -\frac{R_C}{r_e}$$

With C_E removed, the mid-band gain

$$A_{V_2} = \frac{-R_C}{R_E}$$

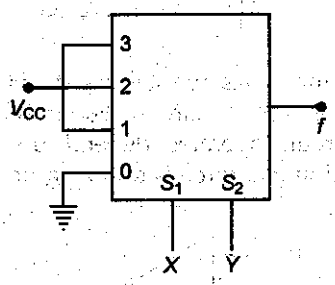
As $R_E >> r_e$

Hence, mid-band frequency gain decreases.

33. (c) The parallel comparator (flash) type is the fastest A/D converter.
In successive type for N-bit comparator, the clock pulse required is N. Hence, slower than parallel.
In counting type for N-bit, 2^N clock pulse required. Hence, slower than above two. The integrating type is slowest among all.

34. (d) The output of NOR or an Ex-NOR gate is 1 when all its inputs are at logic 0.

35. (b)



The truth table is

X	Y	f
0	0	0
0	1	1
1	0	1
1	1	1

$$f = \bar{X}Y + X\bar{Y} + XY$$

$$= \bar{X}Y + X(Y + \bar{Y})$$
$$= \bar{X}Y + X$$
$$f = X + Y$$

36. (c) Closed-loop gain $A_f = \dfrac{A}{1 + A\beta}$

Given, $A_f = 100$

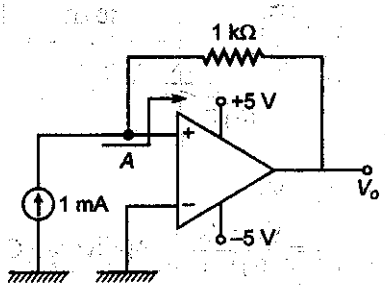
$A = 10^5$

$$1 + A\beta = \frac{10^5}{10^2} = 10^3$$

Upper cut-off frequency

$$f_{Hf} = f_H (1 + A\beta)$$
$$= 10 \times 10^3$$
$$= 10^4 = 10\ \text{kHz}$$

37. (d)



From virtual ground concept,

$$V_A = 0$$

Applying KCL at node A,

$$1\text{mA} = \frac{0 - V_o}{1\text{k}\Omega}$$

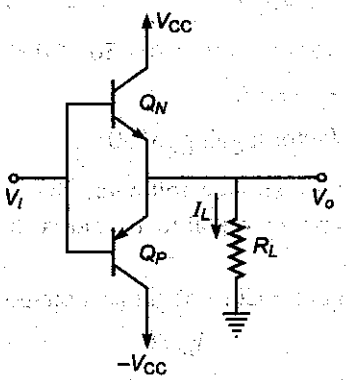
$$V_o = -1\text{V}$$

38. (d)

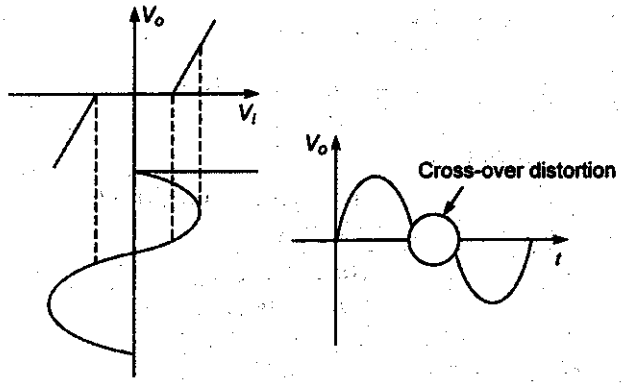
Interrupt	Vector
TRAP	0024 H
INTR	0000H → 038H
RST 7.5	003CH

RST3 is a not vectored interrupt.

39. (b) Class B power amplifier exhibits cross-over distortion.



Class B power amplifier



Transfer characteristics

In class B power amplifier there exists a range of V_i centred around zero where both transistors are cut-off and V_o is zero. This dead band results in the cross-over distortion.

40. (d) Feedback factor $= \dfrac{10}{10 + 90} = \dfrac{1}{10}$

41. (c)

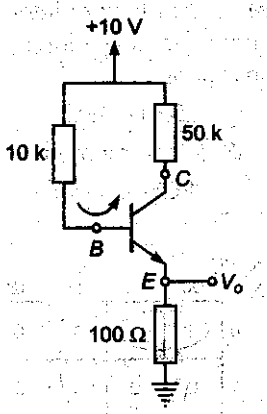
Input			Output	
$J_A = \bar{Q}_B$	$K_A = Q_B$	$T_B = Q_A$	Q_A	Q_B
0	0	0	1	0
1	0	1	1	1
0	1	1	0	0
1	0	0	1	0

42. (c) When $-0.7 < V_i < 5.7$, V_o will follow V_i, because Zener diode and diode D are off.
When $V_i \le -0.7$ V Zener diode is forward biased and $V_o = -0.7$ V
When $V_i \ge 5.7$ V, diode D is forward biased and $V_o = 5.7$ V.

43. (c)

44. (d) $I_C = \dfrac{12.0 - 0.2}{2.2 \times 10^3} = 5.36$ mA (saturation region).

45. (a)



$V_{BE} = 0.7$ V

Applying KVL,

$$10 = 10 I_B + 0.7 + 100 I_E \quad \ldots(i)$$

$$I_E = \beta I_B = 100 I_B$$

From Eq. (i)

$$10 = 10 I_B + 0.7 + 100 \times 100 I_B$$

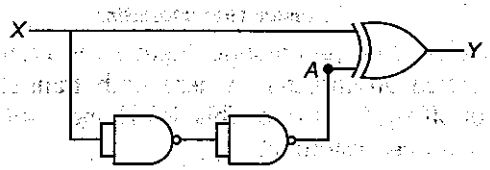
$$I_B = 0.465\,\text{mA}$$

$$V_o = 100 I_E = 100 \beta I_B$$

$$= 100 \times 0.465 \times 10^{-3} \times 100$$

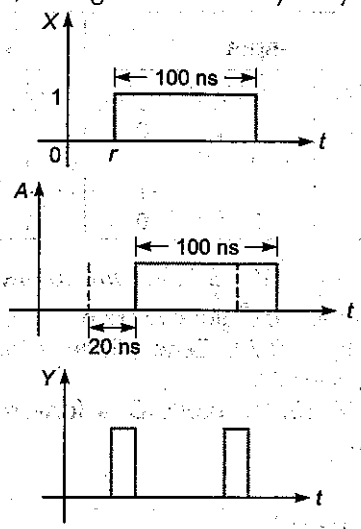
$$= 4.65\ \text{V}$$

46. (a)



The signal $Y = X\bar{A} + \bar{X}A$.

Given that propagation delay of each gate is 10 ns. Hence, the signal at A is delayed by 20 ns w.r.t. X.



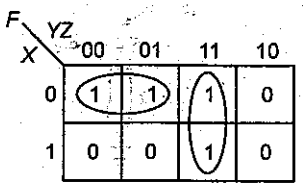
47. (d) When CALL Addr is being executed, this saves the content of program counter (the address of the next instruction) on the stack pointer. The stack pointer is incremented and next address is being fetched to the program counter.

((SP)) ← (PC)

(SP) incremented

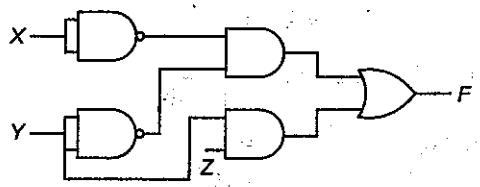
(PC) ← Addr.

48. (b)

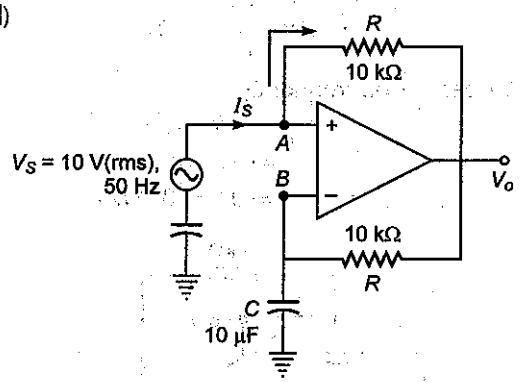


The minimised $F = \bar{X}\bar{Y} + YZ$

49. (d) The circuit will be



50. (d)



$$V_A = V_S$$

$$I_S = \frac{V_S - V_o}{R} \quad \text{(Applying KCL at node } A\text{)}$$

$$V_B = \frac{\frac{1}{j\omega C}}{R + 1/j\omega C} \times V_o$$

(From voltage divider rule)

$$V_B = \frac{1}{1 + j\omega RC} \times V_o$$

From virtual ground concept,

$$V_B = V_S$$

$$I_S = \frac{V_S - V_o}{R}$$

$$= \frac{V_S\,[1 - 1 - j\omega RC]}{R}$$

$$I_S = -j\omega C V_S$$

$$|I_S| = V_S\,\omega C = 10 \times 2\pi \times 50 \times 10 \times 10^{-6}$$

$$= 10\pi\,\text{mA}$$

Due to factor lagging by 90°.

51. (d) In case of emitter follower, the voltage gain is approximately equal to 1 whereas the current gain is high.

Power gain = Current gain × Voltage gain

Hence, $A_p >> 1$

$P_{out} > P_{in} \Rightarrow$ for emitter follower.

In case of transformer,

$$\frac{V_1}{V_2}=\frac{I_2}{I_1}=n$$

$$V_1I_1=V_2I_2$$

$$P_{in}=P_{out}$$

52. (d) XRA A

⇒ Ex-OR operation of content of accumulator with itself, *i.e.*, accumulator is clear. The content of A is 00H.

MVI B F0H

⇒ MOV immediate in register B with F0 the content of B is F0H.

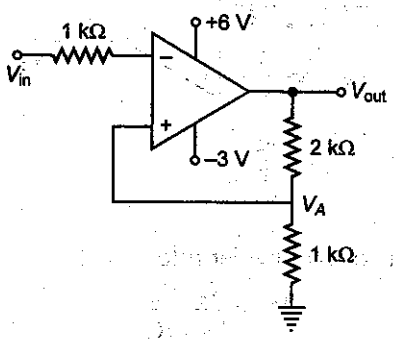
SUB B

⇒ Subtract B from A and result being stored in accumulator.

A ⇒ 00H
B ⇒ F0H
A − B ⇒ 10H

The content of accumulator after being executed is 10H.

53. (d)

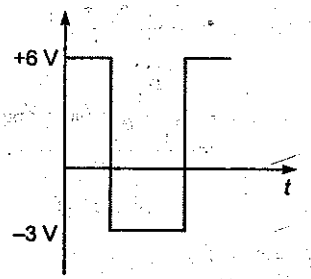


$$V_A=V_o\times\frac{1}{1+2}=\frac{1}{3}V_o$$

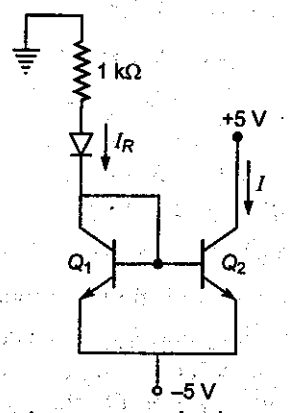
The circuit works as Schmitt trigger.

When $V_{in}<V_A \Rightarrow V_o=+V_{sat}=+6\text{ V}$

$V_{in}>V_A \Rightarrow V_o=-V_{sat}=-3\text{ V}$



54. (c)



As the transistors are perfectly matched, the circuit works as current mirror.

$$I=\frac{\beta}{\beta+2}I_R$$

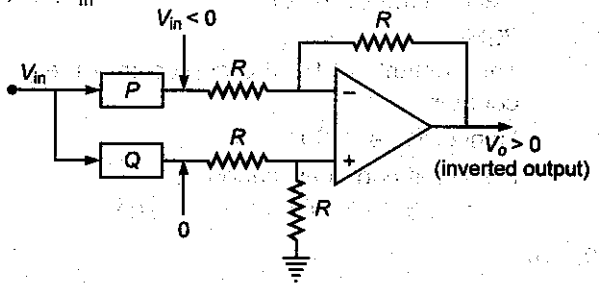
$\beta>>1$. Hence, $I\simeq I_R$

Applying KVL for transistor Q_1

$$0-1\times I_R-V_{BE}=-5$$

$$I_R=\frac{5-V_{BE}}{1}=\frac{5-0.7}{1}=4.3\text{ mA}$$

55. (b) Let $V_{in}>0$

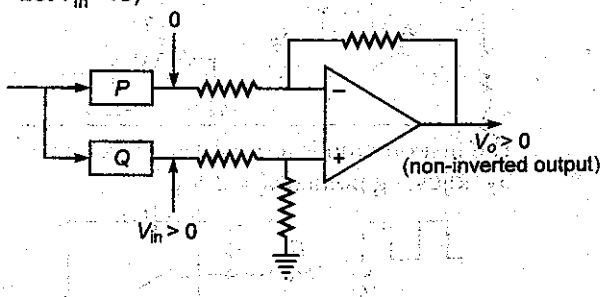


For block *P*,

$V_{in}>0; V_o=-\text{ve}$

$V_{in}<0; V_o=0$

Let $V_{in}<0$,



For block *Q*,

$V_{in}>0; V_o=0$

$V_{in}<0; V_o=+\text{ve}$

56. (c) Outputs of 3 × 8 decoder,

$$D_1 = \overline{X} + \overline{Y} + Z$$
$$D_3 = \overline{X} + Y + Z$$
$$D_5 = X + \overline{Y} + Z$$
$$D_6 = X + Y + \overline{Z}$$

$\therefore$ Output $F = D_1 \cdot D_3 \cdot D_5 \cdot D_6$

$= (\overline{X} + \overline{Y} + Z) \cdot (\overline{X} + Y + Z) \cdot (X + \overline{Y} + Z) \cdot (X + Y + \overline{Z})$

57. (b) The pulse width of the two monoshot outputs Q_1 and Q_2 are T_{ON_1} and T_{ON_2} respectively.

When triggered according to truth table given Q_1 responds to T_{ON_1} and Q_2 for T_{ON_2}, and waveform will be

$$f = \frac{1}{T_{ON_1} + T_{ON_2}}$$

$$\text{Duty cycle} = \left(\frac{T_{ON_2}}{T_{ON_1} + T_{ON_2}} \right)$$

58. (b) DAD SP

The 16-bit contents of the specified register pair are added to the content of the HL register and sum is saved in the HL register.

Hence, HL → 2700 H

PCHL

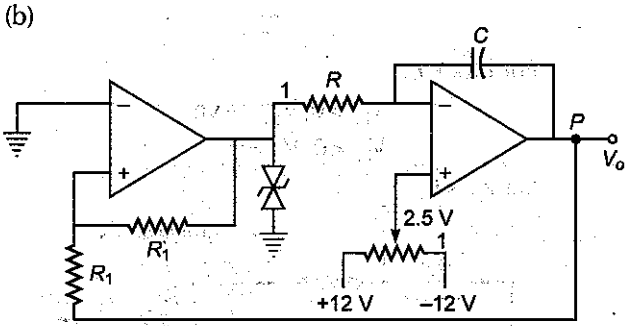
The contents of HL are copied into the program counter.

Hence, PC → 2700 H

Hence, at end of execution

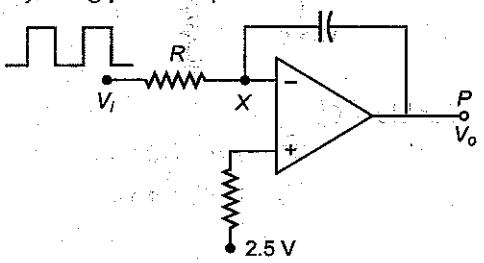
(PC) = 2700 H, (SP) = 2700 H

59. (b)



The first op-amp is a square wave generator.

By adjusting pointer $V_1 = 2.5$ V



By virtual ground concept, $V_x = 2.5$ V

Applying KCL at node X,

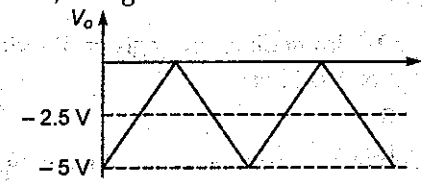
$$\frac{V_i - 2.5}{R} = \frac{d(2.5 - V_o)}{dt}$$

Integrating

$$\frac{1}{RC} \int (V_i - 2.5)\, dt = 2.5 - V_o$$

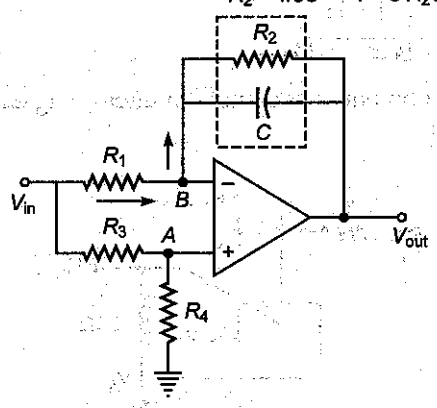
$$V_o = 2.5 - \frac{1}{RC} \int (V_i - 2.5)\, dt$$

Hence, triangular wave will be shifted downward.



60. (c)

$$Z = \frac{R_2 \times 1/sC}{R_2 + 1/sC} = \frac{R_2}{1 + s R_2 C}$$



From voltage divider rule,

$$V_A = \frac{R_4 \cdot V_{in}}{R_3 + R_4}$$

From virtual ground concept,

$$V_A = V_B$$

$$\frac{V_{in} - V_A}{R_1} = \frac{V_A - V_{out}}{Z}$$

$$= \frac{V_{in} - \frac{R_4}{R_3 + R_4} V_{in}}{R_1}$$

$$= \frac{\frac{R_4}{R_3 + R_4} V_{in} - V_{out}}{\frac{-R_2}{1 + SR_2C}}$$

Putting $R_1 = R_2 = R_A$

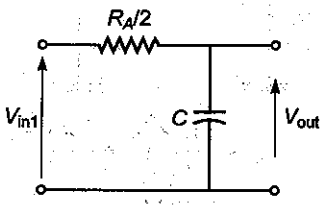
$R_3 = R_4 = R_B$

and simplifying,

$$V_{out} = \frac{1}{1+\frac{1}{sR_AC}} \times \left(\frac{V_{in}}{2}\right)$$

At low frequency V_{out} is zero and at high frequency it exists hence, it is high-pass filter.

61. (d)



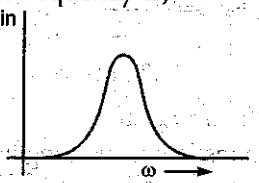
$$V_{out}(s) = \frac{1/sC\, V_{in}(s)}{\frac{R_A}{2}+\frac{1}{sC}} = \frac{2}{1+sR_AC} V_{in1}(s)$$

Putting the value of $V_{in1}(s)$,

$$V_{out}(s) = \frac{2sR_AC}{(1+sR_AC)(1+sR_AC)} \cdot \frac{V_{in}(s)}{2}$$

$$\text{Gain} = \frac{V_{out}(s)}{V_{in}(s)} = \frac{sR_AC}{(1+sR_AC)(1+sR_AC)}$$

Plotting with frequency ω,



62. (b) The circuit is a class B power amplifier.
For class B power amplifier,

$$\text{Efficiency} = \eta = \frac{\pi}{4} \times \frac{V_o}{V_{CC}}$$

where, V_o is peak amplitude.

$$V_o = 7\text{ V}$$

$$V_{CC} = 10, \eta = \frac{\pi}{4} \times \frac{7}{10} \times 100\% = 54.9\% \simeq 55\%$$

63. (d) By putting the individual option.

64. (b) As the Zener diode forward voltage is negligible. At a time one Zener diode is forward biased and other is reverse biased. The output voltage is +5 V and –5 V for other condition.

$$\text{Voltage at positive terminal} = \frac{5\times 100}{110}$$

From virtual ground concept,

$$V_+ = V_-$$

$$\frac{5\times 100}{110} = \text{Voltage across capacitor}$$

$$\frac{5\times 100}{110} = 20\,(1-e^{-t/RC})$$

$$1-e^{-t/RC} = \frac{5\times 100}{110\times 20} = \frac{5}{22}$$

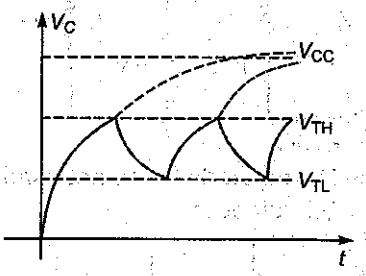
Putting the value, we get

$$R = 1\text{k}\Omega$$

$$C = 0.01\mu\text{F}$$

$$t = 2.57\,\mu\text{s}$$

65. (a) The waveform appears across capacitor.



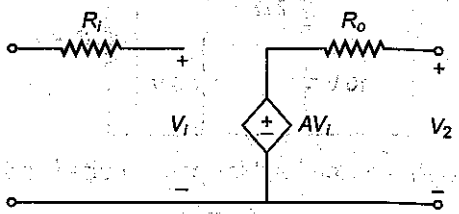
The capacitor starts charging from 0 voltage with time constant $20\,\text{k}\times C$ upto voltage V_{TH}. Then, discharge upto V_{TL} with time constant $10\,\text{k}\times C$. Hence, charging slope is less than discharging slope and voltage at CRO is as option (a).

66. (a) The INT pin is sampled after every machine cycle, but the BRQ is sampled after every instruction cycle.

67. (d) The 2's complement of 11010110 is 00101010 and then divided by 2, then answer is 11101011.

68. (a) For the division operation, the right sequence performed is L_2, R_2, R_1.

69. (a)



Given, $R_i = 1\text{M}\Omega$

$$R_o = 10\,\Omega$$

$$A = 10^6$$

$$V_i = 1\mu\text{V}$$

The output voltage

$$V_o = AV_i$$

$$= 10^6 \times 1\times 10^{-6} = 1\text{V}$$

The Z impedance matrix

$$V_1 = Z_{11}I_1 + Z_{12}I_2$$

$$V_2 = Z_{21}I_1 + Z_{22}I_2$$

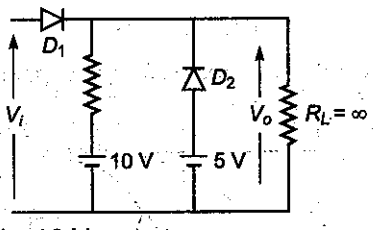
Input impedance

$$Z_{11} = \left.\frac{V_1}{I_1}\right|_{I_2=0} = \frac{V_1}{0} = \infty$$

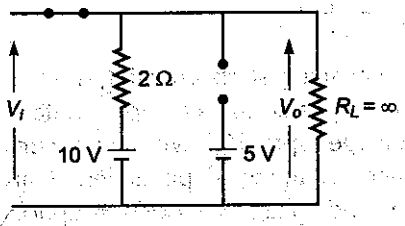
Output impedance

$$Z_{22} = \left.\frac{V_2}{I_2}\right|_{I_1=0} = R_o = 10\,\Omega$$

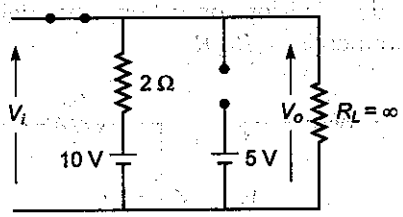
(a)



For V_i <10 V,
Diode D_1 and diode D_2 do not conduct.
Hence, circuit becomes



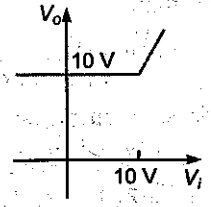
As $R_L = \infty$ the load is open circuited hence, no current flows and V_o =10 V
For V_i > 10 V,
The diode D_1 conducts whereas the diode D_2 does not. Hence,



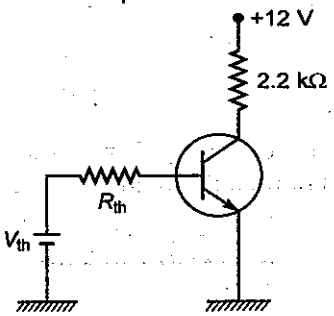
As $R_L = \infty$, the load is open circuited and

$$V_o = V_i$$

Hence,
For V_i <10 V ; V_o =10 V
For V_i >10 V ; $V_o = V_i$
Hence, transfer characteristics



71. (b) The Thevenin's equivalent circuit



where, $V_{th} = \frac{-12\times15}{115} + \frac{5\times100}{5}$

$= 2.78$ V

$$R_{th} = \frac{15\times100}{115} = 13\,\text{k}\Omega$$

$$I_b = \frac{V_{th} - V_{BE}}{R_{th}}$$

$$= \frac{2.78 - 0.7}{13\,\text{k}} = 0.16\text{ mA}$$

$$I_c(\text{active}) = \beta I_b = 0.16\times30 = 4.8\text{ mA}$$

$$I_c(\text{sat}) = \frac{12-0.2}{2.2}$$

$= 5.36$ mA

As I_c(active) > I_c(sat)
Hence, transistor is working in active region.

72. (a) When the output is +ve

$$V_P = \frac{10}{10+10}\times(+V_{sat})$$

$$= \frac{10}{20}\times(+12) = +6\text{ V}$$

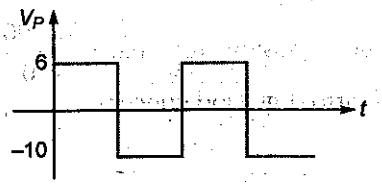
When the output is –ve,

$$V_P = \frac{10}{10+2}(-V_{sat})$$

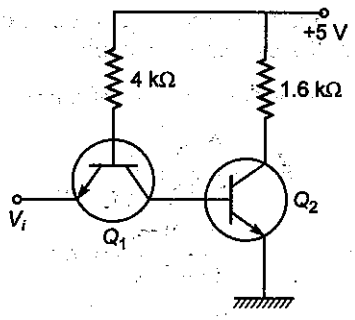
$$= \frac{10}{12}\times(-12)$$

$= -10$ V

Hence, waveform will be



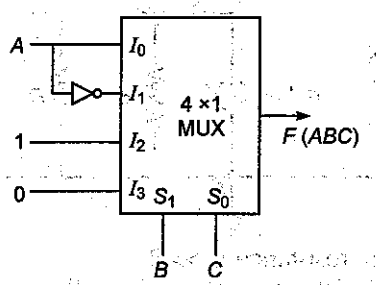
73. (c)



When $V_i = 3$ V, the emitter-base junction of Q_2 will be reverse biased and current will flow through 4 kΩ and through the forward biased base junction of collector of transistor Q_1 into the base of Q_2. In this mode of operator, the collector of transistor Q_1 operates as an emitter and the emitter as a collector. Thus, the transistor Q_1 is said to be operate in inverse ON mode. Transistor Q_2 will be driven into saturation and output V_o will be zero.

74. (c) In the given circuit, the connection from the counter to DAC is not proper.

75. (c)



For 4×1 MUX,

$$F = \bar{S}_1\bar{S}_0I_0 + \bar{S}_1S_0I_1 + S_1\bar{S}_0I_2 + S_1S_0I_3$$
$$= \bar{B}\bar{C}A + \bar{B}CA + B\bar{C}$$
$$= A\bar{B}\bar{C} + A\bar{B}C + B\bar{C}(A + \bar{A}) \quad [(A + \bar{A}) = 1]$$
$$= A\bar{B}\bar{C} + A\bar{B}C + AB\bar{C} + \bar{A}B\bar{C}$$

A	B	C	
0	0	0	...(0)
0	0	1	...(1)
0	1	0	...(2)
0	1	1	...(3)
1	0	0	...(4)
1	0	1	...(5)
1	1	0	...(6)
1	1	1	...(7)

$F = \Sigma(2, 4, 5, 6)$

76. (a) MVI L, 255 D
MVI H, 255 D
The content of H and L is 255.
Loop DCR L
JNZ Loop
L is decremented till the content of L become zero. Hence, number of time DCR L executed is 255.

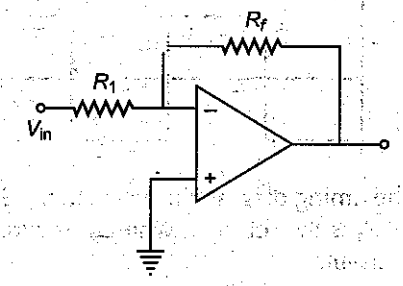
77. (a) To increment the content of memory locator whose address is available in (D,E) register. We have exchange the content of H with D and L with E. Then, we have to increment memory location. Because INR M will increase the content of HL pair.
XCHG
INR M

78. (b)

79. (b) Given $I_E = 1$mA

$$I_B = \frac{I_E}{\beta + 1} = \frac{1\text{mA}}{100 + 1} \simeq 10\,\mu\text{A}$$

80. (c)



For inverting amplifier,

$$\text{Gain} = \frac{-R_f}{R_1}$$
$$R_1 = \frac{-R_f}{\text{Gain}}$$

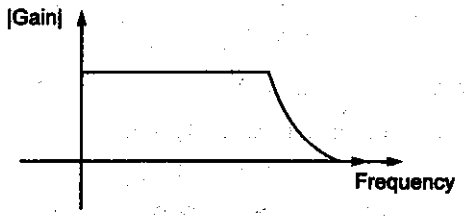
Taking the upper limit value for gain −25,

$$R_1 = \frac{-(-25)}{1\text{M}\Omega} = 40\,\text{k}\Omega$$

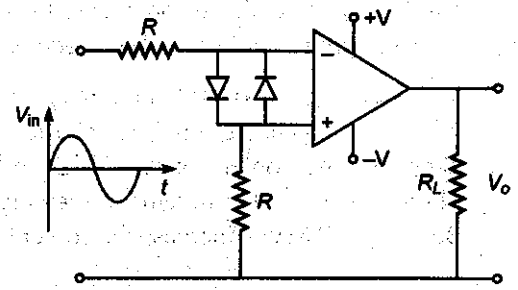
Taking the upper limit value for gain −10,

$$R_1 = \frac{-(-10)}{1\text{M}\Omega} = 100\,\text{k}\Omega$$

81. (b) In direct coupled amplifier, the output of one stage of the amplifier is connected to the input of the next stage directly without using reactive component like capacitor, inductor, transformer and others. As a result, the frequency response of this amplifier is quite flat all the way down to DC (0 Hz).



82. (c)



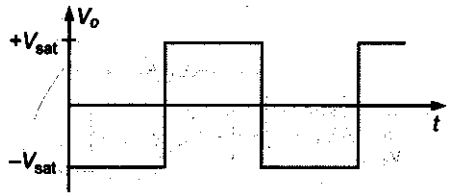
The comparator works as zero-cross detector. The diode works as clamp diode.

When V_{in} passes through zero in positive direction

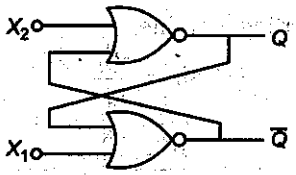
$$V_o = -V_{sat} = -V$$

When V_{in} passes through zero in negative direction

$$V_o = +V_{sat} = +V$$

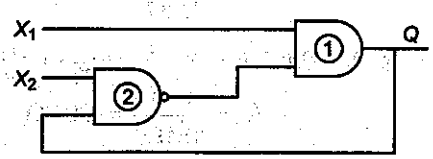


83. (a) By the timing diagram; it seems to the SR latch. The input X_1 is the set input whereas the input X_2 is the reset input.



84. (a) NOR gate can be obtained by using multiple MOS drivers in parallel whereas for NAND gate the MOS drivers are connected in series.

85. (a)



For $X_1 = 1, X_2 = 1$

Consider Q at 1,

Then, second input for Gate 1 is also 1.

and output of Gate 2 becomes zero. That contradict.

Consider Q at 0,

The second input for Gate 1 is also 0 and output of Gate 2 becomes zero. Again contradict.

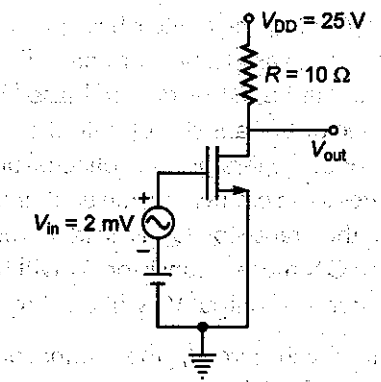
Hence, the system is unstable.

86. (b) The transconductance of MOSFET

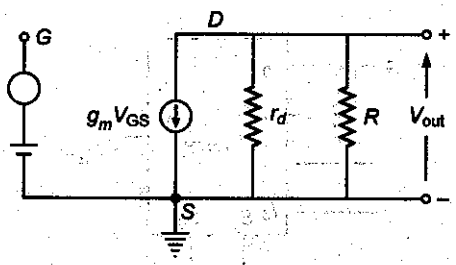
$$g_m = \left.\frac{\Delta I_{DS}}{\Delta V_{GS}}\right|_{V_{DS}\ =\ \text{constant}}$$

$$= \frac{(4-3)\,\text{mA}}{(4-3)\,\text{V}} = 1\,\text{ms}$$

87. (d)



The small circuit model



Drain resistance $r_d >> R$

Hence, the current $g_m V_{GS}$ will pass through R.

$$V_{out} = -g_m V_{GS} R$$

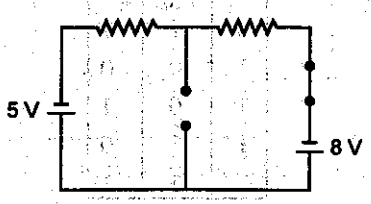
$$= -1\times10^{-3}\times2\times10^{-3}\times10\times10^{3}$$

$$= -20\,\text{mV}$$

$$\text{Voltage gain} = \frac{V_{out}}{V_{in}} = \frac{-20\,\text{mV}}{2\,\text{mV}}$$

$$= -10$$

88. (c) The diode D_1 is reverse biased and diode D_2 is forward biased. Hence, the circuit becomes

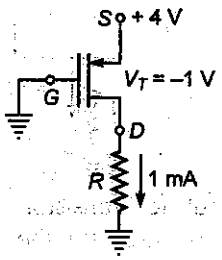


The current through diode D_1 is zero.

89. (c) The input resistance $R_{in} = \frac{h_{fe}}{g_m} = \frac{\beta}{g_m}$

$$= \frac{50}{10\text{ ms}} = 5\text{ k}\Omega$$

90. (d) For PMOS to be biased with linear region, the PMOS must operate in triode region.



For MOS to operate triode region,

$$(V_{GS} - V_T) > V_{DS}$$

$$\Rightarrow \quad V_{GS} = V_G - V_S = -4\text{ V}$$

$$V_{DS} = V_D - V_S = I_D R_D - (4\text{ V})$$

$$V_{DS} = I_D R_D - 4$$

$$\Rightarrow \quad [-4-(-1)] > I_D R_D - 4$$

$$\Rightarrow \quad -3 > I_D R_D - 4$$

$$I_D R_D > 1$$

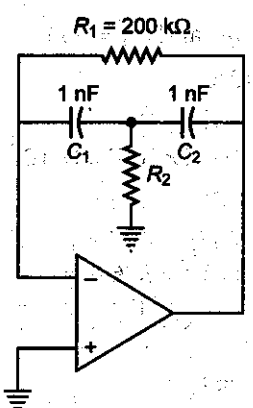
$$\Rightarrow \quad R_D > \frac{1}{I_D}$$

$$\Rightarrow \quad R_D > \frac{1}{1\text{mA}}$$

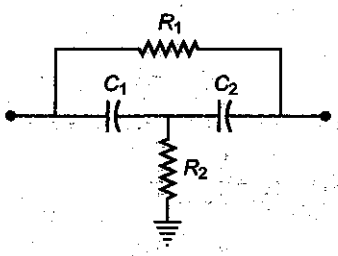
$$\Rightarrow \quad R_D > 1000\,\Omega$$

Hence, $R_D = 1200\,\Omega$

91. (a)



This is a single amplifier biquadratic active filter where, the feedback loop is



The transfer function of the feedback circuit is

$$T(s) = \frac{s^2 + s\left(\frac{1}{C_1} + \frac{1}{C_2}\right)\frac{1}{R_1} + \frac{1}{C_1C_2R_1R_2}}{s^2 + s\left(\frac{1}{R_1C_2} + \frac{1}{R_1C_1} + \frac{1}{C_2R_2}\right) + \frac{1}{C_1C_2R_1R_2}}$$

The polynomial of active filter is equal to numerator of $T(s)$.

$$s^2 + s\frac{\omega_0}{Q} + \omega_0^2 = s^2 + s\left(\frac{1}{C_1} + \frac{1}{C_2}\right)\frac{1}{R_1} + \frac{1}{C_1C_2R_1R_2}$$

$$\omega_0 = \sqrt{\frac{1}{C_1C_2R_1R_2}}$$

$$\frac{1}{Q} = \frac{\sqrt{C_1C_2R_1R_2}}{R_1}\left(\frac{1}{C_1} + \frac{1}{C_2}\right)$$

$$\frac{1}{Q} = \frac{1}{\omega_0 R_1}\left(\frac{1}{C_1} + \frac{1}{C_2}\right)$$

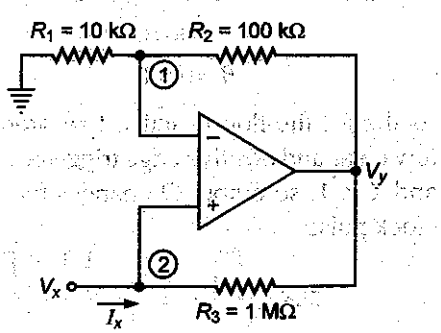
Putting the value,

$$1 = \frac{1}{\omega_0 \times (200\text{ k})}\left(\frac{1}{1\times10^{-9}} + \frac{1}{1\times10^{-9}}\right)$$

$$\omega_0 = \frac{1}{200\text{ k}}(2\times10^9)$$

$$= \frac{1\times10^9}{100\times10^3} = 1000\text{ rad/s}$$

92. (b)



From the concept of virtual ground, the potential of point 1 is V_x. Applying KCL at node 1,

$$\frac{0 - V_x}{10} = \frac{V_x - V_y}{100}$$

$$-10V_x = V_x - V_y$$
$$11V_x = V_y$$

As input current to op-amp is zero.

Hence, $I_x = \dfrac{V_x - V_y}{1\times10^6}$

$$I_x = \frac{V_x - 11V_x}{1\times10^6} = -\frac{10V_x}{1\times10^6}$$

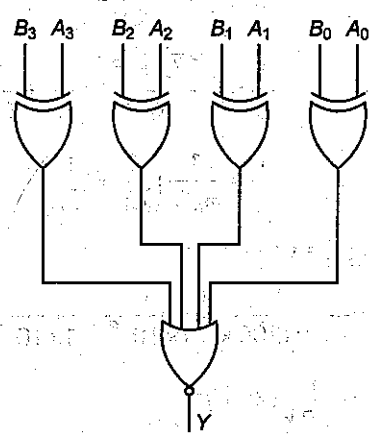
$$\frac{V_x}{I_x} = -10^5\,\Omega = -100\,\text{k}\Omega$$

$$R_{in} = \frac{V_x}{I_x} = -100\,\text{k}\Omega$$

93. (a) $Y = (\overline{A}\cdot BC + D)(\overline{A}\cdot D + \overline{B}\cdot\overline{C})$

$$= \overline{A}BCD + \overline{A}D + \overline{B}\,\overline{C}D$$
$$= \overline{A}D(BC+1) + \overline{B}\,\overline{C}D$$
$$= \overline{A}D + \overline{B}\,\overline{C}D$$

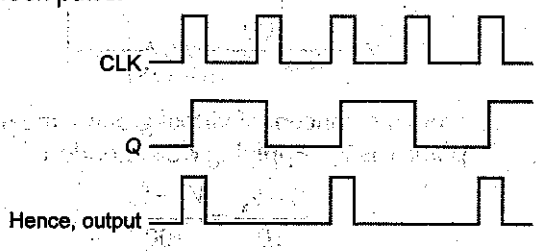
94. (d)



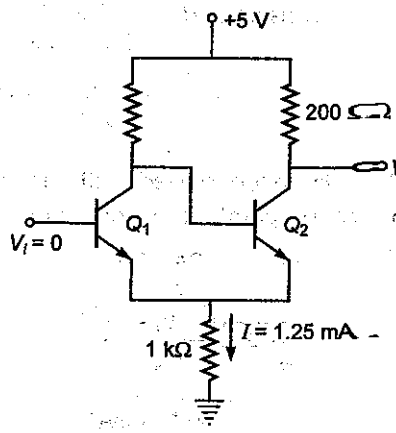
For Y to be zero, any one of the input to NOR required to be 1. For Ex-OR output to be 1, the input bit must be different. Hence, the correct option is

$$A = 0010$$
$$B = 1011$$

95. (b) As the J-K flip-flop is active low preset and active low clear and positive edge triggered. As the input J and K is 1, so output Q changes for positive edge clock pulse.



96. (b)



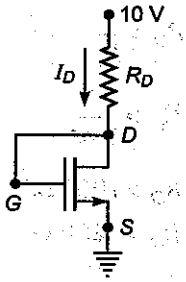
As the input to transistor Q_1 is zero. Hence transistor is in cut-off and the transistor Q_2 is driven by the collector current of transistor Q_1. The output of transistor Q_2

$$V_o = V_{CE\,sat} + I\times1\text{k}\Omega$$
$$= 0.1 + 1.25\times10^{-3}\times1\times10^3$$
$$V_o = 1.35\,\text{V}$$

97. (c)

START MVI A, 14 H	→	Require 1 cycle
SHIFT RLC	→	Require 6 cycle
JNZ SHIFT	→	Require 5 cycle
HALT	→	Require 1 cycle
		Total cycle = 13

98. (a)



For $I_D = 4\,\text{mA}$ and $R_D = 1\text{k}\Omega$

Applying KVL,

$$10 - I_D R_D - V_{GS} = 0$$
$$V_{GS} = 10 - 4\times10^{-3}\times1\times10^3 = 6 \quad \text{V}$$

We know that

$$I_D = K(V_{GS} - V_T)^2$$
$$4\times10^{-3} = K(6-2)^2$$
$$\Rightarrow \quad K = \frac{1}{4}\times10^3$$

For $R_D = 4\,\text{k}\Omega$,

$$V_{GS} = 10 - 4I_D$$
$$I_D = K(V_{GS} - V_T)^2$$

$$\Rightarrow \quad I_D = \frac{1}{4}(10 - 4I_D - 2)^2$$

$$\Rightarrow \quad 4I_D = (8 - 4I_D)^2$$

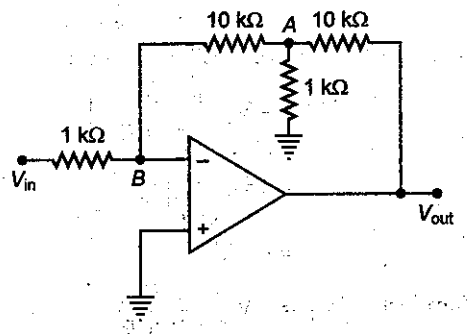
$$\Rightarrow \quad 4I_D = 64 + 16I_D^2 - 64I_D$$

$$\Rightarrow \quad 16I_D^2 - 68I_D + 64 = 0$$

$$\Rightarrow \quad 4I_D^2 - 17I_D + 16 = 0$$

$$I_D = 2.8\text{ mA}$$

99. (d)



From virtual ground concept, $V_B = 0$

Applying KCL at node B,

$$\frac{V_{in} - 0}{1k} = \frac{0 - V_A}{10k}$$

$$V_A = -10\,V_{in}$$

Applying KCL at node A,

$$\frac{0 - V_A}{10k} = \frac{V_A}{1k} + \frac{V_A - V_{out}}{10k}$$

$$-\frac{V_A}{10} = V_A + \frac{V_A}{10} - \frac{V_{out}}{10}$$

$$\Rightarrow \quad \frac{V_{out}}{10} = V_A + \frac{V_A}{10} + \frac{V_A}{10}$$

Putting $V_A = -10\,V_{in}$,

$$\frac{V_{out}}{10} = -10\,V_{in} - V_{in} - V_{in}$$

$$\frac{V_{out}}{V_{in}} = -120$$

100. (a)

101. (c) For astable multi-vibrator,

$$f = \frac{1}{T} = \frac{1}{0.69\,C(R_A + 2R_B)}$$

Given, $f = 10\text{ kHz}$ and $C = 10\text{ nF}$

$$10 \times 10^3 = \frac{1}{0.69 \times 10 \times 10^{-9} \times (R_A + 2R_B)}$$

$$R_A + 2R_B = \frac{1}{0.69 \times 10 \times 10^{-9} \times 10 \times 10^3}$$

$$R_A + 2R_B = \frac{1}{0.69 \times 10^{-4}} = 14.5\text{k}\Omega \qquad \ldots(i)$$

$$\text{Duty cycle} = \frac{R_A + R_B}{R_A + 2R_B} = 0.75 \qquad \text{(Given)}$$

$$\Rightarrow \quad \frac{R_A + R_B}{R_A + 2R_B} = \frac{3}{4}$$

$$\Rightarrow \quad 4R_A + 4R_B = 3R_A + 6R_B$$

$$\Rightarrow \quad R_A - 2R_B = 0 \qquad \ldots(ii)$$

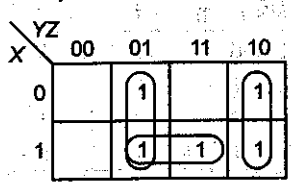
Hence, $R_A = \frac{14.5}{2} = 7.25\,\text{k}\Omega$; $R_B = 3.62\text{ k}\Omega$

102. (b) For dual slope A/D converter,

$$f_{max} = \frac{f_{clock}}{2^N} = \frac{1 \times 10^6}{2^{10}} = 976\text{ Hz} \simeq 1\text{kHz}$$

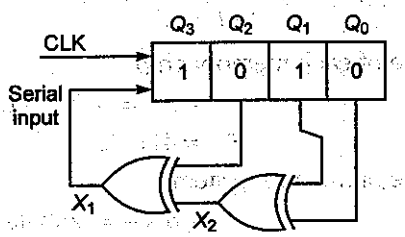
103. (b) $f = \bar{X}Y\bar{Z} + \bar{X}YZ + XY\bar{Z} + X\bar{Y}Z + XYZ$

Making K-map,



Simplified $f = \bar{Y}Z + XZ + Y\bar{Z}$

104. (b)



We can write

$$X_2 = Q_1 \oplus Q_0$$

$$X_1 = Q_3 \oplus X_2$$

$$Q_3 = X_1$$

The sequence

	Q_3	Q_2	Q_1	Q_0	X_1	X_2
1	1	0	1	0	1	1
2	1	1	0	1	0	1
3	0	1	1	0	0	1
4	0	0	1	1	0	0
5	0	0	0	1	1	1
6	0	0	0	0	0	0
7	0	1	0	0	1	0
	1	0	1	0		

Hence, after seven clock pulses 1010 repeats itself.

105. (d) For *X-Y* flip-flop,

X	Y	Q_{n+1}
0	0	1
0	1	Q_n
1	0	$\overline{Q}_n$
1	1	0

For *J-K* flip-flop,

J	K	Q_{n+1}
0	0	Q_n
0	1	0
1	0	1
1	1	$\overline{Q}_n$

and excitation table

$Q(t)$	$Q(t+1)$	J	K	X	Y
0	0	0	X	X	1
0	1	1	X	X	0
1	0	X	1	1	X
1	1	X	0	0	X

Hence, *X-Y* to work *J-K* F/F,

$$J = \overline{Y}$$

$$K = X$$

106. (b) Size of each memory chip

$$= 2^{12} = 4096 \text{ byte}$$

$$= 4 \text{ kbyte}$$

Size of memory system

$$= 8 \times 4 = 32 \text{ kbyte}$$

107. (d) LXIH H, 1FFE

Load 1F in H and FE in L.

MOV B, M

Move content of memory location in register B.

INR L

Increment L, *i.e.* FF

MOV A, M

Move content of memory location in accumulator

ADD B (A → A + B)

INR L

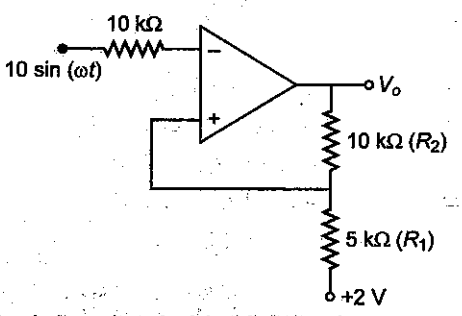
Increment L, *i.e.*, HL → 2000

MOV M, A

XOR A 2000 H

108. (a)

109. (c)



The lower threshold voltage; where

$$V_{TL} = -V_{sat}\left(\frac{R_1}{R_1 + R_2}\right)$$

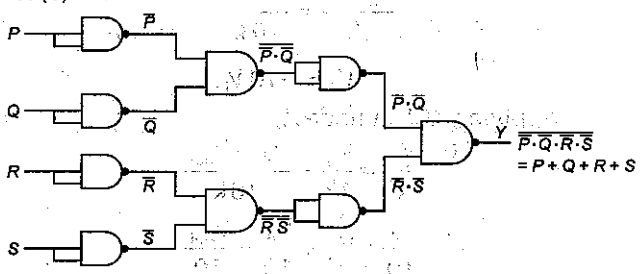
$$= -15\left(\frac{5}{15}\right)$$

$$= -5 \text{ V}$$

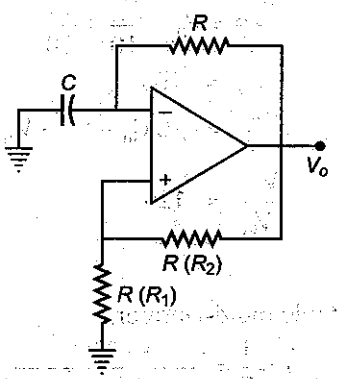
Similarly, $V_{UT} = +V_{sat}\left(\frac{R_1}{R_1 + R_2}\right)$

$$= +5 \text{ V}$$

110. (b)



111. (b)



This is a free running multi-vibrator.

Time period of oscillation $= 2RC \ln\left(\frac{2R_1 + R_2}{R_2}\right)$

$$= 2RC \ln\left(\frac{2R + R}{R}\right)$$

$$= 2RC \ln 3$$

$$\tau = RC \quad \text{(Given)}$$

$$T = 2\tau \ln 3$$

112. (c) MVI A, 10 H; content of A is 10 H.

MVI B, 10 H; content of B is 10 H.

BACK : NOP

ADD B ; content of A is 20 H.

RLC ; content of A × 2 is 40 H.

0	1	0	0	0	0	0	0

JNC BACK *i.e.*, $CY = 0$

NOP

ADD B; content of A is 50 H.

113. (d) The leakage current

$$I = \frac{Q'}{t}$$

$$Q' = \frac{0.5}{100} \times Q = \frac{0.5 \times CV}{100}$$

$$I = \frac{0.5 \times 0.1 \times 10^{-9} \times 5}{100 \times 1 \times 10^{-6}} = 2.5\,\mu A$$

114. (a) The slew rate

$$SR = 2\pi f V_P$$

$$f_{min} = \frac{SR}{2\pi V_{P_1 max}} = \frac{62.8 \times 10^6}{2 \times 3.14 \times 10} = 1MHz$$

115. (c) We know that

$$g = g_{mo}\left(1 - \frac{V_{GS}}{V_P}\right)$$

where, g_{mo} is the maximum transconductance.

$$g_{mo} = \frac{g_m}{\left(1 - \frac{V_{GS}}{V_P}\right)}$$

$$= \frac{1}{1 - \left(\frac{-3}{-5}\right)} = 2.5$$

116. (d) Taking both don't care as 0,

C \ AB	00	01	11	10
0	1	1	ϕ_0	0
1	0	0	ϕ_0	0

Minimal product of sum = AC

117. (b) Given, $I_D = I_S \frac{V}{V_T}$

Taking differentiation,

$$\frac{\partial V}{\partial I_D} = \frac{V_T}{I_S} = \frac{25 \times 10^{-3}}{2 \times 10^{-3}} = 12.5\,\Omega$$

118. (d) In dual slope A/D converter, the counting value

$$n = n_{ref}\left(\frac{V_a}{V_{ref}}\right)$$

It counts upto fixed count n_{ref}.

$$n_{ref} = n\frac{V_{ref}}{V_a} \Rightarrow n_{ref} \propto \frac{1}{V_a}$$

9

Power Electronics

Syllabus

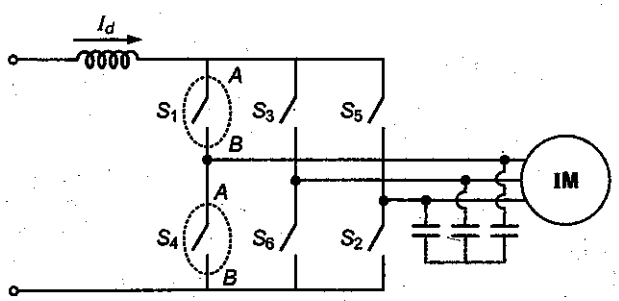
Power Electronics and Drives *Semiconductor power diodes, transistors, thyristors, TRIACs, GTOs, MOSFETs and IGBTs - static characteristics and principles of operation; triggering circuits; phase control rectifiers; bridge converters - fully controlled and half controlled; principles of choppers and inverters; basic concepts of adjustable speed, DC and AC drives.*

GATE 2011 *(1 Mark Questions)*

▼ *Solution Points*

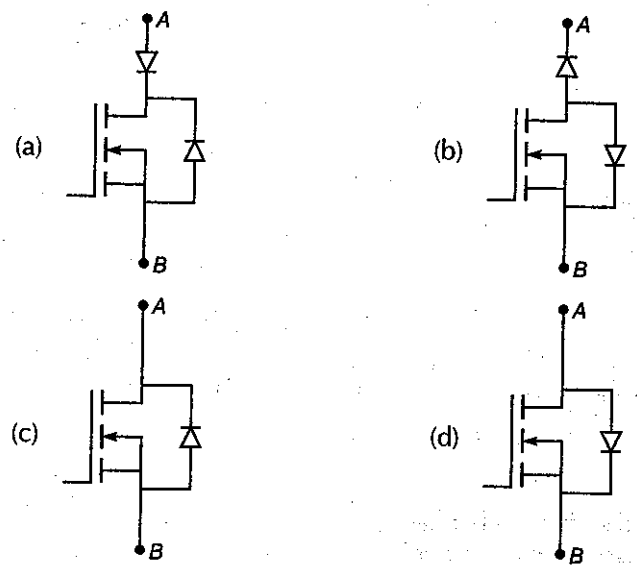
1. Circuit turn-off time of an SCR is defined as the time
(a) taken by the SCR to turn-off
(b) required for the SCR current to become zero
(c) for which the SCR is reverse biased by the commutation circuit
(d) for which the SCR is reverse biased to reduce its current below the holding current

2. A three-phase current source inverter used for the speed control of an induction motor is to be realized using MOSFET switches as shown below. Switches S_1 to S_6 are identical switches.



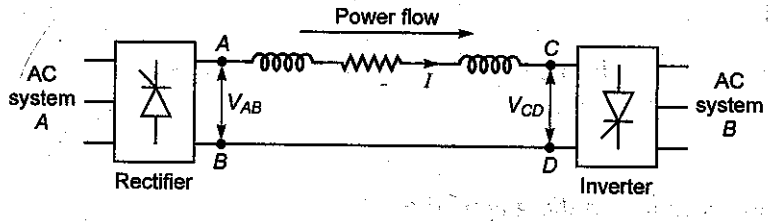
The proper configuration for realizing switches S_1 to S_6 is

▼ *Solution Points*



GATE 2010

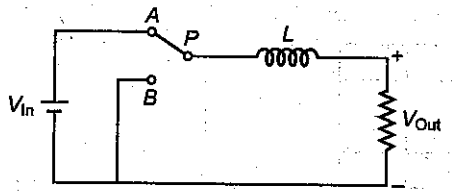
3. Power is transferred from system *A* to system *B* by an HVDC link as shown in the figure. If the voltages V_{AB} and V_{CD} are as indicated in the figure, and $I > 0$, then



(a) $V_{AB} < 0, V_{CD} < 0, V_{AB} > V_{CD}$ (b) $V_{AB} > 0, V_{CD} > 0, V_{AB} > V_{CD}$

(c) $V_{AB} > 0, V_{CD} > 0, V_{AB} < V_{CD}$ (d) $V_{AB} > 0, V_{CD} < 0$

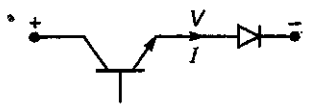
4. The power electronic converter shown in the figure has a single-pole double-throw switch. The pole *P* of the switch is connected alternately to throws *A* and *B*. The converter shown is a

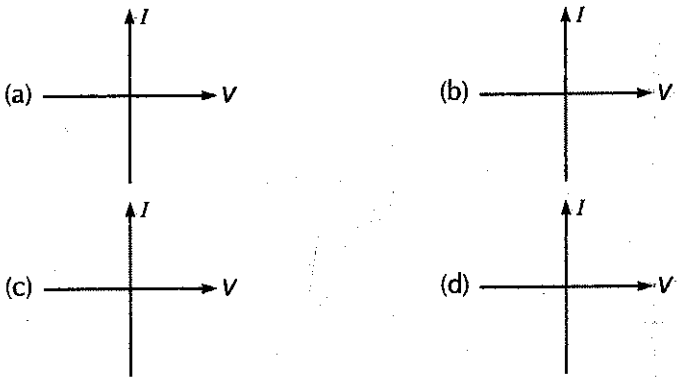


(a) step-down chopper (buck converter)
(b) half-wave rectifier
(c) step-up chopper (boost converter)
(d) full-wave rectifier

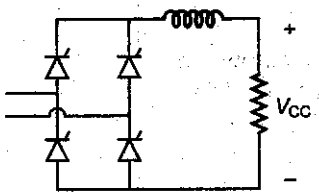
5. Figure shows a composite switch consisting of a power transistor (BJT) in series with a diode. Assuming that the transistor switch and the diode are ideal, the *I-V* characteristics of the composite switch is

▼ *Solution Points*





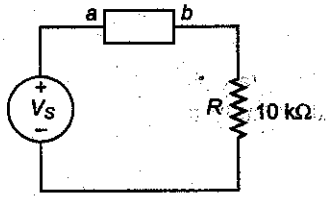
6. The fully controlled thyristor converter in the figure is fed from a single-phase source. When the firing angle is 0°, the DC output voltage of the converter is 300 V. What will be the output voltage for a firing angle of 60°, assuming continuous conduction?

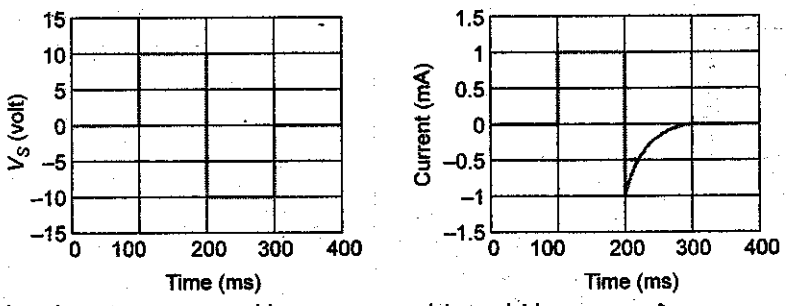


(a) 150 V (b) 210 V (c) 300 V (d) 100π V

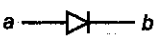
GATE 2009

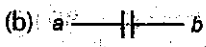
7. The following circuit has a source voltage V_S as shown in the graph. The current through the circuit is also shown.





The element connected between *a* and *b* could be

(a) 

(b) 

(c)

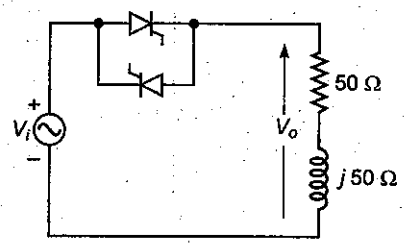
(d)

8. An SCR is considered to be a semi-controlled device because
   (a) it can be turned off but not on with a gate pulse
   (b) it conducts only during one half-cycle of an alternating current wave
   (c) it can be turned on but not off with a gate pulse
   (d) it can be turned on only during one half-cycle of an alternating voltage wave

▼ *Solution Points*

GATE 2008

9. In the single phase voltage controller circuit shown in the figure, for what range of triggering angle (α), the output voltage (V_o) is not controllable?



(a) $0° < \alpha < 45°$ (b) $45° < \alpha < 135°$ (c) $90° < \alpha < 180°$ (d) $135° < \alpha < 180°$

10. A 3-phase voltage source inverter is operated in 180° conduction mode. Which one of the following statements is true?
   (a) Both pole-voltage and line-voltage will have 3rd harmonic components
   (b) Pole-voltage will have 3rd harmonic component but line-voltage will be free from 3rd harmonic
   (c) Line-voltage will have 3rd harmonic component but pole-voltage will be free from 3rd harmonic
   (d) Both pole-voltage and line-voltage will be free from 3rd harmonic components

GATE 2007

11. $x = [x_1\ x_2 \ldots x_n]^T$ is an n-tuple non-zero vector. The $n \times n$ matrix $V = xx^T$
   (a) has rank zero (b) has rank 1 (c) is orthogonal (d) has rank n

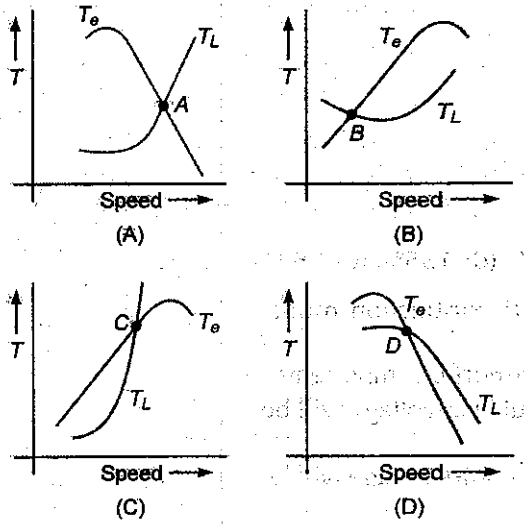
12. A single-phase fully controlled thyristor bridge AC-DC converter is operating at a firing angle of 25° and an overlap angle 10° with constant DC output current of 20 A. The fundamental power factor (displacement factor) at input AC mains is
   (a) 0.78 (b) 0.827 (c) 0.866 (d) 0.9

13. A three-phase fully controlled thyristor bridge converter is used as line commutated inverter to feed 50 kW power at 420 V DC to a three-phase, 415 V (line), 50 Hz AC mains. Consider DC link current to be constant. The rms current of the thyristor is
   (a) 119.05 A (b) 79.37 A (c) 68.73 A (d) 39.68 A

14. Single phase full-wave half-controlled bridge converter feeds an inductive load. The two SCRs in the converter are connected to a common DC bus. The converter has to have a free-wheeling diode

(a) because the converter inherently does not provide for free-wheeling
(b) because the converter does not provide for free-wheeling for high values of triggering angles
(c) or else the free-wheeling action of the converter will cause shorting of the AC supply
(d) or else if a gate pulse to one of the SCRs is missed, it will subsequently cause a high load current in the other SCR

15. The electromagnetic torque T_e of a drive and its connected load torque T_L are as shown below. Out of the operating points A, B, C and D, the stable ones are



(a) A, C, D
(b) B, C
(c) A, D
(d) B, C, D

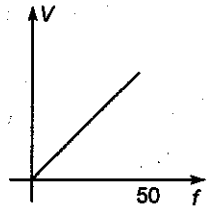
16. "Six MOSFETs connected in a bridge configuration (having no other power device) must be operated as a Voltage Source Inverter (VSI)". This statement is

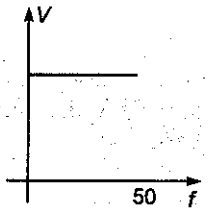
(a) true, because being majority carrier devices, MOSFETs are voltage driven
(b) true, because MOSFETs have inherently anti-parallel diodes
(c) false, because it can be operated both as Current Source Inverter (CSI) or VSI
(d) false, because MOSFETs can be operated as excellent constant current sources in the saturation region

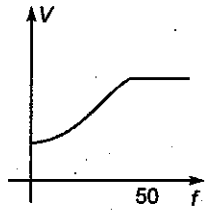
GATE 2006

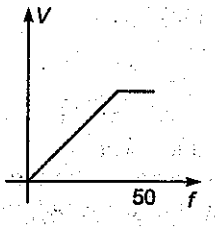
17. The speed of a 3-phase, 440 V, 50 Hz induction motor is to be controlled over a wide range from zero speed to 1.5 times the rated speed using a 3-phase voltage source inverter. It is desired to keep the flux in the machine constant in the constant torque region by controlling the terminal voltage as the frequency changes. The inverter output voltage *vs* frequency characteristic should be

▼ *Solution Points*

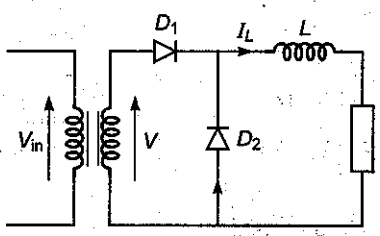
(a)


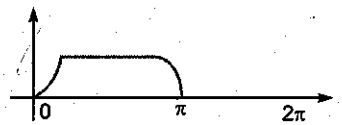
(b)


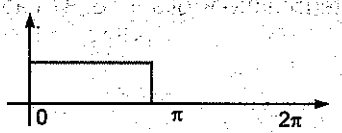
(c)


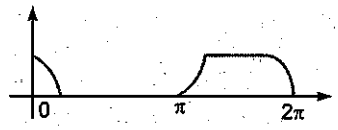
(d)


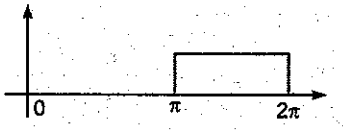
18. A single-phase half wave uncontrolled converter circuit is shown in figure. A 2-winding transformer is used at the input for isolation. Assuming the load current to be constant and $V = V_m \sin \omega t$, the current waveform through diode D_2 will be



(a)


(b)


(c)


(d)


GATE 2005

19. The conduction loss *versus* device current characteristic of a power MOSFET is best approximated by
(a) a parabola
(b) a straight line
(c) a rectangular hyperbola
(d) an exponentially decaying function

20. A three-phase diode bridge rectifier is fed from a 400 V RMS, 50 Hz, three-phase AC source. If the load is purely resistive, then peak instantaneous output voltage is equal to

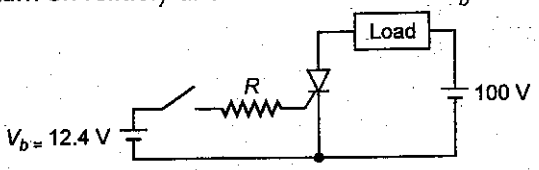
(a) 400 V (b) $400\sqrt{2}$ V (c) $400\sqrt{\frac{2}{3}}$ V (d) $\frac{400}{\sqrt{3}}$ V

21. The output voltage waveform of a three-phase square-wave invertor contains
(a) only even harmonics (b) both odd and even harmonics
(c) only odd harmonics (d) only nipple harmonics

▼ *Solution Points*

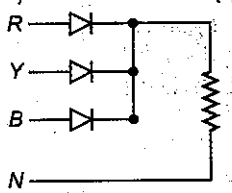
GATE 2004

22. The triggering circuit of a thyristor is shown in figure. The thyristor requires a gate current of 10 mA, for guaranteed turn-on. The value of R required for the thyristor to turn on reliably under all conditions of V_b variation is



(a) 10000 Ω (b) 1600 Ω (c) 1200 Ω (d) 800 Ω

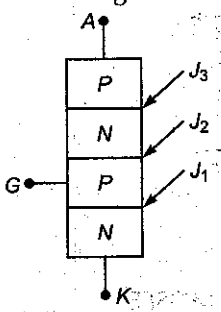
23. The circuit in figure shows a 3-phase half-wave rectifier. The source is a symmetrical, 3-phase four wire system. The line-to-line voltage of the source is 100 V. The supply frequency is 400 Hz. The ripple frequency at the output is



(a) 400 Hz (b) 800 Hz (c) 1200 Hz (d) 2400 Hz

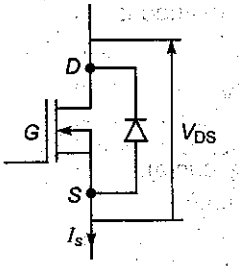
GATE 2003

24. Figure shows a thyristor with the standard terminations of anode (A), cathode (K), gate (G) and the different junctions named J_1, J_2 and J_3. When the thyristor is turned-on and conducting



(a) J_1 and J_2 are forward biased and J_3 is reverse biased
(b) J_1 and J_2 are forward biased and J_2 is reverse biased
(c) J_1 is forward biased and J_2 and J_3 is reverse biased
(d) J_1, J_2 and J_3 are all forward biased

25. Figure shows a MOSFET with an integral body diode. It is employed as a power switching device in the on and off states through appropriate control. The on and off states of the switch are given on the V_{DS}, I_S plane by



▼ *Solution Points*

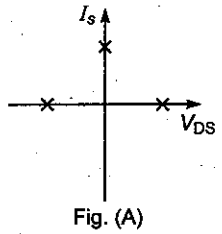


Fig. (A)

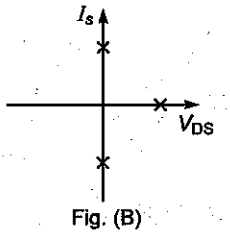


Fig. (B)

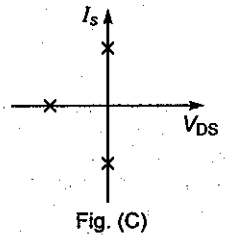


Fig. (C)

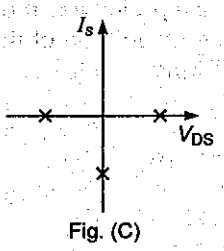


Fig. (C)

(a) Fig. A (b) Fig. B (c) Fig. C (d) Fig. D

26. The speed/torque regimes in a DC motor and the control methods suitable for the same are given respectively in Group II and Group I.

Group I	Group II
P. Field control	1. Below base speed
Q. Armature control	2. Above base speed
	3. Above base torque
	4. Below base torque

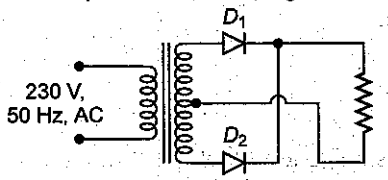
The match between the control method and the speed/torque regime is as follows

(a) P-1; Q-3 (b) P-2; Q-4 (c) P-2; Q-3 (d) P-1; Q-4

27. A Bipolar Junction Transistor (BJT) is used as a power control switch by biasing it in line cut-off region (off state) or in the saturation region (on state). In the on state, for the BJT

(a) both the base-emitter and base-collector junctions are reverse biased
(b) the base-emitter junction is reverse biased and the base-collector junction is forward biased
(c) the base-emitter junction is forward biased and the base-collector junction is reverse biased
(d) both the base-emitter and base-collector junctions are forward biased

28. The circuit in figure shows a full-wave rectifier. The input voltage is 230 V (rms) single-phase AC. The peak reverse voltage across the diodes D_1 and D_2 is



(a) $100\sqrt{2}$ V (b) 100 V (c) $50\sqrt{2}$ V (d) 50 V

29. A fully controlled natural commutated 3-phase bridge rectifier is operating with a firing angle $\alpha = 30°$. The peak to peak voltage ripple expressed as a ratio of the peak output DC voltage at the output of the converter bridge is

(a) 0.5 (b) $\frac{\sqrt{3}}{2}$ (c) $\left(1-\frac{\sqrt{3}}{2}\right)$ (d) $\sqrt{3}-1$

▼ *Solution Points*

GATE 2002

30. A six pulse thyristor rectifier bridge is connected to a balanced 50 Hz three phase AC source. Assuming that the DC output current of the rectifier is constant, the lowest frequency harmonic component in the AC source line current is

(a) 100 Hz (b) 150 Hz (c) 250 Hz (d) 300 Hz

31. A step down chopper is operated in the continuous conduction mode in steady state with a constant duty ratio D. If V_o is the magnitude of the DC output voltage and if V_i is the magnitude of the DC input voltage, the ratio V_o/V_i is given by

(a) D (b) $1-D$ (c) $\frac{1}{1-D}$ (d) $\frac{D}{1-D}$

GATE 2001

32. The main reason for connecting a pulse transformer at the output stage of a thyristor triggering circuit is to

(a) amplify the power of the triggering pulse
(b) provide electrical isolation
(c) reduce the turn on time of the thyristor
(d) avoid spurious triggering of the thyristor due to noise

33. AC to DC circulating current dual converters are operated with the following relationship between their triggering angles (α_1 and α_2)

(a) $\alpha_1 + \alpha_2 = 180°$ (b) $\alpha_1 + \alpha_2 = 360°$
(c) $\alpha_1 - \alpha_2 = 180°$ (d) $\alpha_1 + \alpha_2 = 90°$

34. In case of an armature controlled separately excited DC motor drive with closed-loop speed control, an inner current loop is useful because it

(a) limits the speed of the motor to a safe value
(b) helps in improving the drive energy efficiency
(c) limits the peak current of the motor to the permissible value
(d) reduces the steady state speed error

GATE 2000

35. A three phase semiconverter feeds the armature of a separately excited DC motor, supplying a non-zero torque. For steady state operation, the motor armature current is found to drop to zero at certain instances of time. At such instances, the voltage assumes a value that is

(a) equal to the instantaneous value of the AC phase voltage
(b) equal to the instantaneous value of the motor back emf
(c) arbitrary
(d) zero

36. A thyristorised, three phase, fully controlled converter feeds a DC load that draws a constant current. Then the input AC line current to the converter has

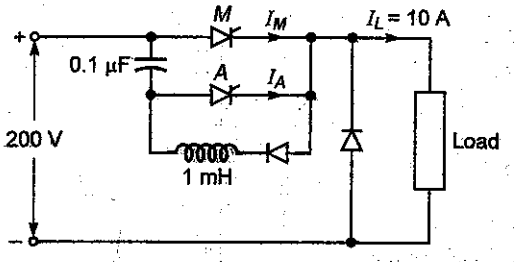
(a) an rms value equal to the DC load current
(b) an average value equal to the DC load current
(c) a peak value equal to the DC load current
(d) a fundamental frequency component, whose rms value is equal to the DC load current

GATE 2011

(2 Marks Questions)

▼ *Solution Points*

37. A voltage commutated chopper circuit, operated at 500 Hz, is shown below.



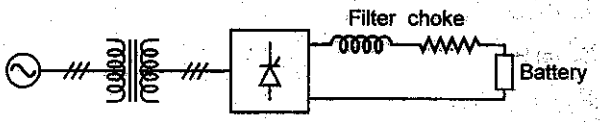
If the maximum value of load current is 10 A, then the maximum current through the main (M) and auxiliary (A) thyristors will be

(a) $I_{M\max}$ = 12 A and $I_{A\max}$ = 10 A (b) $I_{M\max}$ = 12 A and $I_{A\max}$ = 2 A

(c) $I_{M\max}$ = 10 A and $I_{A\max}$ = 12 A (d) $I_{M\max}$ = 10 A and $I_{A\max}$ = 8 A

Statements for Linked Answer Questions 38 and 39

A solar energy installation utilizes a three-phase bridge converter to feed energy into power system through a transformer of 400 V/400 V, as shown below.



The energy is collected in a bank of 400 V battery and is connected to converter through a large filter choke of resistance 10 Ω.

38. The maximum current through the battery will be

(a) 14 A (b) 40 A (c) 80 A (d) 94 A

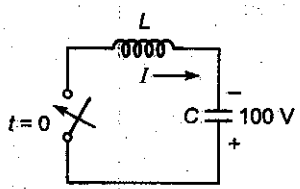
39. The kVA rating of the input transformer is

(a) 53.2 kVA (b) 46.0 kVA (c) 22.6 kVA (d) 19.6 kVA

GATE 2010

Statements for Linked Answer Questions 40 and 41

The LC circuit shown in the figure has an inductance L = 1mH and a capacitance C = 10 μF.



40. The initial current through the inductor is zero, while the initial capacitor voltage is 100 V. The switch is closed at t = 0. The current I through the circuit is

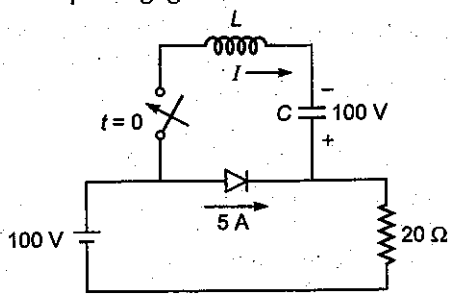
(a) $5\cos(5\times10^3 t)$ A (b) $5\sin(10^4 t)$ A

(c) $10\cos(5\times10^3 t)$ A (d) $10\sin(10^4 t)$ A

41. The LC circuit given above is used to commutate a thyristor, which is initially carrying a current of 5 A as shown in the figure below. The values and initial

▼ *Solution Points*

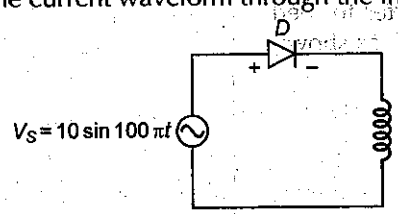
conditions of L and C are 1 mH and 10 μF respectively. The switch is closed at $t = 0$. If the forward drop is negligible, the time taken for the device to turn off is



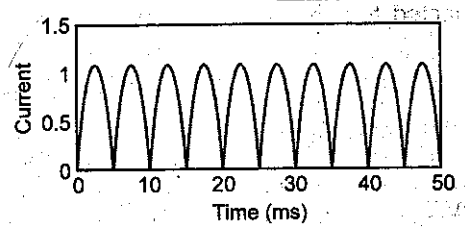
(a) 52 μs (b) 156 μs (c) 312 μs (d) 26 μs

GATE 2009

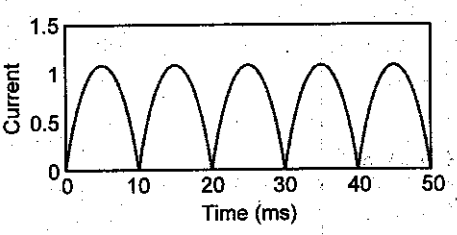
42. The circuit shows an ideal diode connected to a pure inductor and is connected to a purely sinusoidal 50 Hz voltage source. Under ideal conditions the current waveform through the inductor will look like



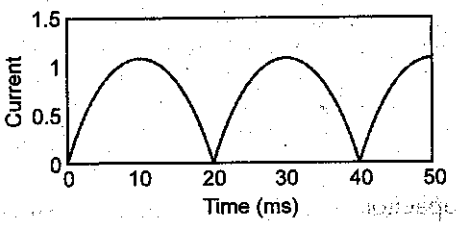
(a)



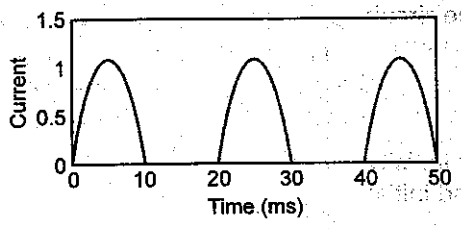
(b)



(c)

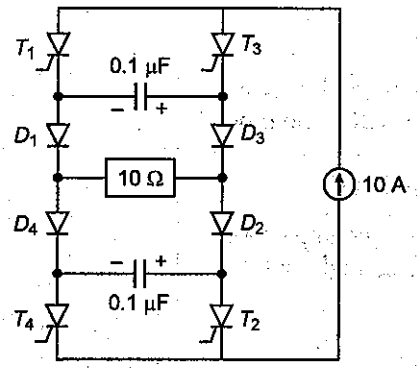


(d)



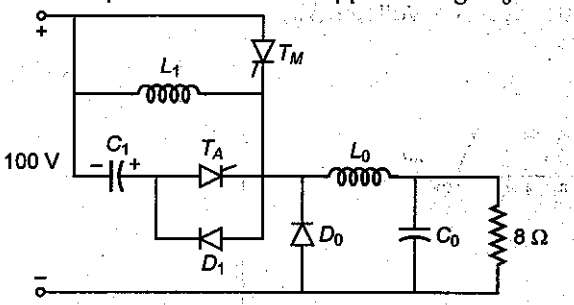
▼ *Solution Points*

43. The current source inverter shown in figure, is operated by alternately turning on thyristor pairs (T_1, T_2) and (T_3, T_4). If the load is purely resistive, the theoretical maximum output frequency obtainable will be



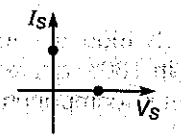
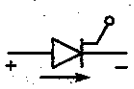
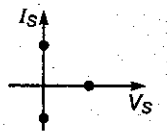
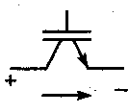
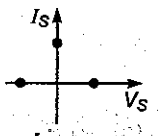
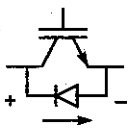
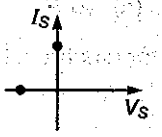
(a) 125 kHz (b) 250 kHz (c) 500 kHz (d) 50 kHz

44. In the chopper circuit shown, the main thyristor (T_M) is operated at a duty ratio of 0.8, which is much larger the commutation interval. If the maximum allowable reapplied dV/dt on T_M is 50 V/μs, what should be the theoretical minimum value of C_1? Assume current ripple through L_0 to be negligible.



(a) 0.2 μF (b) 0.02 μF (c) 2μF (d) 20 μF

45. Match the switch arrangements on the top row to the steady-state *V-I* characteristics on the lower row. The steady state operating points are shown by large black dots.

List I	List II
A.	1. 
B. 	2. 
C. 	3. 
D. 	4. 

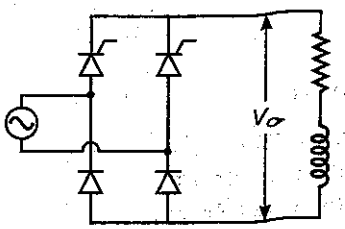
(a) A–1, B–2, C–3, D–4 (b) A–2, B–4, C–1, D–3
(c) A–4, B–3, C–1, D–2 (d) A–4, B–3, C–2, D–1

▼ *Solution Points*

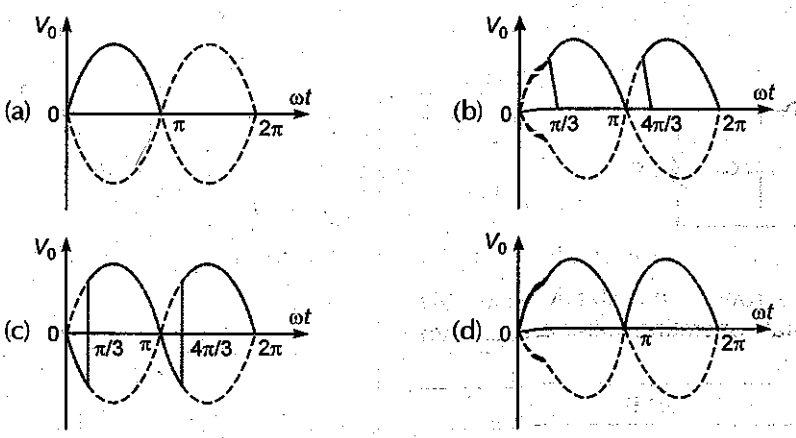
GATE 2008

46. A single-phase fully controlled bridge converter supplies a load drawing constant and ripple free load current. If the triggering angle is 30°, the input power factor will be
(a) 0.65 (b) 0.78 (c) 0.85 (d) 0.866

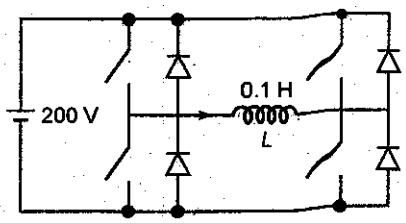
47. A single-phase half controlled converter shown in the figure is feeding power to highly inductive load. The converter is operating at a firing angle of 60°.



If the firing pulses are suddenly removed, the steady state voltage (V_0) waveform of the converter will become



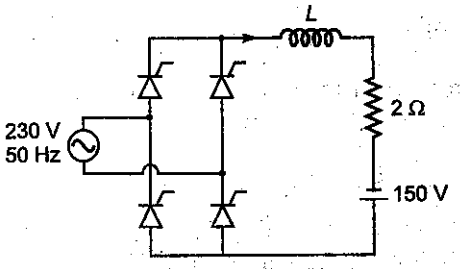
48. A single-phase voltage source inverter is feeding a purely inductive load as shown in the figure. The inverter is operated at 50 Hz in 180° square wave mode. Assume that the load current does not have any DC component. The peak value of the inductor current I_0 will be



(a) 6.37 A (b) 10 A (c) 20 A (d) 40 A

49. A single-phase fully controlled converter bridge is used for electrical braking of a separately excited DC motor. The DC motor load is represented by an equivalent circuit as shown in the figure.

▼ *Solution Points*



Assume that the load inductance is sufficient to ensure continuous and ripple free load current. The fringe angle of the bridge for a load current of I_0 =10 A will be

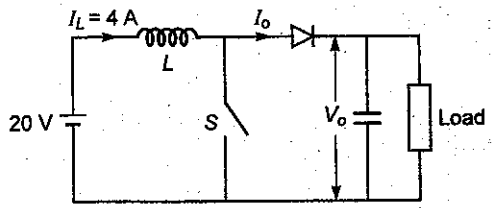
(a) 44° (b) 51° (c) 129° (d) 136°

50. A three-phase fully controlled bridge converter is feeding a load drawing a constant and ripple free load current of 10 A at a firing angle of 30°. The approximate Total Harmonic Distortion (%THD) and the rms value of fundamental component of the input current will respectively be

(a) 31% and 6.8 A (b) 31% and 7.8 A

(c) 66% and 6.8 A (d) 66% and 7.8 A

51. In the circuit shown in the figure the switch is operated at a duty cycle of 0.5. A large capacitor is connected across the load. The inductor current is



assumed to be continuous. The average voltage across the load and the average current through the diode will respectively be

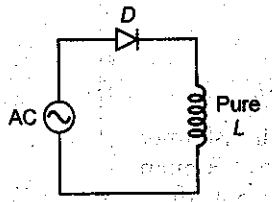
(a) 10 V, 2 A (b) 10 V, 8 A (c) 40 V, 2 A (d) 40 V, 8 A

GATE 2007

52. A single-phase voltage source inverter is controlled in a single-phase width modulated mode with a pulse width of 150° in each half cycle. Total harmonic distortion is defined as $\text{THD} = \frac{\sqrt{V_{rms}^2 - V_1^2}}{V_1} \times 100$, where V_1 is the rms value of the fundamental component of the output voltage. The THD of output AC voltage waveform is

(a) 65.65% (b) 48.34% (c) 31.83% (d) 30.49%

53. In the circuit of adjacent figure, the diode connects the AC source to a pure inductance *L*. The diode conducts for



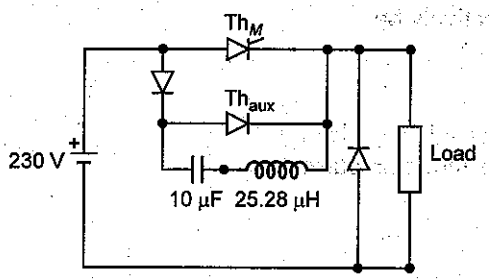
(a) 90° (b) 180° (c) 270° (d) 360°

▼ *Solution Points*

54. A voltage source inverter is used to control the speed of a three-phase, 50 Hz, squirrel cage induction motor. Its slip for rated torque is 4%. The flux is maintained at rated value. If the stator resistance and rotational losses are neglected, then the frequency of the impressed voltage to obtain twice the rated torque at starting should be
(a) 10 Hz (b) 5 Hz (c) 4 Hz (d) 2 Hz

55. A three-phase, 440 V, 50 Hz AC mains fed thyristor bridge is feeding a 440 V DC, 15 kW, 1500 rpm separately excited DC motor with a ripple free continuous current in the DC link under all operating conditions. Neglecting the losses, the power factor of the AC mains at half the rated speed, is
(a) 0.354 (b) 0.372 (c) 0.90 (d) 0.995

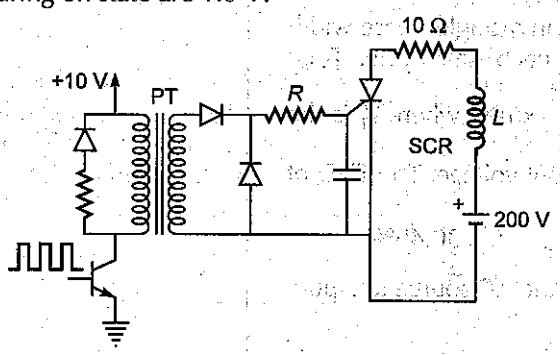
56. The circuit in the figure is a current commutated DC-DC chopper where, Th_M is the main SCR and Th_{aux} is the auxiliary SCR. The load current is constant at 10 A. Th_M is on. Th_{aux} is triggered at $t=0$. Th_M is turned off between



(a) $0 < t \leq 25\,\mu s$ (b) $25\,\mu s < t \leq 50\,\mu s$
(c) $50\,\mu s < t \leq 75\,\mu s$ (d) $75\,\mu s < t \leq 100\,\mu s$

Common Data for Questions 57 and 58

1 Pulse Transformer (PT) is used to trigger the SCR in the adjacent figure. The SCR is rated at 1.5 kV, 250 A with $I_L = 250$ mA, $I_H = 150$ mA, and $I_{G,max} = 150$ mA, $I_{G,min} = 100$ mA. The SCR is connected to an inductive load, where $L = 150$ mH in series with a small resistance and the supply voltage is 200 V DC. The forward drops of all transistors/diodes and gate-cathode junction during on state are 1.0 V.



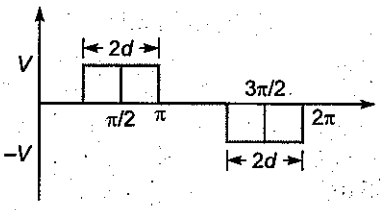
57. The resistance R should be
(a) 4.7 kΩ (b) 470 Ω (c) 47 Ω (d) 4.7 Ω

58. The minimum approximate volt-second rating of the pulse transformer suitable for triggering the SCR should be (volt-second rating is the maximum of product of the voltage and the width of the pulse that may be applied)
(a) 2000 μV-s (b) 200 μV-s (c) 20 μV-s (d) 2.0 μV-s

GATE 2006

▼ *Solution Points*

59. A single-phase inverter is operated in PWM mode generating a single-pulse of width $2d$ in the centre of each half cycle as shown in figure. It is found that the output voltage is free from 5th harmonic for pulse width 144°. What will be percentage of 3rd harmonic present in the output voltage (V_{03}/V_{01max})?

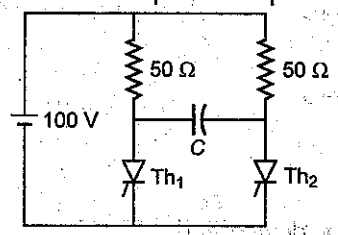


(a) zero (b) 19.6% (c) 31.7% (d) 53.9%

60. A 3-phase fully controlled bridge converter with free-wheeling diode is fed from 400 V, 50 Hz AC source and is operating at a firing angle of 60°. The load current is assumed constant at 10 A due to high load inductance. The input displacement factor (IDF) and the input power factor (IPF) of the converter will be

(a) IDF = 0.867 IPF = 0.828
(b) IDF = 0.867 IPF = 0.552
(c) IDF = 0.5 IPF = 0.478
(d) IDF = 0.5 IPF = 0.318

61. A voltage commutation circuit is shown in figure. If the turn off time of the SCRs is 50 μs and a safety margin of 2 is considered, then what will be the approximate minimum value of capacitor required for proper commutation?

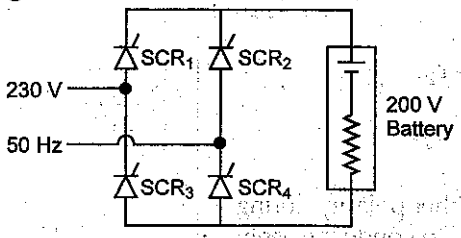


(a) 2.88 μF (b) 1.44 μF (c) 0.91 μF (d) 0.72 μF

62. A solar cell of 350 V is feeding power to an AC supply of 440 V, 50 Hz through a 3-phase fully controlled bridge converter. A large inductance is connected in the DC circuit to maintain the DC current at 20 A. If the solar cell resistance is 0.5 Ω, then each thyristor will be reverse biased for a period of

(a) 125° (b) 120° (c) 60° (d) 55°

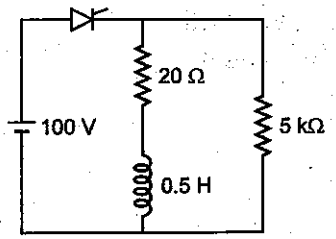
63. A single-phase bridge converter is used to charge a battery of 200 V having an internal resistance of 0.2 Ω as shown in figure. The SCRs are triggered by a constant DC signal. If SCR_2 gets open circuited, then what will be the average charging current?



(a) 23.8 A (b) 15 A (c) 11.9 A (d) 3.54 A

▼ *Solution Points*

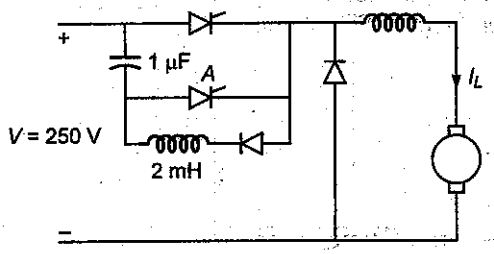
64. An SCR having a turn on time of 5 μs, latching current of 50 mA and holding current of 40 mA is triggered by a short duration pulse and is used in the circuit shown in figure. The minimum pulse width required to turn the SCR on will be



(a) 251 μs (b) 150 μs (c) 100 μs (d) 5 μs

Statements for Linked Answer Questions 65 and 66

A voltage commutated chopper operating at 1 kHz is used to control the speed of DC motor as shown in figure. The load current is assumed to be constant at 10 A.



65. The minimum time in μs for which the SCR_A should be on is

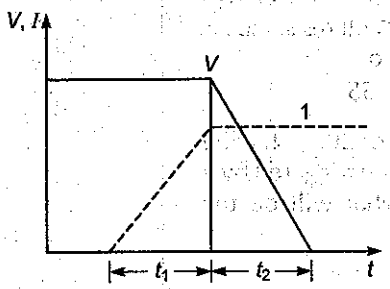
(a) 280 (b) 140 (c) 70 (d) zero

66. The average output voltage of the chopper will be

(a) 70 V (b) 47.5 V (c) 35 V (d) zero

GATE 2005

67. The figure shows the voltage across a power semi-conductor device and current through the device during a switching transitions. Is the transition turn on transition or a turn off transition? What is the energy lost during the transition?



(a) Turn on, $\frac{VI}{2}(t_1 + t_2)$ (b) Turn off, $VI(t_1 + t_2)$

(c) Turn on, $VI(t_1 + t_2)$ (d) Turn off, $\frac{VI}{2}(t_1 + t_2)$

68. An electronic switch S is required to block voltages of either polarity during its off state as shown in the Fig. (a). The switch is required to conduct in only one direction during its on state as shown in the Fig. (b).

▼ *Solution Points*

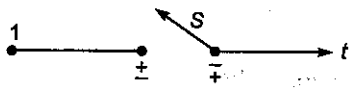


Fig. (a)

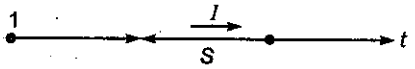
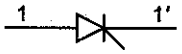
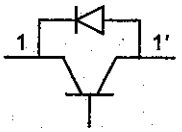
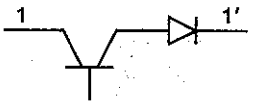


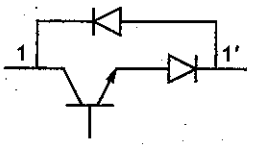
Fig. (b)

Which of the following is/are valid realization of the switch *S*?

(P) 

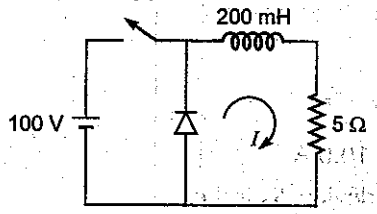
(Q) 

(R) 

(S) 

(a) Only *P* (b) *P* and *Q* (c) *P* and *R* (d) *R* and *S*

69. The given figure shows a step-down chopper switched at 1 kHz with a duty ratio $D = 0.5$. The peak-peak ripple in the load current is closed to

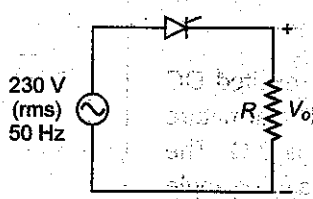


(a) 10 A (b) 0.5 A (c) 0.125 A (d) 0.25 A

70. An electric motor, developing a starting torque of 15 N-m, starts with a load torque of 7 N-m on its shaft. If the acceleration at start is 2 rad/s^2, the moment of inertia of the system must be (neglecting viscous and coulomb friction)

(a) 0.25 kg-m^2 (b) 0.25 N-m^2 (c) 4 kg-m^2 (d) 4 N-m^2

71. Consider a phase-controlled converter shown in the figure. The thyristor is fired at an angle α in every positive half cycle of the input voltage. If the peak value of the instantaneous output voltage equals 230 V, the firing angle α is closed to

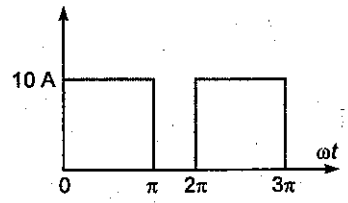


(a) 45° (b) 135° (c) 90° (d) 83.6°

GATE 2004

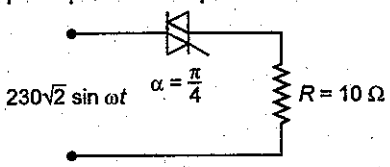
▼ *Solution Points*

72. A MOSFET rated for 15 A, carries a periodic current as shown in figure. The on state resistance of the MOSFET is 0.15 Ω. The average on state loss in the MOSFET is



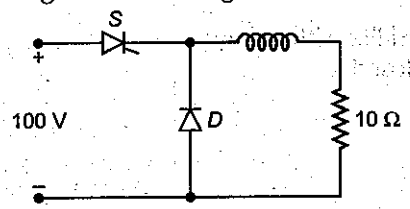
(a) 33.8 W (b) 15.0 W (c) 7.5 W (d) 3.8 W

73. The TRIAC circuit shown in figure controls the AC output power to the resistive load. The peak power dissipation in the load is



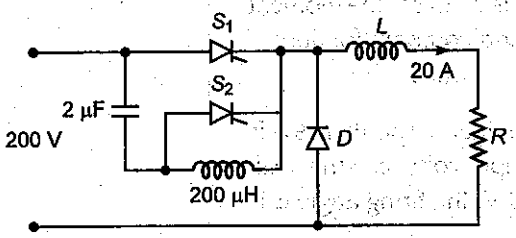
(a) 3968 W (b) 5290 W (c) 7935 W (d) 10580 W

74. Figure shows a chopper operating from a 100 V DC input. The duty ratio of the main switch S is 0.8. The load is sufficiently inductive so that the load current is ripple free. The average current through the diode D under steady state is



(a) 1.6 A (b) 6.4 A (c) 8.0 A (d) 10.0 A

75. Figure shows a chopper. The device S_1 is the main switching device. S_2 is the auxiliary commutation device. S_1 is rated for 400 V, 60 A. S_2 is rated for 400 V, 30 A. The load current is 20 A. The main device operates with a duty ratio of 0.5. The peak current through S_1 is



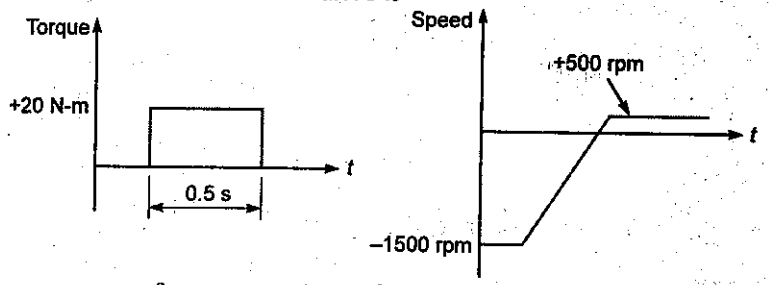
(a) 10 A (b) 30 A (c) 20 A (d) 40 A

76. A single-phase half-controlled rectifier is driving a separately excited DC motor. The DC motor has a back emf constant of 0.5 V/rpm. The armature current is 5 A without any ripple. The armature resistance is 2 Ω. The converter is working from a 230 V, single-phase AC source with a firing angle of 30°. Under this operating condition, the speed of the motor will be

(a) 339 rpm (b) 359 rpm (c) 366 rpm (d) 386 rpm

▼ *Solution Points*

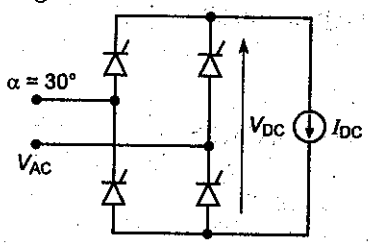
77. A variable speed drive rated for 1500 rpm, 40 N-m is reversing under no load. Figure shows the reversing torque and the speed during the transient. The moment of inertia of the drive is



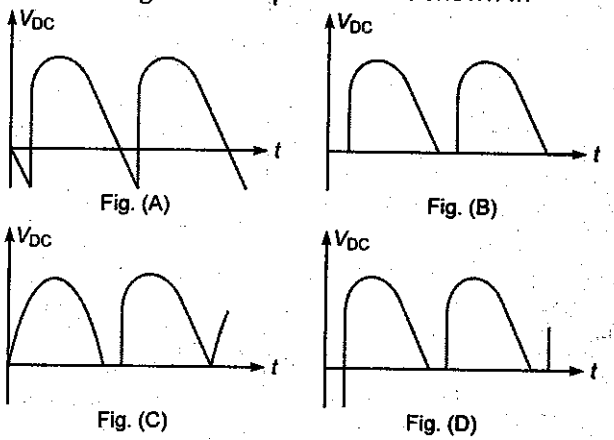
(a) 0.048 kg-m^2 (b) 0.064 kg-m^2 (c) 0.096 kg-m^2 (d) 0.128 kg-m^2

GATE 2003

78. A phase-controlled half-controlled single-phase converter is shown in figure. The control angle $\alpha = 30°$.

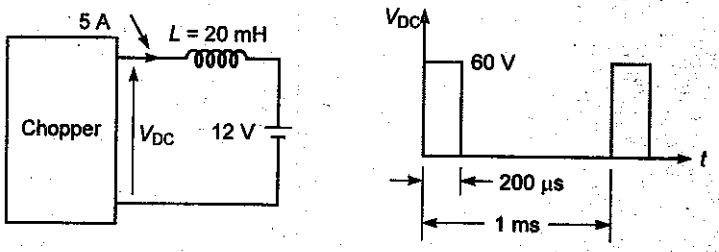


The output DC voltage waveshape will be as shown in



(a) Fig. A (b) Fig. B (c) Fig. C (d) Fig. D

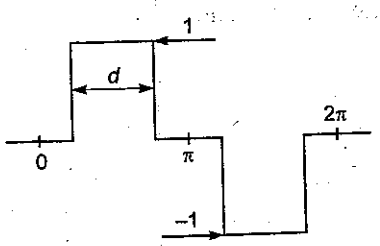
79. A chopper is employed to charge a battery as shown in figure. The charging current is 5 A. The duty ratio is 0.2. The chopper output voltage is also shown in figure. The peak to peak ripple current in the charging current is



(a) 0.48 A (b) 1.2 A (c) 2.4 A (d) 1 A

▼ *Solution Points*

80. An inverter has a periodic output voltage with the output waveform as shown in figure.



When the conduction angle $\alpha = 120°$, the rms fundamental component of the output voltage is

(a) 0.78 V (b) 1.10 V (c) 0.90 V (d) 1.27 V

81. With reference to the output waveform given in figure, the output of the converter will be free from 5th harmonic when

(a) $\alpha = 72°$ (b) $\alpha = 36°$ (c) $\alpha = 150°$ (d) $\alpha = 120°$

82. An AC induction motor is used for a speed control application. It is driven from an inverter with a constant V/f control. The motor name-plate details are as follows

V: 415 V_{Ph}: 3 f: 50 Hz N:2850 rpm

The motor is run with the inverter output frequency set at 40 Hz, and with half the rated slip. The running speed of the motor is

(a) 2400 rpm (b) 2280 rpm (c) 2340 rpm (d) 2790 rpm

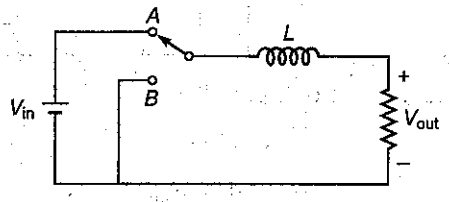
Answers with Explanations

1. (b) The turn-off time is the minimum value of time interval between the instants when the on-state current has decreased to zero and the instant when the thyristor is capable of withstanding forward voltage without turning-on.

2. (a) If MOSFETs are used for the current source inverter, the reverse blocking capability can at present only be achieved with the diode connected in series to the MOSFETs. Also the diodes are needed in CSI, so as to prevent the capacitor from discharging the load.

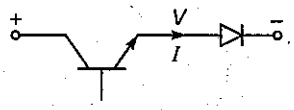
3. (c)

4. (a)

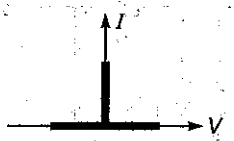


The circuit shown in step down (buck converter), because the output voltage is less than input voltage only. The voltage transfer function $M_V = \frac{V_o}{V_S}$ is less than unity.

5. (c)

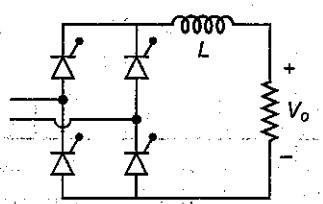


Assuming the transistor switch and diode are ideal, the *I-V* characteristics will be



The composite switch carries in one direction or in both directions, *i.e.*, forward conducting, bidirectional conducting.

6. (a)



Given that DC output voltage

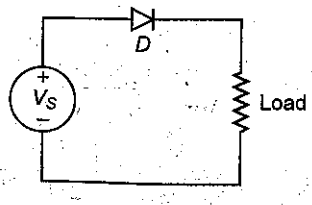
$$V_{DC} = \frac{2V_m}{\pi} = 300 \text{ V}$$

Firing angle = 60°

$$\text{Output voltage } V_o = \frac{2V_m}{\pi}\cos\alpha = \frac{2V_m}{\pi}\cos 60°$$

$$= 300 \times \cos 60° = 300 \times \frac{1}{2} = 150 \text{ V}$$

7. (a) The element connected is diode.

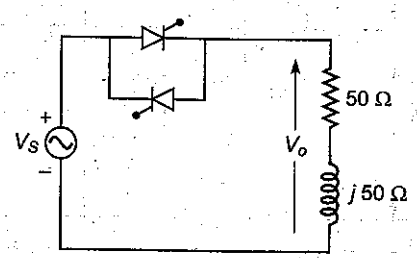


When V_S has positive voltage (pulse), the diode conducts and forward current flows.

When V_S has negative voltage (pulse), the diode is off and due to charge storage initially, the reverse current flow which decays and finally zero current value exists.

8. (c) Thyristors are generally turned from a forward block to forward conducting state by applying specific gate pulse but it cannot be turned off with the gate pulse. Hence, it is termed on as semi-controllable device.

9. (a)



The load impedance angle

$$= \tan^{-1}\left(\frac{\omega L}{R}\right) = \tan^{-1}\left(\frac{50}{50}\right) = \tan^{-1} 1$$

$$\phi = 45°$$

Triggering angle (α) must satisfy the condition $\phi \le \alpha < \pi$ for output voltage controllable.

Hence, $0° < \alpha < 45°$ not satisfies condition.

10. (d) Third harmonics are absent in pole voltage and line voltage because of the factor cos $(n\pi/f)$.

11. (d) $V = xx^T$ is the $n \times n$ matrix

Hence, has the rank 'n'.

12. (a) Given $V_m = 230\sqrt{2}$

I_o = output DC current

$$= \frac{V_m}{\omega L_S}[\cos\alpha - \cos(\alpha+\mu)]$$

$$= \frac{230\times\sqrt{2}}{2\pi\times 50\, L_S}[\cos 25° - \cos(25+10)°]$$

$$L_S = 0.0045\,\text{H}$$

$$V_o = \frac{2V_m\cos\alpha}{\pi} - \frac{\omega L_S I_o}{\pi}$$

$$= 178.74\,\text{V}$$

$$\text{Power factor} = \frac{\text{Active power}}{\text{Apparent power}}$$

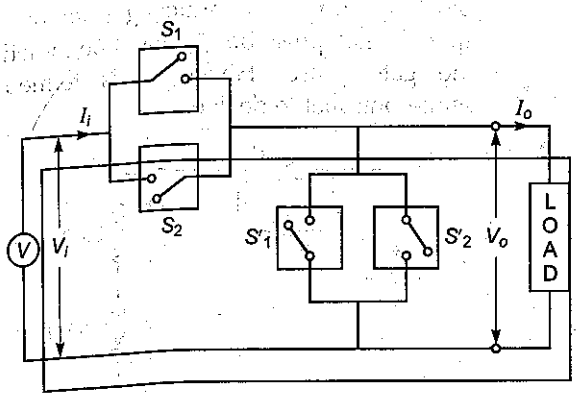
$$= \frac{P}{S} = \frac{V_o I_o}{V_S I_S} = \frac{178.25\times 20}{230\times 20} = 0.78$$

13. (c) $P_D = V_D I_D = \sqrt{3}\, V I_{rms}$

$$I_{rms} = \frac{P_D}{\sqrt{3}\,V}$$

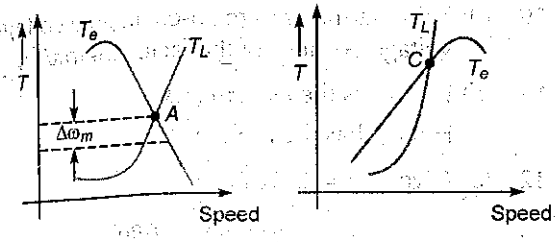
$$= \frac{50\times 1000}{\sqrt{3}\times 420} = 68.73\,\text{A}$$

14. (c)



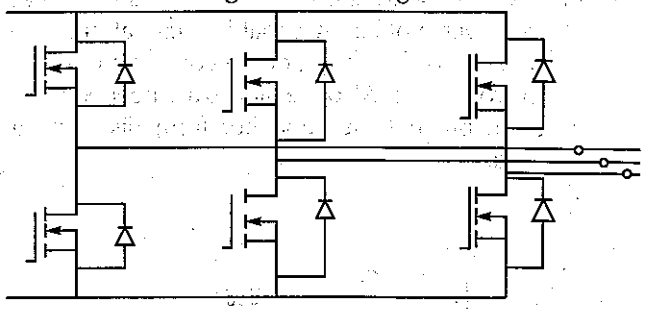
Here SCRs S_1 and S_2 connected in antiparallel are turned on and off many times during the positive and negative half cycles of the input voltage respectively. S_1' and S_2' provide the free-wheeling path for the load current when S_1 and S_2 are off or else the free-wheeling action of the converter will cause shorting of the AC supply.

15. (c) The stable operating points are



The equilibrium point will be termed as stable when the operation will be restored to it after a small departure from it due to a disturbance in the motor or load. Let the disturbance causes a reduction $\Delta\omega_m$ in speed. At new speed, motor torque is greater than the load torque, consequently, motor will be accelerate and operation will restore to A.

16. (b) The circuit diagram of a voltage source inverter

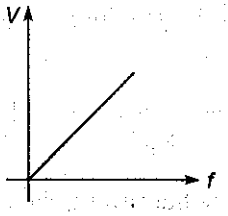


17. (a) The inverter output voltage

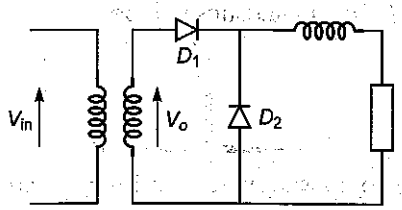
$$V = 4.44\,NB_mAf$$

$$\frac{V}{f} = 4.44\,NB_mA$$

To maintain B_m (maximum flux density) constant. Hence, $\frac{V}{f}$ should remain constant.



18. (d)



For first half cycle, diode D_1 conducts and D_2 is in off condition. Hence, the current through diode D_1 is zero and also for $\theta = 0$ there is no current.

19. (a) The total conduction (on-state) power loss for a given MOSFET with forward current I_D and on-resistance $R_{DS(on)}$ is given by

$$P_{on,\,diss} = I_D^2\, R_{DS(on)}$$

20. (b) Since, the load is purely resistive.

Peak instantaneous voltage $= \sqrt{2} \times V_{rms} = 400\sqrt{2}$

21. (c) Square wave having half wave symmetry and hence contains only odd harmonics.

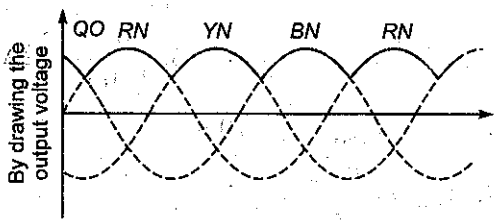
22. (b) To turn on the thyristor, the minimum current required is 10 mA.

The value R be choosen for maximum value of voltage to provide 10 mA current.

$V_{D,max} = 16$ V

$$R = \frac{V_{D,max}}{I_g} = \frac{16}{10\,\text{mA}} = 1600\,\Omega$$

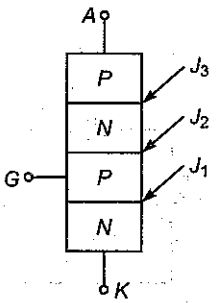
23. (c)



Frequency of ripple $= 3f = 3 \times 400 = 1200$ Hz

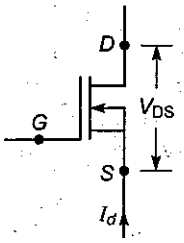
Clearly, from V_0, ripple frequency is 1200 Hz.

24. (d)

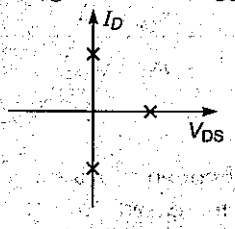


When a positive voltage is applied to anode (with respect to a cathode), the function J_3 is in forward blocking state (J_3 forward biased). The centre junction J_2 is reverse biased. When a positive gate current is injected into the device, J_1 becomes forward biased and electrons are injected from the n-emitter into p-base. Some of these electrons diffuse across the p-base and collected in the n-base. The addition of these collected holes in the p-base acts same as gate current. This regenerative process will take place and will cause the increase in charge carrier until J_2 is forward biased and the thyristor is latched in its on state.

25. (b)



The MOSFET carries in both directions but block only in one direction, *i.e.*, bidirectional carrying, forward blocking. Hence, the $V_{DS} - I_D$ characteristics



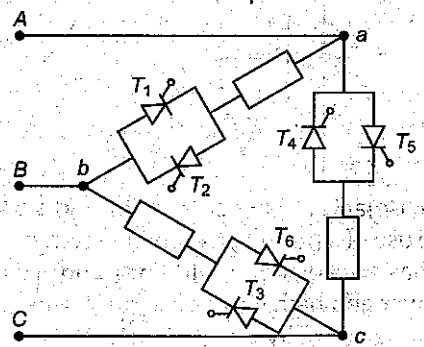
26. (b) Armature voltage control provides speed control only below base (rated) speed because the armature voltage can't be allowed to exceed rated value. For speed control above base speed, field flux control is employed.

27. (d) In the saturation mode both the base-emitter junction and base-collector junction are forward biased and the base-emitter junction and base-collector junction are reverse biased.

28. (a) Peak-inverse voltage across full-wave rectifier $= (2V_m)$, $(V_m = 50\sqrt{2})$

$\therefore$ Peak inverse voltage $= 100\sqrt{2}$

29. (a) Let us consider an example of load connected.



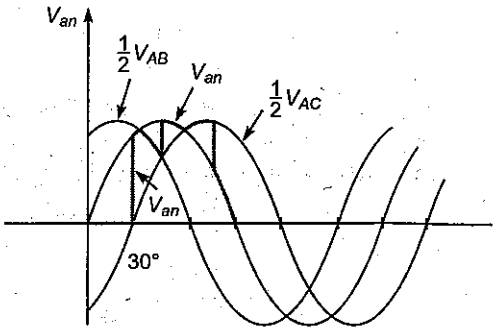
Depending upon firing angle α there are operating mode.

For $0 \le \alpha \le 60°$ is known as mode 2/3. There are period when three SCRs conducting, one in each phase for either direction and periods when just two SCRs conduct.

For $\alpha = 30°$,

$$V_{an} = \frac{1}{2} V_{AB} = \frac{1}{2} V_{AC}$$

$$\frac{V_{an}}{V_{AB}} = \frac{1}{2}$$

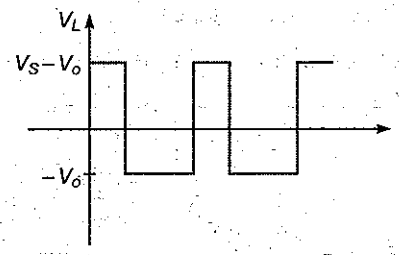


30. (d) The ripple frequency of a six pulse thyristor rectifier bridge is increased to six times to the input frequency. Given input frequency 50 Hz. Hence, ripple frequency will be 300 Hz. Hence, lowest frequency component in the source line content is 300 Hz.

31. (a) The state of the converter in which the induction current is never zero for any period of time is called the continuous conduction mode. If D is duty ratio, then we can write

$$(V_S - V_o) DT = -V_o (1 - D) T$$

$$\frac{V_o}{V_S} = D$$



32. (b) For isolation between control and power circuits, the use of a pulse transformer is common. Normally it has two or three windings and it is small, low power and high frequency transformer and the turn ratio is normally 1 : 1 or 2 : 1.

Hence, pulse transformer is used to provide electrical isolation.

33. (a) In AC-to-DC circulating current dual converters, two converters are controlled in such a way that if α_1 is the delay (triggering) angle of the positive current converter and delay angle of the negative current converter is α_2. Then

$$\alpha_1 + \alpha_2 = 180°$$

34. (c) The closed-loop speed control schemes are provided with inner current control loop, in order to limit the current within a safe limit and also accelerate and decelerate the drive at maximum permissible current and torque during transient operation.

35. (b) When motor armature current is found drop to zero at such instances, the voltage has value that is equal to the instantaneous value of motor back emf.

36. (c) In three-phase, fully controlled converter, the input AC line current to the converter has a peak value equal to DC load current.

37. (a) $I_{M\max} = I_0 + I_{C\,\text{Peak}} = I_0 + V_S \sqrt{\frac{C}{L}}$

$$= 10 + 200 \sqrt{\frac{0.1\mu}{1\text{m}}} = 12\,\text{A}$$

$$I_{A\max} = I_0 = 10\,\text{A}$$

38. (a) $(V_0)_{\max} = \frac{3\sqrt{6}\, V_{pL}}{\pi} = I_A R_A + E$ $(\because \cos\alpha = 1)$

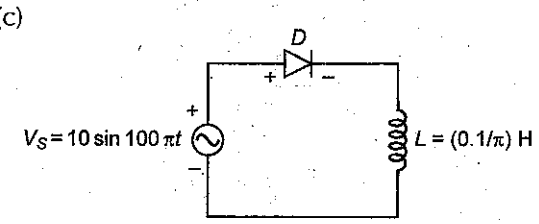
$$540.2 = I_A (10) + 400$$

$$\Rightarrow \quad I_A = 14\,\text{A}$$

39. (c) $\because$ kVA rating $\sqrt{3}\, V_L I_L$

40. (a) **41.** (a)

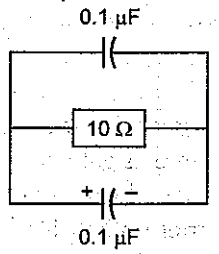
42. (c)



The current through inductor $= \frac{1}{L} \int V_L\, dt$

where, V_L is the voltage across inductor.

43. (c) For one complete cycle, the charging and discharging take place.



Time constant $= R C_{eq} = R(2C)$

$$= 2 \times 0.1\mu \times 10 = 2\mu\text{s}$$

$$\text{Maximum output frequency} = \frac{1}{2\times10^{-6}} = 500\ \text{kHz}$$

44. (a) Output voltage $V_o = DV_S$

$$= 0.8\times100 = 80\ \text{V}$$

$$\text{Current} = \frac{80}{8} = 10\ \text{A}$$

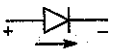
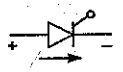
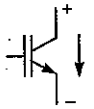
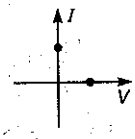
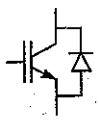
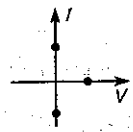
$$C\left.\frac{dV}{dt}\right|_{T_M} = 10$$

$$C\times\frac{50}{10^{-6}} = 10$$

$$C = \frac{10\times10^{-6}}{50}$$

$$= 0.2\ \mu\text{F}$$

45. (c)

Symbol	Device	Operating point	Remark
	Diode		Forward conducting, reverse blocking
	GTO		Forward conducting, bidirectional blocking
	BJT		Forward conducting, forward blocking
	FET		Bidirection carrying, Forward blocking

Diode works as half-wave rectifier.

46. (b) For single-phase fully controlled bridge converter,

Input power factor

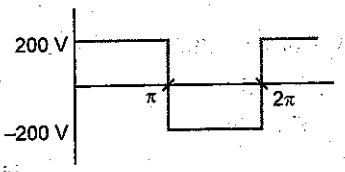
$$= \frac{2\sqrt{2}}{\pi}\cos\alpha = \frac{2\sqrt{2}}{\pi}\cos30° = 0.78$$

47. (b) Even if firing pulse is removed, the thyristor will be on condition because due to high inductive load the load current will always be greater than the holding current of thyristor.

48. (a) The peak value of inductor current

$$= \frac{2V_S}{3R} = \frac{2\times200}{3\times\omega\times L}$$

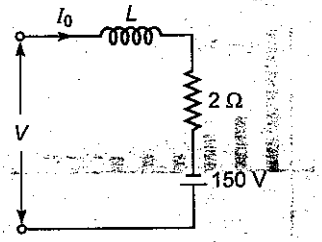
$$= \frac{2\times200}{3\times2\pi\times50\times0.1} = 6.37\ \text{A}$$



49. (c) Let α be the firing angle.

Output of single phase fully controlled bridge converter $= \frac{2V_m\cos\alpha}{\pi}$

Now, the circuit can be approximated.



Applying KVL,

$$V = 2I_a - 150$$

$$\frac{2V_m\cos\alpha}{\pi} = 2\times10 - 150$$

$$\cos\alpha = -\frac{130\times\pi}{2\times230\sqrt{2}}$$

$$\alpha = 128.8° \simeq 129°$$

50. (b) Total rms $= I_a = \sqrt{\frac{2}{3}}\times10 = 8.16\ \text{A}$

(Fundamental) $I_{a_1} = 0.78\times10 = 7.8\ \text{A}$

$$\text{THD} = \sqrt{\frac{1}{DF^2} - 1}$$

where, $DF = \frac{I_{a_1}}{I_a} = \frac{0.78\times10}{0.816\times10} = 0.955$

$$\text{THD} = \sqrt{\frac{1}{0.955^2} - 1}$$

$$= 31\%$$

51. (c) When switch S is opened, $I_0 < I_L = 4\ \text{A}, V_0 = 20\ \text{V}$

When switch S is closed, $I_D = 0, V_0 = 0$

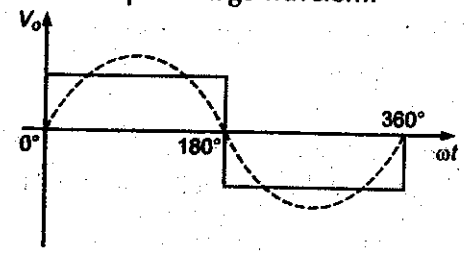
Since, duty cycle is 0.5 therefore $\frac{V_S}{1-S}$

$$\text{Average current} = \frac{0+4}{2} = 2\ \text{A}$$

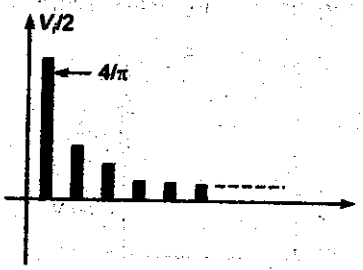
$$\text{Average voltage} = \frac{20}{1-0.5} = 40\ \text{V}$$

52. (b) Given THD $= \dfrac{\sqrt{V_{rms}^2 - V_1^2}}{V_1}$

The AC output voltage waveform



The AC output voltage spectrum



The fundamental AC voltage amplitude

$$V_1 = \frac{4}{\pi} \times \frac{V_i}{2} = \frac{4}{\pi}\frac{\sqrt{2}V_{rms}}{2}$$

$$= \frac{4}{\pi}\frac{V_{rms}}{\sqrt{2}}$$

$$\text{THD} = \sqrt{\frac{V_{rms}^2 - \left(\frac{4}{\pi}\right)^2\left(\frac{V_{rms}}{\sqrt{2}}\right)^2}{\left(\frac{4}{\pi}\frac{V_{rms}}{\sqrt{2}}\right)^2}}$$

$$= \sqrt{\frac{\pi^2 - 8}{8}} = 48.34\%$$

53. (b) **54.** (a)

55. (a) Output voltage,

$$V_o = K_m \omega_m$$

$$\Rightarrow \quad K_m = \frac{440}{50\pi} = 2.8$$

$$V_t = \frac{3\sqrt{2} \times 440}{\pi}(0.3700)$$

$$= 219.85$$

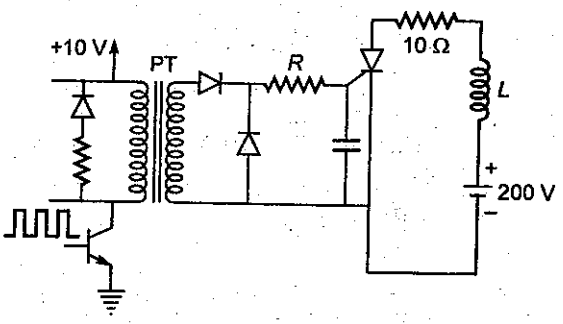
$$I_a = \frac{15000}{440} = 34.090$$

$$I_{Sr} = I_0\sqrt{\frac{2}{3}} = 27.83$$

$$\text{PF} = \frac{V_t \cdot I_a}{\sqrt{3}V_S I_{Sr}} = 0.354$$

56. (c)

57. (c)



Given that 1 V drop in primary side and subsequently 1 V drop in diode and in gate cathode junction. Hence, maximum voltage to thyristor

$$V_m = 10 - 1 - 1 - 1 = 7 \text{ V}$$

Given $I_{G,\max} = 150 \text{ mA}$

Hence, $R = \dfrac{7\text{V}}{150\,\text{mA}} = 46.67\,\Omega$

58. (a) Volt-second rating = Maximum voltage × Gate pulse width

Given maximum voltage = 10 V

Gate pulse width = Time taken to reach current to latch current of thyristor

The current $I = \dfrac{V}{R}[1 - e^{-t/15\times10^{-3}}]$

$$I = \frac{200}{10}[1 - e^{-t/15\times10^{-3}}] \qquad \ldots(i)$$

At $t = T, I_a = I_L = 0.25$ A

From Eq. (i),

$$0.25 = 20\,[1 - e^{-t/15\times10^{-3}}]$$

$$\Rightarrow \quad \frac{0.25}{20} = 1 - e^{-t/15\times10^{-3}}$$

$$\Rightarrow 12.5\times10^{-3} = 1 - e^{-t/15\times10^{-3}}$$

$$t \simeq 200\,\mu\text{s}$$

Rating $\simeq 2000\,\mu$V-s

59. (b) The output voltage can be expressed as

$$V_o = \sum_{n=1}^{\infty} \frac{4V_S}{n\pi} \sin nd \sin n\omega t$$

Given pulse width $= 2d = 144°$

$$d = 72°$$

The fundamental component

$$V_{o1} = \frac{4V_S}{\pi} \sin 72° \sin \omega t$$

The third harmonic component

$$V_{o3} = \frac{4V_S}{3\pi}\sin(3\times 72°)\sin 3\omega t$$

$$\left.\frac{V_{o3}}{V_{o1}}\right|_{max} = \frac{\frac{4V_S}{3\pi}\times\sin(3\times 72°)}{\frac{4V_S}{\pi}\times\sin(72°)}\times 100\% = 19.6\%$$

60. (c) IDF (Input Displacement Factor) = cos α; where α is the firing angle.

Given $\alpha = 60°$

$$\text{IDF} = \cos 60° = \frac{1}{2} = 0.5$$

$$\text{Distortion factor} = \frac{I_S\ (\text{Fundamental})}{I_S}$$

$$= \frac{4\times 10}{\pi\times\sqrt{2}}\,\frac{\sin 60°}{10\times\sqrt{2/3}} = 0.955$$

Input power factor = Distortion factor × cos α

$$= 0.955\times\cos 60°$$

$$= 0.478$$

61. (a) The time constant of *R-C* circuit must be $50\,\mu s \times 2 = 100\,\mu s$

$$RC\ln(2) = 100\times 10^{-6}$$

$$C = \frac{100\times 10^{-6}}{50\times 0.693}$$

$$= 2.88\,\mu F$$

62. (a) The power supply feeding to converter

$$= 350 - 20\times 0.5 = 340\ V$$

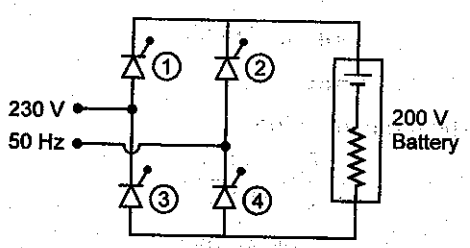
Hence, we can write

$$\frac{3\times 440\times\sqrt{2}}{\pi}\cos\alpha = 340$$

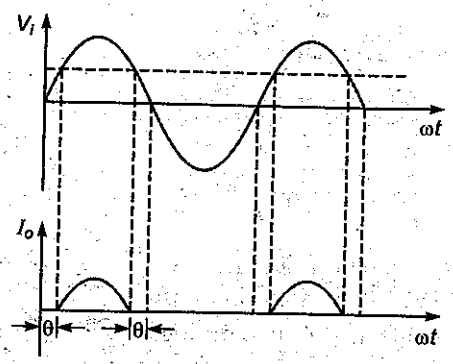
$$\cos\alpha = 55°$$

Hence, each thyristor will be reverse biased for $180° - 55° = 125°$

63. (c)



When SCR_2 gets open circuited, circuit becomes half wave rectifier.



$$I_{avg} = \frac{1}{2\pi\theta}\int_{\theta_1}^{\pi-\theta_1}(V_m\sin\omega t - \theta)\,d\theta$$

$$E = V_m\sin\theta_1$$

$$\theta_1 = \sin^{-1}\left(\frac{E}{V_m}\right) = \sin^{-1}\left(\frac{200}{230\times\sqrt{2}}\right)$$

$$= 38° = 0.66\ \text{rad}$$

$$I_{avg} = \frac{1}{2\pi R}[2V_m\cos\theta - E(\pi - 2\theta_1)]$$

$$= \frac{1}{2\pi\times 2}[2\times 230\times\sqrt{2}\cos 38° - 200(\pi - 2\times 0.66)]$$

$$= 11.9\ A$$

64. (b) Minimum gate pulse width = Time required by I_a to rise upto I_L (latch current)

$$\text{Current through resistor} = \frac{100}{5\times 10^3} = 20\ mA$$

$$\text{Current in } R\text{-}L \text{ branch} = \frac{100}{20}[1 - e^{-40t}]$$

$$I = 20\times 10^{-3} + 5(1 - e^{-40t})$$

At $t = T;\ I = I_L = 0.05$

$$0.05 = 0.02 + 5(1 - e^{-40t})$$

$$T = 150\,\mu s$$

65. (b) Minimum time required for charging

$$= \frac{1}{2}\times\text{time period}$$

$$= \frac{1}{2}\times\frac{2\pi}{\omega_0} = \frac{\pi}{\omega_0}$$

$$= \frac{\pi}{1/\sqrt{LC}}$$

$$= \pi\sqrt{LC}$$

$$= 3.14\times\sqrt{2\times 10^{-3}\times 10^{-6}}$$

$$= 140\,\mu s$$

66. (c) $T_{total} = \frac{1}{f} = \frac{1}{10^3} = 10^{-3}\ s$

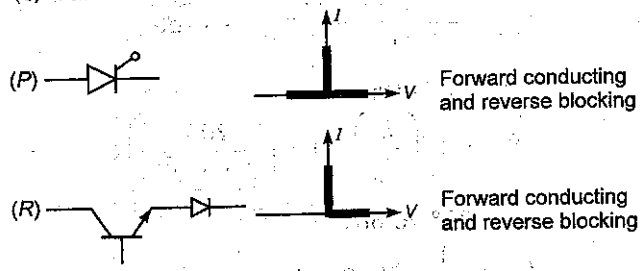
$$\text{Average output} = \frac{T_{on}}{T_{total}} \times V$$

$$= \frac{140 \times 10^{-6}}{1 \times 10^{-3}} \times 250$$

$$= 35 \text{ V}$$

67. (a) As the voltage drop to zero and current established. Hence, it is a turn on transition.
Average power loss during transition $= VI(t_1 + t_2)/2$

68. (c) Valid realizations of switch are P and R.



69. (c) $T_{total} = \frac{1}{1 \times 10^3} = 10^{-3}\text{s}$

$$\text{Time constant} = \frac{L}{R} = \frac{200 \text{ mH}}{5} = 40 \text{ ms}$$

$$\text{Ripple}|_{max} = \frac{V_S}{4fL} = \frac{100}{4 \times 10^3 \times 200 \times 10^{-3}}$$

$$= 0.125 \text{ A}$$

70. (c) Net torque $= T_S - T_L = 15 - 7 = 8$ N-m

$$I\alpha = 8 \text{ N-m}$$

$$I = \frac{8}{2} = 4 \text{ kg-m}^2$$

71. (b) Given $V_{peak} = V_m \sin\alpha$

$$\Rightarrow \quad 230 = 230\sqrt{2}\sin\alpha$$

$$\Rightarrow \quad \sin\alpha = \frac{1}{\sqrt{2}}$$

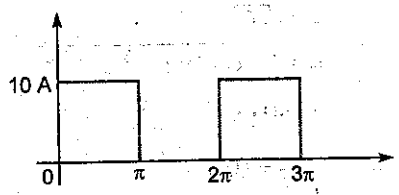
$$\alpha = 45°, 135°$$

For $\alpha < 90°$,

$$V_m = V_{peak}$$

Hence, $\alpha = 135°$

72. (c)



The average power losses $= \frac{1}{T}\int_0^{T/2} I^2R\,dt$

$$= \frac{1}{(2\pi/\omega)}\int_0^{\pi/\omega} I^2R\,dt$$

$$= I^2R\left(\frac{\pi/\omega}{2\pi/\omega}\right)$$

$$= \frac{1}{2} \times 10^2 \times 0.15$$

$$= 7.5 \text{ W}$$

73. (d) $V_{DC} = 230\sqrt{2}\left[\frac{1}{\alpha\pi}\left[(2\pi - \alpha) + \frac{\sin 2\alpha}{2}\right]\right]^{1/2}$

$$= 317.8 \text{ V}$$

$$\text{Loss} = \frac{V^2}{R} = 10100 \text{ W} \cong 10580 \text{ W}$$

74. (c) Average current through the diode

$$= \frac{\alpha V_S}{R} = \frac{0.8 \times 100}{10} = 8 \text{ A}$$

75. (d) Peak current through S_1 is given by

$$= I_0 + V_0\sqrt{\frac{C}{L}}$$

$$= 20 + 200\sqrt{\frac{2 \times 10^{-6}}{200 \times 10^{-6}}} = 40 \text{ A}$$

76. (d) $V = \frac{V_m}{2\pi}(1 + \cos\alpha) = \frac{230\sqrt{2}}{2\pi}(1 + \cos 30°)$

$$= 96.6 \text{ V}$$

$$E = \text{Back emf} = V - I_aR_a$$

Speed of motor $= 386$ rpm

77. (c) From the waveform,

The angular acceleration

$$\alpha = \left(\frac{500 - (-1500)}{0.5}\right) \times \frac{2\pi}{60} \text{ rad/s}$$

$$= 418.67 \text{ rad/s}$$

$$T = I\alpha$$

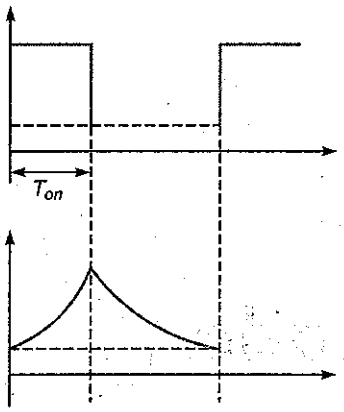
The moment of inertia

$$I = \frac{T}{\alpha} = \frac{40}{418.67}$$

$$= 0.096 \text{ kg-m}^2$$

78. (a)

79. (a)



During T_{on}, V-t area across $L = L(I_{max} - I_{min})$

$L(I_{max} - I_{min}) = (60 - 12)\, T_{on}$

$$\Delta I = \frac{48 \times 0.2 \times 10^{-3}}{20 \times 10^{-3}} = 0.48 \text{ A}$$

80. (a) Output voltage

$$V_o = \sum_{n=1}^{\infty} \frac{4V_S}{n\pi} \sin nd \sin n\omega t \sin \frac{n\pi}{2}$$

$$\alpha = 120°$$

$$\Rightarrow \quad 2d = 120°$$

$$d = 60°$$

RMS value of fundamental component

$$= \frac{4V_S}{\sqrt{2}\,\pi} \sin d$$

$$= \frac{4V_S}{\sqrt{2}\,\pi} \sin 60°$$

$$= 0.78 \text{ V}$$

81. (a)

82. (c) Given, $N_{S_1} = 3000$ rpm

$$N_1 = 2850 \text{ rpm}$$

$$S_{FL} = \frac{3000 - 2850}{3000} = 0.05$$

By V/f control,

$$N_{S_2} = 3000\left(\frac{40}{50}\right)$$

$$= 2400 \text{ rpm}$$

New running speed of motor $= 2400\left(1 - \frac{0.05}{2}\right)$

$$= 2340 \text{ rpm}$$

10

General Aptitude

GATE 2011 *(1 Mark Questions)*

▼ *Solution Points*

1. Choose the word from the options given below that is most nearly opposite in meaning to the given word.
Amalgamate
(a) Merge (b) Split (c) Collect (d) Separate

2. Which of the following options is the closest in the meaning to the word below?
Inexplicable
(a) Incomprehensible (b) Indelible
(c) Inextricable (d) Infallible

3. If $\log P = \left(\frac{1}{2}\right)\log Q = \left(\frac{1}{3}\right)\log R$, then which of the following options is true?
(a) $P^2 = Q^3R^2$ (b) $Q^2 = PR$ (c) $Q^2 = R^3P$ (d) $R = P^2Q^2$

4. Choose the most appropriate word(s) from the options given below to complete the following sentence.
I contemplated ..... Singapore for my vacation but decided against it.
(a) to visit (b) having to visit
(c) visiting (d) for a visit

5. Choose the most appropriate word from the options given below to complete the following sentence.
If you are trying to make a strong impression on your audience, you cannot do so by being understated, tentative or ..... .
(a) hyperbolic (b) restrained
(c) argumentative (d) indifferent

▼ *Solution Points*

6. Choose the most appropriate word from the options given below to complete the following sentence.
It was her view that the country's problems had been ..... by foreign technocrats, so that to invite them to come back would be counter-productive.
(a) identified (b) ascertained
(c) exacerbated (d) analysed

7. Choose the word from the options given below that is most nearly opposite in meaning to the given word.
Frequency
(a) Periodicity (b) Rarity (c) Gradualness (d) Persistency

8. Choose the most appropriate word from the options given below to complete the following sentence.
Under ethical guidelines recently adopted by the Indian Medical Association, human genes are to be manipulated only to correct diseases for which ..... treatments are unsatisfactory.
(a) similar (b) most (c) uncommon (d) available

9. The question below consists of a pair of related words followed by four pairs of words. Select the pair that best expresses the relation in the original pair.
Gladiator : Arena
(a) Cancer : Stage (b) Commuter : Train
(c) Teacher : Classroom (d) Lawyer : Courtroom

10. There are two candidates *P* and *Q* in an election. During the campaign, 40% of the voters promised to vote for *P*, and rest for *Q*. However, on the day of election 15% of the voters went back on their promise to vote for *P* and instead voted for *Q*. 25% of the voters went back on their promise to vote for *Q* and instead voted for *P*. Suppose *P* lost by 2 votes, then what was the total number of voters?
(a) 100 (b) 110 (c) 90 (d) 95

GATE 2010

11. Which of the following options is the closest in meaning to the word given below? Circuitous
(a) Cyclic (b) Indirect (c) Confusing (d) Crooked

12. The question below consists of a pair of related words followed by four pairs of words. Select the pair that best expresses the relation in the original pair.
Unemployed : Worker
(a) Fallow : Land (b) Unaware : Sleeper
(c) Wit : Jester (d) Renovated : House

13. Choose the most appropriate word from the options given below to complete the following sentence.
If we manage to ..... our natural resources, we would leave a better planet for our children.
(a) uphold (b) restrain (c) cherish (d) conserve

14. Choose the most appropriate word from the options given below to complete the following sentence.

▼ *Solution Points*

His rather casual remarks on politics ..... his lack of seriousness about the subject.
(a) masked (b) belied (c) betrayed (d) conserve

15. 25 persons are in a room, 15 of them play hockey, 17 of them play football and 10 of them play both hockey and football. Then, the number of persons playing neither hockey nor football is
(a) 2 (b) 17 (c) 13 (d) 3

16. Which of the following options is the closest in meaning to the word given below?
Mirth
(a) Anger (b) Merriment (c) Audacity (d) Blunder

17. Which of the following options is the closest in meaning to the word given below?
Ruminate
(a) Run fast (b) Reprimand (c) Think deeply (d) Spend lavishly

18. Which of the following options is the farthest in meaning to the word given below?
Confidant
(a) Turncoat (b) Arrogant (c) Confederate (d) Firm

19. Which of the following options is the farthest in meaning to the word given below?
Perfunctory
(a) Quick (b) Slow (c) Careful (d) Loud

20. Which of the following options is the closest in meaning to the word given below?
Prepossessing
(a) Economical (b) Pleasing (c) Selfish (d) Wise

Directions (Q. 21-25). Each of these questions consists of a pair of related words followed by four pairs of words. Select the pair which best expresses the relation in the original pair.

21. Day : Week
(a) Week : Year (b) Second : Time
(c) Time : Duration (d) Week : Month

22. Arc : Curve
(a) Triangle : Base (b) Rectangle : Square
(c) Revolution : Distance (d) Square : Polygon

23. Jackal : Dog
(a) Crow : Bat (b) Orange : Lemon
(c) Tiger : Wolf (d) Ant : Antepol

24. Error : Mistake
(a) Connection : Retaliation (b) Literature : Poetry
(c) Music : Art (d) Doubt : Suspicion

25. Hockey : Game
(a) King : Rule (b) Constitution : Assembly
(c) Book : Read (d) Latin : Language

▼ *Solution Points*

Directions (Q. 26-30). Choose the most appropriate word from the options given below to complete each sentence.

26. No man had a more ..... love for literature or a higher respect for it than Dr. Samuel Johnson.
(a) animated (b) adroit (c) ardent (d) arduous

27. My father was too ..... to open the heavy door.
(a) faint (b) feeble (c) timid (d) faltering

28. The government should provide attractive tax ..... to create the market for quality goods.
(a) revenues (b) structuret (c) incentives (d) controls

29. The Hubble Space Telescope will search for planets around other stars, a key to the ..... for extraterrestrial life, and examine interstellar dust and gases out of which stars are born.
(a) perception (b) discovery (c) enquiry (d) quest

30. He knew everything better than anybody else, and it was an effront to his ..... vanity that you should disagree with him.
(a) overstrung (b) overweening
(c) overwhelming (d) overwrought

31. A train travels from *A* to *B* at the rate of 60 km/h and from *B* to *A* at the rate of 40 km/h, then the average rate for the whole journey is
(a) 46 km/h (b) 48 km/h (c) 50 km/h (d) 52 km/h

32. A watch is offered for sale at ₹ 115 and if that price is reduced by 5%, the dealer who is selling it will still make $9\frac{1}{4}$% profit. How much did the watch cost him?
(a) ₹ 129.25 (b) ₹ 109 (c) ₹ 105 (d) ₹ 100

33. A company buys equal number of red pencils and green pencils. It uses $\frac{7}{8}$ of the red pencils and $\frac{5}{6}$ of the green pencils. Find what fraction of pencils bought remain unused?
(a) $\frac{7}{24}$ (b) $\frac{17}{24}$ (c) $\frac{13}{48}$ (d) $\frac{7}{48}$

34. If a man takes 2 h to row 7 km upstream or 15 km downstream, what is the speed of the current (in km/h)?
(a) 2 (b) 10.5 (c) 3.5 (d) 7.5

35. A reduction of 30% in the price of sugar enables a housewife to buy 6 kg more sugar for ₹ 20. What was the price per kg of sugar before the price reduction?
(a) ₹ $2\frac{1}{3}$ (b) ₹ $3\frac{1}{3}$ (c) ₹ $3\frac{1}{7}$ (d) ₹ $1\frac{3}{4}$

36. Printer *A* can complete 50 pages in 12 min. Printer *B* can print the same manuscript in 30 min. In how many minutes will it take for both printers to print a 100 page manuscript?
(a) $17\frac{1}{7}$ (b) $20\frac{1}{7}$ (c) 24 (d) 42

▼ *Solution Points*

37. The areas of two squares are in the ratio 3 : 1, what is the ratio for their perimeters?
(a) 3 : 1 (b) $1 : \sqrt{3}$
(c) $\sqrt{3} : 1$ (d) 1 : 3

38. The average weight of 24 students in a class is 40 kg. If the weight of the teacher is included, the weight is increased by 500 g. The weight of the teacher is
(a) 52 kg (b) 52.5 kg
(c) 69 kg (d) 53.3 kg

39. A man wishes to divide his monthly savings of ₹ 846 between his two sons and one daughter in the ratio $\frac{1}{4} : \frac{1}{5} : \frac{1}{3}$ respectively. How much did his daughter get?
(a) 270 (b) 216
(c) 360 (d) 300

40. In a fraction, the numerator 8 is less than the denominator. If 3 is added to the numerator, the fraction becomes $\frac{2}{3}$. Then, the fraction is
(a) $\frac{7}{15}$ (b) $\frac{2}{5}$
(c) $\frac{3}{11}$ (d) $\frac{7}{16}$

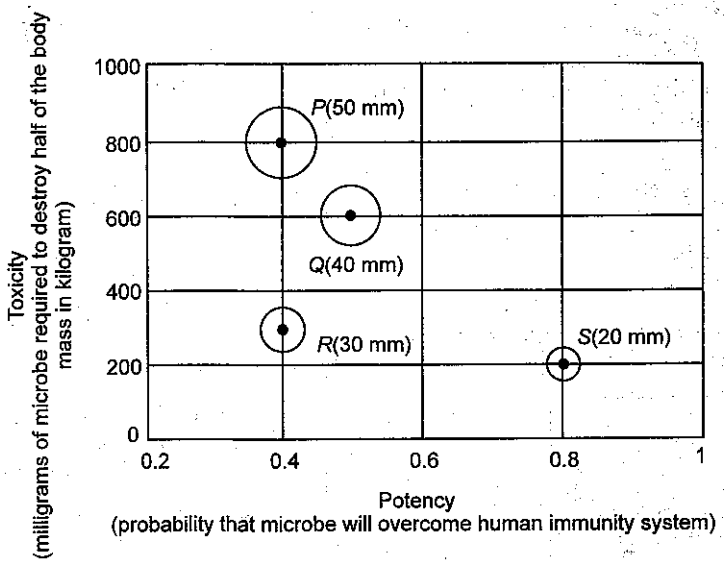
GATE 2011 *(2 Marks Questions)*

41. A container originally contains 10 L of pure spirit. From this container 1 L of spirit is replaced with 1 L of water. Subsequently, 1 L of the mixture is again replaced with 1 L of water and this process is repeated one more time. How much spirit is now left in the container?
(a) 7.58 L (b) 7.84 L (c) 7 L (d) 7.29 L

42. Few school curriculum include a unit on how to deal with bereavement and grief, and yet all students at some point in their lives suffer from losses through death and parting.
Based on the above passage which topic would not be included in a unit on bereavement?
(a) How to write a letter of condolence
(b) What emotional stages are passed through in the healing process
(c) What the leading causes of death are
(d) How to give support to a grieving friend

43. *P, Q, R* and *S* are four types of dangerous microbes recently found in a human habitat. The area of each circle with its diameter printed in brackets represents the growth of a single microbe surviving human immunity system within 24 h of entering the body. The danger to human beings varies proportionately with the toxicity, potency and growth attributed to a microbe shown in the figure ahead.

▼ *Solution Points*



A pharmaceutical company is contemplating the development of a vaccine against the most dangerous microbe. Which microbe should the company target in its first attempt?

(a) P (b) Q (c) R (d) S

44. The variable cost (V) of manufacturing a product varies according to the equation $V = 4q$, where q is the quantity produced. The fixed cost (F) of production of same product reduces with q according to the equation $F = 100/q$. How many units should be produced to minimize the total cost ($V + F$)?

(a) 5 (b) 4 (c) 7 (d) 6

45. A transporter receives the same number of orders each day. Currently, he has some pending orders (backlog) to be shipped. If he uses 7 trucks, then at the end of the 4th day he can clear all the orders. Alternatively, if he uses only 3 trucks, then all the orders are cleared at the end of the 10th day. What is the minimum number of trucks so that there will be no pending order at the end of the 5th day?

(a) 4 (b) 5 (c) 6 (d) 7

46. Three friends R, S and T shared toffees from a bowl. R took 1/3rd of the toffees, but returned four to the bowl. S took 1/4th of what was left but returned three toffees to the bowl. T took half of the remainder but returned two back into the bowl. If the bowl had 17 toffees left, how many toffees were originally there in the bowl?

(a) 38 (b) 31 (c) 48 (d) 41

47. Given that $f(y) = |y|/y$ and q is any non-zero real number, the value of $|f(q) - f(-q)|$ is

(a) 0 (b) –1 (c) 1 (d) 2

48. The sum of n terms of the series $4 + 44 + 444 + \ldots$ is

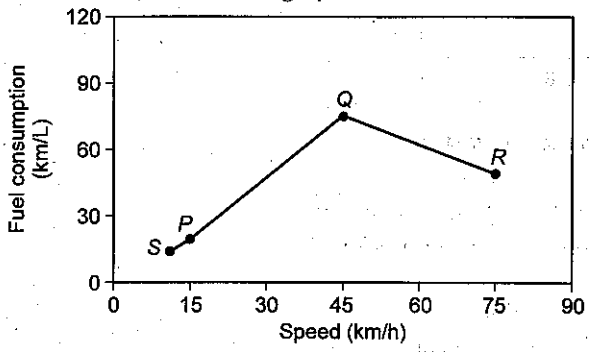
(a) $\left(\frac{4}{81}\right)(10^{n+1} - 9n - 1)$ (b) $\left(\frac{4}{81}\right)(10^{n-1} - 9n - 1)$

(c) $\left(\frac{4}{81}\right)(10^{n+1} - 9n - 10)$ (d) $\left(\frac{4}{81}\right)(10^{n} - 9n - 10)$

▼ *Solution Points*

49. The horse has played a little known but very important role in the field of medicine. Horses were injected with toxins of diseases until their blood built up immunities. Then a serum was made from their blood. Serums to fight with diphtheria and tetanus were developed this way.
It can be inferred from the passage, that horses were
(a) given immunity to diseases
(b) generally quite immune to diseases
(c) given medicines to fight toxins
(d) given diphtheria and tetanus serums

50. The fuel consumed by a motorcycle during a journey while travelling at various speeds is indicated in the graph below.



The distances covered during four laps of the journey are listed in the table below.

Lap	Distance (km)	Average speed (km/h)
P	15	15
Q	75	45
R	40	75
S	10	10

From the given data, we can conclude that the fuel consumed per kilometre was least during the lap
(a) *P* (b) *Q* (c) *R* (d) *S*

GATE 2010

51. Modern warfare has changed from larger scale clashes of armies to suppression, of civilian populations. Chemical agents that do their work silently appear to be suited to such warfare; and regretfully, there exist people in military establishment who think that chemical agents are useful tools for their cause.
Which of the following statements best sums up the meaning of the above passage?
(a) Modern warfare has resulted in civil strife
(b) Chemical agents are useful in modern warfare
(c) Use of chemical agents in warfare would be undesirable
(d) People in military establishments like to use chemical agents in war

▼ *Solution Points*

52. If 137 + 276 = 435, how much is 731+ 672?
(a) 534 (b) 1403 (c) 1623 (d) 1513

53. 5 skilled workers can build a wall in 20 days; 8 semi-skilled workers can build a wall in 25 days; 10 unskilled workers can build a wall in 30 days. If a team has 2 skilled, 6 semi-skilled and 5 unskilled workers, how long will it take to build the wall?
(a) 20 days (b) 18 days (c) 16 days (d) 15 days

54. Given digits 2, 2, 3, 3, 3, 4, 4, 4, 4, how many distinct 4 digit numbers greater than 3000 can be formed?
(a) 50 (b) 51 (c) 52 (d) 54

55. Hari (H), Gita (G), Irfan (I) and Saira (S) are siblings (*i.e.*, brothers and sisters). All were born on 1st January. The age difference between any two successive siblings (that is born one after another) is less than 3 yr. Given the following facts :
i. Hari's age + Gita's age > Irfan's age + Saira's age.
ii. The age difference between Gita and Saira is 1 yr. However, Gita is not the oldest and Saira is not the youngest.
iii. There are not twins.
In what order were they born (oldest first)?
(a) HSIG (b) SGHI (c) IGSH (d) IHSG

56. Instead of walking along the adjacent sides of a rectangular field, a boy takes a short cut along the diagonal of the field and saves a distance equal to half the longer side. The ratio of the shorter side to that of the longer side is
(a) 1 : 2 (b) 2 : 3 (c) 1 : 4 (d) 3 : 4

57. The proportion of milk and water in two samples is 5 : 2 and 7 : 5. If a mixture comprising of equal quantities of the two samples is made, the proportion of milk and water in the mixture is
(a) 12 : 7 (b) 7 : 12 (c) 109 : 59 (d) 59 : 109

58. An express train moving at a speed of 80 km/h, overtakes a goods train twice as long as the express train and moving in the same direction at a speed of 40 km/h, in 54 s. The time taken by express train to go through a station 400 m long is
(a) 27 s (b) 54 s
(c) 18 s (d) None of these

59. A child was asked to add first few natural numbers (that is, 1+ 2+ 3 + ...) so long his patience permitted. As he stopped, he gave the sum as 575. When the teacher declared the result wrong the child discovered he had missed one number in the sequence during addition. The number he missed was
(a) less than 10 (b) 10
(c) 15 (d) more than 15

60. A colony of bacteria in a container grows by each bacterium splitting into eight next generation bacteria. However, because of environmental conditions only 50% of the bacteria in a generation can split as above. A colony of first generation was put in container and it was found that the number of seventh generation bacteria was 4096 million. What was the size of the first generation population initially put in the container?
(a) 2 million (b) 8 million (c) 1 million (d) 4 million

▼ *Solution Points*

61. N persons stand on the circumference of a circle at distinct points. Each possible pair of persons, not standing next to each other, sings a two-minute song one pair after the other. If the total time taken for singing is 28 min, what is N?
(a) 5 (b) 7 (c) 9 (d) None of these

62. Ram has to travel from Hyderabad to Chennai which is a certain distance apart. 23% of the distance was travelled by bus, 50% of the remaining by train and rest of the distance 231 km by taxi. Find the distance between Hyderabad and Chennai in km.
(a) 600 (b) 462 (c) 231 (d) 856

63. A and B can do a piece of work in 45 days and 40 days respectively. They began to do the work together but A leaves after some days and then B completed the remaining work in 23 days. The number of days after which A left the work was how many days?
(a) 6 days (b) 9 days (c) 12 days (d) 15 days

64. A dishonest businessman professes to sell his articles at cost price but he uses false weight with which he cheats by 10% while buying and by 10% while selling. Find his profit percentage.
(a) 20% (b) 21% (c) 22.22% (d) 25%

65. In nuts and bolts factory, one machine produces only nuts at the rate of 100 nuts per minute and needs to be cleaned for 5 min after production of every 1000 nuts. Another machine produces only bolts at the rate of 75 bolts per minute and needs to be cleaned for 10 min after production of every 1500 bolts. If both the machines start production at the same time, what is the minimum duration required for producing 9000 pairs of nuts and bolts?
(a) 130 min (b) 135 min (c) 170 min (d) 180 min

66. Consider a 99 digit number created by writing side by side the first fifty four natural numbers as follows

1 2 3 4 5 6 7 8 9 10 11 12 13.....53 54

The above number when divided by 8 will leave a remainder of
(a) 6 (b) 4 (c) 2 (d) 0

67. The distance between two stations A and B is 600 km. One train leaves station A towards station B at the average speed of 54 km/h. After an hour another train left station B towards station A at the average speed of 66 km/h. The distance from station A where the two trains meet is?
(a) 272.7 km (b) 299.7 km (c) 333.3 km (d) 387.3 km

68. Amol was asked to calculate the arithmetic mean of ten positive integers each of which had two digits. By mistake, he interchanged the two digits say a and b, in one of these ten integers. As a result, his answer for the arithmetic mean was 1.8 more than what it should have been. Then $(b - a)$ equals.
(a) 1 (b) 2 (c) 3 (d) None of these

69. In a number system, the product of 44 and 11 is 1034. The number 3111 of this system, when converted to the decimal number system, becomes
(a) 406 (b) 1086 (c) 213 (d) 691

70. Three consecutive positive integers are raised to the first, second and third powers respectively and then added. The sum so obtained is a perfect square whose square root equals the total of the three original integers. Which of the following best describes the minimum, say m, of these three integers?
(a) $1 \le m \le 3$ (b) $4 \le m \le 6$ (c) $7 \le m \le 9$ (d) $10 \le m \le 12$

▼ *Solution Points*

71. Speed of a railway engine is 42 km/h when no compartment is attached and the reduction in speed is directly proportional to the square root of the number of compartments attached. If speed of the train carried by this engine is 24 km/h when 9 compartments are attached, then maximum number of compartments that can be carried by the engine is

(a) 49 (b) 48 (c) 46 (d) 47

72. There is a leak in the bottom of a cistern. When the cistern had no leak, it was filled in 2.5 h. It now takes half an hour longer. If the cistern is full of water, how long will it take in leaking itself empty, in case the water leaks out at double the rate after half the cistern becomes empty?

(a) 15 h (b) 11 h 15 min

(c) 11 h 25 min (d) 7.5 h

73. A certain basketball team that has played 2/3 of its games has a record of 17 wins and 3 losses. What is the greatest number of the remaining games that the team can lose and still win at least 3/4 of the total games played?

(a) 4 (b) 7 (c) 5 (d) 6

74. Two typists undertake to do a job. The second typist begins working one hour after the first. Three hours after the first typist has begun working there is still 9/20th of the work to be done. When the assignment is completed, it turns out that each typist has done half the work. How many hours would it take each one to do the whole job individually?

(a) 12, 8 h (b) 8, 5.6 h (c) 10, 8 h (d) 5, 4 h

75. A student took five papers in an examinations, where the full marks were the same for each paper. His marks in these papers were in the proportion of 6 : 7 : 8 : 9 : 10. In all papers together, the candidate obtained 60% of the total marks. Then, the number of papers in which he got more than 50% marks is

(a) 2 (b) 3 (c) 4 (d) 5

76. Company Alpha buys free-travel coupons from people who are awarded the coupons by Bravo Airlines for flying frequently on Bravo Airplanes. The coupons are sold to people who pay less for the coupons than they would pay by purchasing tickets from Bravo. This marketing of coupons results in lost revenue for Bravo.

To discourage the buying and selling of free-travel coupons, it would be best for Bravo Airlines to restrict the

(a) number of coupons that a person can be awarded in a particular year

(b) limiting the use of the coupons to those who were awarded the coupons and members of the immediate families

(c) days that the coupons can be used from Monday through Friday

(d) amount of time that the coupons can be used after they are issued

77. Opponents of laws that require automobile drivers and passengers to wear seat belts argue that in a free society people have the right to take risks as long as the people do not harm others as a result of taking the risks. As a result, they conclude that it should be each person's decision whether or not to wear a seat belt.

Which of the following, if true, most seriously weaken the conclusion drawn above?

(a) Many new cars are built with seat belts that automatically fasten when someone sits in the front seat

(b) Automobile insurance rates for all automobile owners are higher because of the need to pay for the increased injuries or deaths of people not wearing seat belts

▼ *Solution Points*

(c) Passengers in airplanes are required to wear seat belts during take-offs and landings

(d) The rate of automobile fatalities in states that do not have mandatory seat belt laws is greater than the rate of fatalities in states that do have such laws

Common Data for Questions 78 and 79

The average life expectancy for the United States population as a whole is 73.9 yr, but children born in Hawaii will live an average of 77 yr, and those born in Louisiana, 71.7 yr. If a newly wed couple from Louisiana were to begin their family in Hawaii, their children would live longer than if the couple began their family in Louisiana.

78. Which of the following, if true, would most significantly strengthen the conclusion drawn in the passage?

(a) 25% of all Louisianians who move to Hawaii live longer than 22 yr

(b) Over the last decade, average life expectancy has risen at a higher rate for Louisianians than for Hawaiians

(c) Environmental factors tending to favour longevity are abundant in Hawaii and less numerous in Louisiana

(d) As population density increases in Hawaii, life expectancy figures for the state are likely to be revised downward

79. Which of the following, if true, would most seriously weaken the conclusion drawn in the passage?

(a) Insurance company statisticians do not believe that moving to Hawaii will significantly lengthen the average Louisiana life

(b) The governor of Louisiana has falsely alleged that statistics of his state are inaccurate

(c) The longevity attributed to Hawaii's current population is attributed mostly to genetically determined factors

(d) 30% of all Louisianians can expect to live longer than 77 yr

80. Blood banks will shortly start to screen all donors for NANB hepatitis. Although the new screening tests are estimated to disqualify upto 5% of all prospective blood donors, they will still miss two-thirds of donors carrying NANB hepatitis. Therefore, about 10% of actual donors will still supply NANB contaminated blood.

The argument above depends on which of the following assumptions?

(a) Donors carrying NANB hepatitis do not, in a large percentage of cases, carry other infections for which reliable screening tests are routinely performed

(b) Donors carrying NANB hepatitis do not, in a large percentage of cases, develop the disease themselves at any point

(c) The number of donors who would be disqualified by tests for NANB hepatitis has been underestimated

(d) The incidence of NANB hepatitis is lower among the potential blood donors then it is in the population at large

Answers with Explanations

1. (b) Amalgamate means combine or unite to form an organization or structure when two companies or organizations merge and become finally one organization, process to becoming a one is amalgamation. So, the best option here is split. Separate on the other hand, although a close synonym, it is too general to be the best antonym in the given question while merge is the synonym, collect is not related.

2. (a) Inexplicable means not explicable, that cannot be explained, understood, or accounted for. So, the best synonym here is incomprehensible.

3. (b) $\log P = \frac{1}{2}\log Q = \frac{1}{3}\log R = k$

$\therefore \quad P = b^k, Q = b^{2k}, R = b^{3k}$

Now, $Q^2 = b^{4k} = b^{3k}\, b^k = PR$

4. (c) Contemplate is a transitive verb and hence is followed by a gerund. Hence, the correct usage of contemplate is verb + ing form.

5. (b) The tone of the sentence clearly indicates a word that is similar to understated is needed for the blank. Alternatively, the word should be antonym of strong (fail to make strong impression). Therefore, the best choice is restrained which means controlled/reserved/timid.

6. (c) **7.** (a) **8.** (b) **9.** (b)

10. (a) $P = 40$ votes

$Q = 60$ votes

P loses 15% of 40 votes, *i.e.*, 6 votes

and Q loses 25% of 60 votes, *i.e.*, 15 votes

So, finally P got $= 40 - 6 + 15 = 49$ votes

Q got $= 60 - 15 + 6 = 51$ votes

Difference $= 51 - 49 = 2$ votes

So, the total votes is 100.

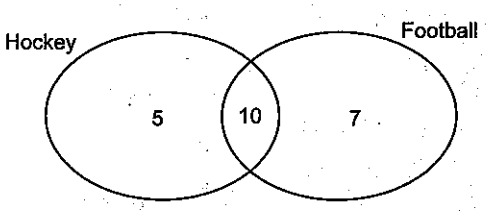
11. (b) Circuitous means round about or indirect.

12. (a) Relationship between the given word is that first word tells the negative quality of record.
So, option (a) is correct because fallow means uncultivated land which is a negative quality.

13. (d) Conserve is the most appropriate word because it means protect or safe.

14. (c) Betrayed is the most appropriate word because it means to tell identity unintentionally.

15. (d)



From the Venn-diagram, it is clear that 5 persons play only hockey.

7 persons play only football.

Total number of persons playing either game $= 5 + 10 + 7 = 22$.

So, number of persons playing neither hockey nor football

$= 25 - 22 = 3$

16. (b) Mirth means laughter or merriment.

17. (c) Ruminate means musing on a subject or think deeply.

18. (a) Confidant means a person trusted with knowledge on one's private affairs while turncoat means a person who changes sides in a conflict.

19. (c) Perfunctory means something done in a careless manner.

20. (b) Prepossessing means attractive or appealing or pleasing.

21. (d) Immediate upper unit of time after day is week. Similarly, immediate upper unit of time after week is month.

22. (d) An arc is a curve, similarly a square is a polygon.

23. (b) As jackal and dog belong to same class, similarly orange and lemon belong to same class.

24. (d) As error and mistake are synonyms to each other, similarly doubt and suspicion are synonyms to each other.

25. (d) Hockey is a game, similarly Latin is a language.

26. (c) Ardent means intense and it is used for love.

27. (b) The sense in this sentence is that due to physical weakness my father was unable to open the heavy door. So, feeble is the best word to complete the sentence.

28. (c) Incentives means rewads or concession. So, it is the best word to complete the sentence.

29. (d) For can be used only with quest which means search.

30. (b) Overweening means too much pride. So, it is the best word to complete the sentence.

31. (b) As the train travels equal distance on both sides.

So, average speed $= \frac{2\times 60\times 40}{60+40}$

$= \frac{4800}{100} = 48$ km/h

32. (d) Marked price of watch = ₹ 115

SP of watch $= 115\times 0.95 =$ ₹ 109.25

$\therefore$ CP of watch $= \frac{109.25}{109.25}\times 100 =$ ₹ 100

33. (a) Let company buys x number of each type of pencil

$\therefore$ Used pencils $= \frac{7}{8}x + \frac{5}{6}x$

$= \frac{21x+20x}{24} = \frac{41x}{24}$

$\therefore$ Unused pencils $= 2x - \frac{41x}{24}$

$= \frac{7}{24}x$

So, fraction of pencils remains unused $= \frac{7}{24}$

34. (a) Let the speed of man in still water be x and speed of current be y.

$\therefore$ Upstream speed $= x - y$

$\Rightarrow \quad x - y = \frac{7}{2}$ ...(i)

$\therefore$ Downstream speed $= x + y$

$\Rightarrow \quad x + y = \frac{15}{2}$ ...(ii)

On solving Eqs. (i) and (ii), we get

$x = 5.5$ km/h and $y = 2$ km/h

35. (d) Let the initial price be ₹ x per kg

$\therefore$ Initial quantity of sugar for ₹ 20 $= \frac{20}{x}$ kg

Now, the price of sugar $= 0.7x$ per kg

$\therefore$ New quantity of sugar for ₹ 20

$\frac{20}{0.7x} - \frac{20}{x} = 6$

$\Rightarrow \quad \frac{20-14}{0.7x} = 6$

$\Rightarrow \quad 6 = 4.2x$

$\Rightarrow \quad x = \frac{6}{4.2} = \frac{60}{42}$

$= 1\frac{3}{7}$ per kg

36. (a) Printer A can print 50 pages in 12 min.

So, printer A can print 100 pages in 24 min.

Printer B can print 50 pages in 30 min.

So, printer B can print 100 pages in 60 min.

So, both printers can print 100 pages in $\frac{24\times 60}{24+60}$ min

$= \frac{1440}{84}$ min

$= 17\frac{1}{7}$ min

37. (c) Ratio of areas of two squares = 3 : 1

$\therefore$ Ratio of sides of two squares $= \sqrt{3} : 1$

$\therefore$ Ratio of perimeters of two squares $= \sqrt{3} : 1$

38. (b) Average weight of 24 students = 40 kg

Total weight of 24 students = 960 kg

Average weight of 24 students and the teacher = 40.5 kg

Total weight of 24 students and the teacher

= 1012.5 kg

$\therefore$ Weigth of teacher $= 1012.5 - 960 = 52.5$

39. (c) Ratio of amount $= \frac{1}{4} : \frac{1}{5} : \frac{1}{3}$

$= 15 : 12 : 20$

So, amount of daughter $= \frac{20}{15+12+20}\times 846$

$= \frac{20}{47}\times 846 =$ ₹ 360

40. (a) Let the denominator be x.

So, the numerator $= x - 8$

Now, $\frac{x-8+3}{x} = \frac{2}{3}$

$\Rightarrow \quad 3x - 15 = 2x$

$\Rightarrow \quad x = 15$

$\therefore \quad$ Fraction $= \frac{15-8}{15} = \frac{7}{15}$

41. (d) $10\left(\frac{10-1}{10}\right)^3 = 10\left(\frac{9}{10}\right)^3 = 10\times\frac{729}{1000} = \frac{729}{100}$

$\therefore \quad \frac{729}{100}\times 1 = 7.29$ L

42. (c) The given passage clearly deals with how to deal with bereavement and grief. So, after the tragedy occurs and not about precautions. Therefore, irrespective of the causes of death, a school student rarely gets into details of causes—which is beyond the scope of the context. Rest all are important in dealing with grief.

43. (d) By observation of the table, we can say *S*.

	P	Q	R	S
Requirement	800	600	300	200
Potency	0.4	0.5	0.4	0.8

44. (a) Checking with all options in formula

$(4q + 100/q)$ *i.e.*, $(V + F)$.

Option (a) gives the minimum cost.

45. (c) Let each truck carries 100 units.

$2800 = 4n + e$ (n = normal)

$3000 = 10n + e$ (e = excess/pending)

$\therefore \quad n = \frac{100}{3}, \; e = \frac{8000}{3}$

$$5 \text{ days} \Rightarrow 500x = \frac{5.100}{3} + \frac{8000}{3}$$

$$x = \frac{8500}{3 \times 500} = 5.6 \text{ days}$$

So, $x > 5$ days

So, minimum possible days = 6

Hint : 7 trucks × 4 days × 100 units = 2800

46. (c) Let the total toffee is x.

R took 1/3, *i.e.*, $\frac{x}{3}$ but returned 4.

Finally R taken $= \left(\frac{x}{3} - 4\right)$

Remaining toffee $= x - \left(\frac{x}{3} - 4\right)$

$$= x - \frac{x}{3} + 4 = \frac{2x}{3} + 4$$

S took $\frac{1}{4}$, *i.e.*, $\left(\frac{2x}{3} + 4\right)\frac{1}{4}$ but returned 3.

So, finally S taken $= \left(\frac{2x}{3} + 4\right)\frac{1}{4} - 3 = \frac{x}{6} - 2$

Remainder $= \left(\frac{2x}{3} + 4\right) - \left(\frac{x}{6} - 2\right)$

$$= \frac{2x}{3} + 4 - \frac{x}{6} + 2 = \frac{3x}{6} + 6 = \frac{x}{2} + 6$$

T took half of the remainder, *i.e.*, $\left(\frac{x}{2} + 6\right)\frac{1}{2}$ but return 2 toffees.

So, T finally taken $= \left(\frac{x}{2} + 6\right)\frac{1}{2} - 2 = \frac{x}{4} + 1$

Remaining $= \left(\frac{x}{2} + 6\right) - \left(\frac{x}{4} + 1\right)$

$$= \frac{x}{2} + 6 - \frac{x}{4} - 1 = \frac{x}{4} + 5$$

and $\frac{x}{4} + 5 = 17$

$$x = (17 - 5) \times 4 = 48$$

47. (d) **48.** (c) **49.** (a) **50.** (b)

51. (d) From the statement of the passage, it is clear that people in military establishments think that chemical agents are useful tool for their cause.
It is not given in the passage that chemical agents are useful or undesirable in modern warfare.

52. (a) 137 + 276 = 435
If 137 and 276 are written in reverse order, then 731 and 672 are obtained.
So, 731 + 672 = reverse order of 435 = 534

53. (d) 5 skilled workers can build a wall in 20 days.
∴ 2 skilled workers can build the wall in

$$\frac{20 \times 5}{2} \text{ days} = 50 \text{ days}$$

8 semi-skilled workers can build a wall in 25 days.
∴ 6 semi-skilled workers can build the wall in

$$\frac{25 \times 8}{6} \text{ days} = \frac{100}{3} \text{ days}$$

10 unskilled workers can build a wall in 30 days.
∴ 5 unskilled workers can build the wall in 60 days.
One day work of a team of 2 skilled, 6 semi-skilled and 5 unskilled workers

$$= \frac{1}{50} + \frac{3}{100} + \frac{1}{60}$$

$$= \frac{6 + 9 + 5}{300} = \frac{20}{300} = \frac{1}{15}$$

So, this team can complete the work in 15 days.

54. (b) As the number should be greater than 3000.
So, thousand's place can be filled by 3 or 4, *i.e.*, 2 ways.
Hundred's place can be filled by 2, 3 or 4, *i.e.*, 3 ways.

Tenth place can be filled by 2, 3 or 4, *i.e.*, 3 ways.
First place can be filled by 2, 3 or 4, *i.e.*, 3 ways.
But when all 2's and 3's are used, *i.e.*, 333, 322, and 422 then there is a limitation for first place.
So, total three such types of numbers have limitation.
Total number of numbers greater than 3000
$$= 2\times 3\times 3\times 3 - 3$$
$$= 54 - 3 = 51$$

55. (b) The age difference between any two successive siblings should be at least 1 yr and at most 2 yr.
So, Gita and Saira are successive siblings.
If Gita is older than Saira,
Then, Hari's age + Gita's age > Irfan's age + (Gita's age – 1)
$\Rightarrow$ Hari's age > Irfan's age – 1
As there is at least 1 yr age difference.
So, Hari is older than Irfan.
If Gita is younger than Saira,
Then, Hari's age + Gita's age > Irfan's age + (Gita's age + 1)
$\Rightarrow$ Hari's age > Irfan's age + 1
So, Hari is older than Irfan in this case also.
As Gita is not the oldest.
So, correct order is SGHI.

56. (d) Let shorter side be x and longer side be y.
$$\therefore \quad \text{Length of diagonal} = \sqrt{x^2 + y^2}$$
$$(x + y) - \sqrt{x^2 + y^2} = \frac{1}{2}y$$
$$\Rightarrow \quad x + \frac{1}{2}y = \sqrt{x^2 + y^2}$$
$$\Rightarrow x^2 + \frac{1}{4}y^2 + 2\times\frac{1}{2}xy = x^2 + y^2$$
$$\Rightarrow \quad xy = \frac{3}{4}y^2$$
$$\Rightarrow \quad x = \frac{3}{4}y$$
$$\Rightarrow \quad \frac{x}{y} = \frac{3}{4}$$

57. (c) LCM of (5 + 2) and (7 + 5) = 84
Let 84 L of each sample is mixed.
$$\text{Quantity of milk in this mixture} = \frac{5}{7}\times 84 + \frac{7}{12}\times 84$$
$$= 109 \text{ L}$$
$$\text{Quantity of water in this mixture} = \frac{2}{7}\times 84 + \frac{5}{12}\times 84$$
$$= 59 \text{ L}$$
$\therefore$ Required ratio = 109 : 59

58. (a) Let the length of express train be x metre.
As both trains are running in opposite directions.
So, relative speed = 80 – 40 = 40 km/h
$$= 40\times\frac{5}{18} \text{ m/s}$$
$$\text{Distance} = \text{speed} \times \text{time}$$
$$x + 2x = 40\times\frac{5}{18}\times 54$$
$$\Rightarrow \quad 3x = 40\times 5\times 3$$
$$\Rightarrow \quad x = 200 \text{ m}$$
$$\therefore \quad \text{Required time} = \frac{200 + 400}{80\times\frac{5}{18}}$$
$$= \frac{600\times 18}{80\times 5} = 27 \text{ s}$$

59. (d) Let the child was asked to add first n natural numbers and the number missed was x.
$$\therefore \quad (1 + 2 + 3 + \ldots n) - x = 575$$
$$\Rightarrow \quad \frac{n(n+1)}{2} - x = 575$$
For $n = 33$,
$$\frac{n(n+1)}{2} = 561$$
For $n = 34$,
$$\frac{n(n+1)}{2} = 595$$
So, $$x = 595 - 575 = 20$$
So, x is more than 15.

60. (c) Bacteria are increasing by four times in next generation.
So, they are forming a GP with common ratio 4.
Let the size of first generation be a million.
$$\therefore \quad a(r)^{7-1} = 4096$$
$$\Rightarrow \quad a\times 4^6 = 4096$$
$$\Rightarrow \quad a = \frac{4096}{4096} = 1$$

61. (b) Each person can be paired with $(N - 3)$ persons. (*i.e.*, excluding himself and the adjacent two persons)
So, total number of pairs $= \frac{N(N-3)}{2}$
$$\text{Total time taken for ringing} = \frac{N(N-3)}{2}\times 2$$
$$\Rightarrow \quad N(N-3) = 28$$
$$\Rightarrow \quad N^2 - 3N - 28 = 0$$

$\Rightarrow \quad (N-7)(N+4)=0$

$\Rightarrow \quad N=7$ as $N\neq -4$

62. (a) Let the distance between Hyderabad and Chennai be x km.

$\therefore$ Distance travelled by bus $=0.23x$

Distance travelled by train $=\frac{1}{2}\times 0.77x$

$=0.385x$

$\therefore$ Remaining distance $=x-(0.23x+0.385x)$

$=0.385x$

$0.385x=231$

$\therefore \quad x=\frac{231}{0.385}=600$ km

63. (b) 1 day work of A and $B=\frac{1}{45}+\frac{1}{40}=\frac{17}{360}$

So, A and B together can complete the work in $\frac{360}{17}$ days.

23 days work of $B=\frac{23}{40}$

$\therefore$ Work done by A and $B=1-\frac{23}{40}=\frac{17}{40}$

So A and B can do $\frac{17}{40}$ part of work in $\frac{17}{40}\times\frac{360}{17}$ days $=9$ days

So, A left the work after 9 days.

64. (c) In place of 100 g weight, he uses 110g weight while buying and 90 g weight while selling.

So, he gained 20 g after selling 90 g.

$\therefore$ Profit percentage $=\frac{20}{90}\times 100=22.22\%$

65. (c) Time required to produce 1000 nuts $=10$ min

Now, machine needs to be cleaned for 5 min.

So, time required to produce 8000 nuts

$=8\times 15=120$ min

Time required to prodcue 900 nuts $=130$ min

Time required to produce 1500 bolts $=20$ min

Now, machine needs to be cleaned for 10 min.

So, time required to prodcue 7500 bolts $=150$ min

Time required to produce 900 bolts $=170$ min

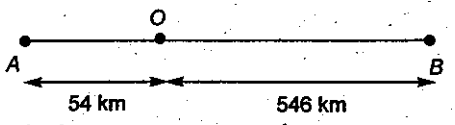
So, time required to produce 9000 pairs of nuts and bolts $=170$ min

66. (c) We know that, if a number formed from the last 3 digits of a number is divisible by 8, then the whole number is also divisible by 8.

So, required remainder

= Remainder when 354 is divided by $8=2$

67. (b)



In 1h, first train covers 54 km.

As trains are running in opposite directions.

So, relative speed $=54+66=120$ km/h

$\therefore$ Required time $=\frac{546}{120}=4.55$ h

Distance travelled by first train before the meeting of trains $=54+4.55\times 54=299.7$ km

68. (b) Arithmetic mean of 10 integers is increased by 1.8.

So, sum of these 10 integers in increased by 18.

$(10b+a)-(10a+b)=18$

$\Rightarrow \quad 9b-9a=18$

$\Rightarrow \quad b-a=2$

69. (a)

$$\begin{array}{r} 4\ 4 \\ \times\ 1\ 1 \\ \hline 4\ 4 \\ 4\ 4 \\ \hline 1\ 0\ 3\ 4 \end{array}$$

From the above calculation it is clear that the base of given number system is 5.

$(3111)_5=3\times 5^3+1\times 5^2+1\times 5^1+1\times 5^0$

$=375+25+5+1$

$=406$

70. (a) Three positive integers are $m,(m+1)$ and $(m+2)$

$m+(m+1)^2+(m+2)^3=(m+m+1+m+2)^2$

$\Rightarrow \quad m+m^2+1+2m+m^3+8+6m(m+2)=(3m+3)^2$

$\Rightarrow \quad m^3-2m^2-3m=0$

$\Rightarrow \quad m^2-2m-3=0$

$\Rightarrow \quad (m-3)(m+1)=0$

So, $\quad m=3$ as $m\neq -1$

71. (b) $S=42-k\sqrt{n}$

where, S is the speed of the engine.

k is the constant.

n is the number of compartment

$24=42-k\sqrt{9}$

$\Rightarrow \quad 3k=18$

$\Rightarrow \quad k=6$

$S=42-6\sqrt{n}$

For $n=49$, $S=0$

So, maximum number of compartments that can be carried by the engine is 48.

72. (c) Let leak can empty the cistern in x hour at normal rate.

$$\frac{1}{2.5}-\frac{1}{x}=\frac{1}{3}$$

$$\Rightarrow \quad \frac{1}{x}=\frac{1}{2.5}-\frac{1}{3}$$

$$\Rightarrow \quad x=15\text{ h}$$

Now, half the cistern becomes empty in 7.5 h after this rate of emptying doubles.

So, remaining half part of cistern empty in $\frac{7.5}{2}$ h.

$$\therefore \text{ Total time}=\left(7.5+\frac{7.5}{2}\right)\text{h}$$

$$=11.25\text{ h}=11\text{h }25\text{ min}$$

73. (a) $\frac{2}{3}$ of total games $=17+3$

$$\therefore \text{ Total number of games}=20\times\frac{3}{2}=30$$

$\frac{3}{4}$ of total games $=\frac{3}{4}\times 30=22.5$

So, team has to win at least 23 games.

So, team can lose at most 4 more games.

74. (c) Let first typist takes x hour to complete the job while second typist takes y hour to complete the job.

When first typist typed for 3 h, second typist typed for 2 h.

Total work was completed $=\frac{11}{20}$

$$\therefore \quad \frac{3}{x}+\frac{2}{y}=\frac{11}{20} \qquad \ldots(i)$$

Since, both typists had done half the work.

So, first typist typed for $\frac{x}{2}$ h while second typist typed for $\frac{y}{2}$ h.

But $$\frac{x}{2}-\frac{y}{2}=1 \qquad \ldots(ii)$$

On solving Eqs. (i) and (ii), we get

$$x=10\text{ h} \quad \text{and} \quad y=8\text{ h}$$

75. (c) Let the marks obtained in five papers be $6x, 7x, 8x, 9x$ and $10x$.

$$\frac{6x+7x+8x+9x+10x}{5}=60\% \text{ of total marks}$$

$$\Rightarrow \quad \frac{40x}{5}=60\% \text{ of total marks}$$

$$\Rightarrow \quad 8x=60\% \text{ of total marks}$$

$$\Rightarrow \quad x=7.5\% \text{ of total marks.}$$

So, except the subject in which he got $6x$ marks, in other 4 subjects he got more than 50% of marks.

76. (b) The loss for Bravo is due to the marketing of the coupons. These coupons are used by other persons in place of those who were awarded the coupons. So, the use of coupons should be limited to those who were awarded the coupons and their immediate family members.

77. (b) The conclusion of the paragraph is that, automobile drivers and passengers not wearing seat belts should be allowed to do it as we are living in a free society where people can take risks on themselves. But option (b) goes against it.

78. (c) It is given in the paragraph that life expectancy in Hawaii is more than that of in Louisiana. This may be due to some environmental factors tending to favour longevity are more in Hawaii and less in Louisiana.

79. (c) From the paragraph it can be concluded that if couples from other places began their family in Hawaii then their children will live longer. This is because in Hawaii geography and living conditions are much better than other places. But this is not because genetically determined factrors as given in option (c). If it is true then only those person have higher life expectancy who have their roots in Hawaii.

80. (a) From the paragraph it can be concluded that about 10% of actual donors will supply NANB contaminated blood even after the screening of all donors. It should be confirmed that these donors do not have any other infections for which reliable screening tests are routinely performed.

❐

GATE

Crack Papers (1-3)

GATE Exam

(Graduate Aptitude Test in Engineering)

Crack Paper-1 **(Fully Solved)**

1 Mark Questions

1. Find the z-transform of $a^n \cos n\theta$.

(a) $\dfrac{(z - a\cos\theta)}{z^2 - 2za\cos\theta}$ (b) $\dfrac{z(z - a)}{z^2 - 2za + a^2}$

(c) $\dfrac{z(z - a\cos\theta)}{z^2 - 2za\cos\theta + a^2}$ (d) None of these

2. Matrix $\begin{bmatrix} 8 & x & 0 \\ 4 & 0 & 2 \\ 12 & 6 & 0 \end{bmatrix}$ will become singular, if value of x is

(a) 4 (b) 6 (c) 8 (d) 12

3. The number of vertices of odd degree is always

(a) even number (b) odd number

(c) prime number (d) zero

4. There are 9 objects and 9 boxes. Out of 9, 5 objects cannot fit to be put in 3 small boxes. How many arrangements can be made, such that each object can be put in one box only?

(a) 17280 (b) 17820 (c) 17208 (d) 17028

5. What values of A, B, C and D satisfy the following simultaneous Boolean equations?

$$\overline{A} + AB = 0,\ AB = AC$$

$$AB + A\overline{C} + CD = \overline{CD}$$

(a) $A = 1,\ B = 0,\ C = 0,\ D = 1$

(b) $A = 1,\ B = 1,\ C = 0,\ D = 0$

(c) $A = 1,\ B = 0,\ C = 1,\ D = 1$

(d) $A = 1,\ B = 0,\ C = 0,\ D = 0$

6. The functionally complete set is

(a) $\{\overline{\ }, \wedge, \vee\}$ (b) $\{\downarrow, \wedge\}$ (c) $\{\uparrow\}$ (d) $\{\rightarrow, \wedge\}$

7. The function $C(t) = e^{at}u(t) * e^{bt}u(t)$ is

(a) $\dfrac{1}{a-b}(e^{at} - e^{bt})\,u(t)$ (b) $\dfrac{1}{a+b}(a^{at} - e^{bt})\,u(t)$

(c) $\dfrac{1}{a-b}(e^{-at} - e^{-bt})\,u(t)$ (d) $\dfrac{1}{a+b}(e^{-at} + e^{-bt})\,u(t)$

8. For a distortionless transmission,

1. amplitude response is constant.
2. phase response is linear with frequency.

Which of the following is/are correct?

(a) Only 1 (b) Only 2

(c) Both 1 and 2 (d) Neither 1 nor 2

9. Hilbert transform is known as

(a) $-\pi$ phase shifter (b) $-\dfrac{\pi}{2}$ phase shifter

(c) π phase shifter (d) 2π phase shifter

10. The VSWR can have any value between

(a) 0 and 1 (b) -1 and $+1$

(c) 0 and ∞ (d) 1 and ∞

11. If the closed-loop transfer function of a control system is given as $T(s) = \dfrac{(s-5)}{(s+2)(s+3)}$, then it is

(a) an unstable system

(b) an uncontrollable system

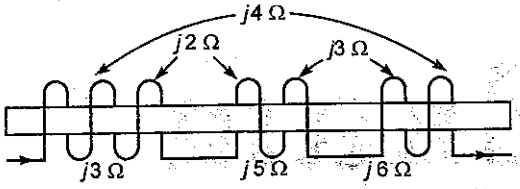
(c) a minimum phase system

(d) a non-minimum phase system

12. The open-loop DC gain of a unity negative feedback system with closed-loop transfer function $\dfrac{s+4}{s^2+7s+13}$ is

(a) $\dfrac{4}{13}$ (b) $\dfrac{4}{9}$ (c) 4 (d) 13

13. The equivalent inductive reactance of the circuit is



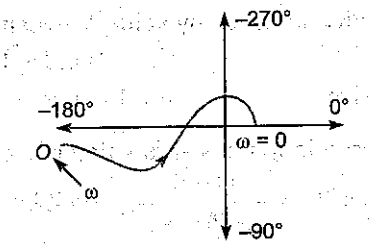
(a) $j15\Omega$ (b) $j20\Omega$ (c) $j10\Omega$ (d) $j12\Omega$

14. In series *R-L-C* circuit at resonance frequency, which of the following statements is correct?
(a) Voltage across capacity is greater than voltage across inductor
(b) Voltage across inductor is greater than voltage across capacitor
(c) Voltage across inductor is equal to voltage across capacitor
(d) Current is minimum

15. The Fourier transform of a function is equal to its two sided Laplace transform evaluated
(a) on the real axis of the *s*-plane
(b) on a line parallel to the real axis of the *s*-plane
(c) on the imaginary axis of *s*-plane
(d) on a line parallel to the imaginary axis of the *s*-plane

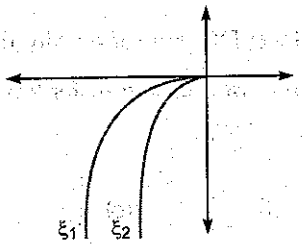
16. A 220 V three-phase, two pole, 50 Hz induction motor is running at a slip of 5%. Then speed of rotor in revolution per min, is
(a) 3000 rev/min (b) 150 rev/min
(c) 285 rev/min (d) 2850 rev/min

17. The polar plot is shown below. The type of system is



(a) type 0 (b) type 1 (c) type 2 (d) type 3

18. The polar plot of $G(j\omega)$ for different damping ratio for the system is shown below.



Which of the following is correct?
(a) $\xi_1 > \xi_2$ (b) $\xi_1 < \xi_2$
(c) $\xi_1 = \xi_2$ (d) Can't be determined

19. The root locus of the system $G(s)\,H(s) = \dfrac{K}{s(s+2)(s+3)}$ has the breakaway point located at
(a) (– 0.5, 0) (b) (– 2.548, 0)
(c) (– 4, 0) (d) (– 0.784, 0)

20. Convolution of $x(t+5)$ with impulse function $\delta(t-7)$ is equal to
(a) $x(t-12)$ (b) $x(t+12)$
(c) $x(t-2)$ (d) $x(t+2)$

21. Which of the following cannot be the Fourier series expansion of a periodic signal?
(a) $x(t) = 2\cos t + 3\cos 3t$
(b) $x(t) = 2\cos \pi t + 7\cos t$
(c) $x(t) = \cos t + \cos 3t$
(d) $x(t) = 2\cos 15\,\pi t + \sin 35\,\pi t$

22. The Laplace transform of continuous-time signal $x(t)$ is $X(s) = \dfrac{5-s}{s^2-s-2}$. If the Fourier transform of this signal exists, then $x(t)$ is
(a) $e^{2t}u(t) - 2e^{-t}u(t)$ (b) $-e^{2t}u(-t) + 2e^{-t}u(t)$
(c) $e^{2t}u(t) - 2e^{-t}u(t)$ (d) $e^{2t}u(-t) - 2e^{-t}u(t)$

2 Marks Questions

23. From a box containing 4 white and 6 black balls, 3 balls are transferred to another empty box. From new box a ball is drawn and it is black. What is the probability that out of 3 balls transferred 2 are white and 1 black?
(a) $\frac{1}{4}$ (b) $\frac{1}{5}$ (c) $\frac{1}{6}$ (d) $\frac{1}{7}$

24. If ϕ and ψ are functions satisfying Laplace's equation

$$s = \frac{\partial\phi}{\partial y} - \frac{\partial\psi}{\partial x} \quad \text{and} \quad t = \frac{\partial\phi}{\partial x} + \frac{\partial\psi}{\partial y}$$

Is $(s + it)$ analytic?
(a) Yes (b) No
(c) Insufficient data (d) None of these

25. The singular solution of $p = \tan(px - y)$ is
(a) $y = x\sqrt{\dfrac{1-x}{x}} - \tan^{-1}\sqrt{\dfrac{1-x}{x}}$
(b) $y = x\left[\sqrt{\dfrac{1-x}{x}} - \tan^{-1}\sqrt{\dfrac{1-x}{x}}\right]$
(c) $y = \sqrt{1-x} - \tan^{-1} x$
(d) $y = x^2\sqrt{1-x} - \tan^{-1}\sqrt{1-x}$

26. If $f(0)=2$ and $f(x)=\frac{1}{5-x^2}$, then lower and upper bound of $f(t)$ estimated by the mean value theorem are

(a) 1.9, 2.2 (b) 2.2, 2.25

(c) 2.25, 2.5 (d) None of these

27. Newton-Raphson iteration formula for finding $\sqrt[3]{C}$, where $C>0$, is

(a) $x_{n+1}=\frac{2x_n^3+\sqrt[3]{C}}{3x_n^2}$ (b) $x_{n+1}=\frac{2x_n^3-\sqrt[3]{C}}{3x_n^2}$

(c) $x_{n+1}=\frac{2x_n^3+C}{3x_n^2}$ (d) $x_{n+1}=\frac{2x_n^2-C}{3x_n^2}$

28. If on an average one ship of every ten is wrecked, find the probability that out of 5 ships expected to arrive, 4 at least will arrive safely.

(a) $\left(\frac{9}{10}\right)^4$ (b) $\frac{7}{5}\left(\frac{9}{10}\right)^4$

(c) $\frac{5}{7}\left(\frac{9}{10}\right)^4$ (d) None of these

29. Evaluate $f(x)$ satisfying the equation $\begin{vmatrix} f(x) & f'(x) \\ f'(x) & f''(x) \end{vmatrix}=0,\ f(0)=1, f'(0)=2.$

(a) $f(x)=e^{2x}$ (b) $f(x)=e^{x}$

(c) $f(x)=e^{-2x}$ (d) $f(x)=e^{x^2}$

30. The value of the improper integral $\int_0^1 x\log x\,dx$ is

(a) $\frac{1}{4}$ (b) 0 (c) $-\frac{1}{4}$ (d) 1

31. Consider an abrupt *p-n* junction. Let V_{bi} be the built-in potential of this junction and V_R be the applied reverse bias. If the junction capacitance (C_j) is 1pF for $V_{bi}+V_R=1$V, then for $V_{bi}+V_R=4$ V, C_j will be

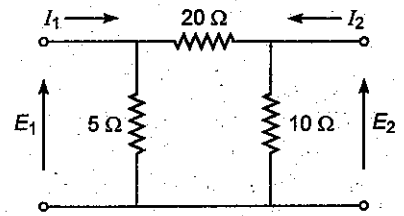
(a) 4 pF (b) 2 pF (c) 0.25 pF (d) 0.5 pF

32. A silicon sample *A* is doped with 10^{18} atom/cm^3 of boron. Another sample *B* of identical dimension is doped with 10^{18} atom/cm^3 phosphorus. The ratio of electron to hole mobility is 3. The ratio of conductivities of the sample *A* to *B* is

(a) 3 (b) $\frac{1}{3}$

(c) $\frac{2}{3}$ (d) $\frac{3}{2}$

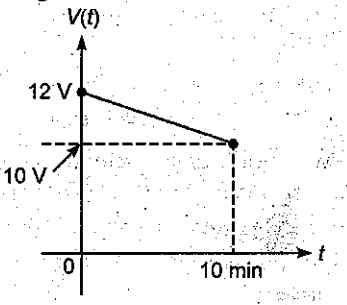
33. The admittance parameter Y_{12} in the 2-port network in figure is



(a) – 0.02 mho (b) 0.1 mho

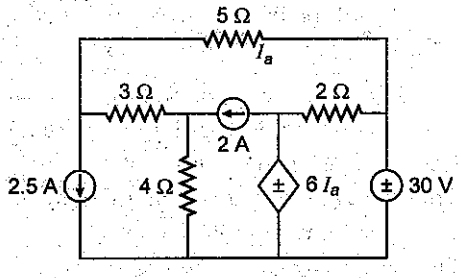
(c) – 0.05 mho (d) 0.05 mho

34. A fully charged mobile phone with a 12 V battery is good for a 10 min talk-time. Assume that, during the talk-time, the battery delivers a constant current 0.2 A and its voltage drops linearly from 12 V to 10 V as shown in figure. How much energy does the battery deliver during this talk-time?



(a) 220 J (b) 12 kJ (c) 13.2 kJ (d) 14.4 J

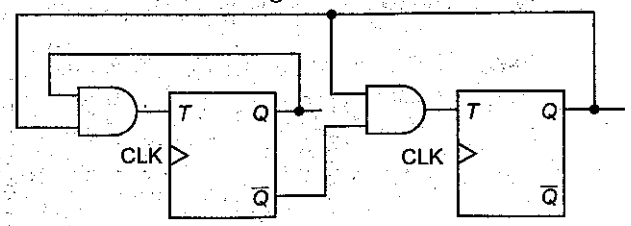
35. The power being dissipated in the 2 Ω resistor in the circuit of figure is



(a) 76.4 W (b) 305.6 W

(c) 52.5 W (d) 210.0 W

36. The circuit shown in figure



(a) is a MOD-2 counter

(b) is a MOD-3 counter

(c) generates sequence 00, 10, 01, 00 .....

(d) generates sequence 00, 10, 00, 10, 00 .....

37. A 4 bit modulo-6 ripple counter uses *J-K* flip-flop. If the propagation delay of each flip-flop is 50 ns, the maximum clock frequency that can be used is equal to
(a) 5 MHz (b) 10 MHz
(c) 4 MHz (d) 20 MHz

38. A battery has a short circuit current of 30 A and an open circuit voltage of 24 V. If the battery is connected to an electric bulb of resistance 2 Ω, the power dissipated by the bulb is
(a) 80 W (b) 1800 W (c) 112.5 W (d) 228 W

39. Which of the following meters does not exhibit square law response?
(a) Moving coil (b) Moving iron
(c) Electro dynamo meter (d) Hot wire instrument

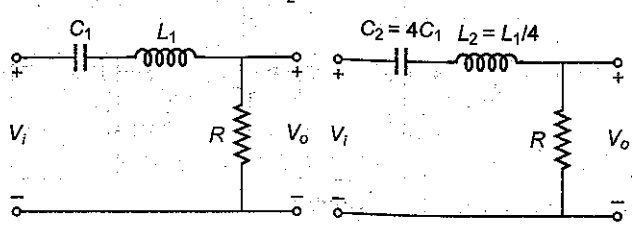
40. A certain oscilloscope with 4 cm × 4 cm screen has its own sweep output fed to its input. If the *X* and *Y* sensitivities are same, what will the oscilloscope display?
(a) Triangular wave (b) Straight line
(c) Sine wave (d) Circle

41. The sine wave output of a function generator is fed to both the horizontal (*X*) and vertical (*Y*) inputs of a CRO. What will be the pattern on the cathode ray screen?
(a) A circle
(b) An ellipse
(c) A straight line with 45° slope
(d) Sinusoidal

42. What is the reading of 0.5245 on 1V range in four and a half digit voltmeter displayed as?
(a) 0.5245 (b) 00.524 (c) 000.52 (d) 0000.5

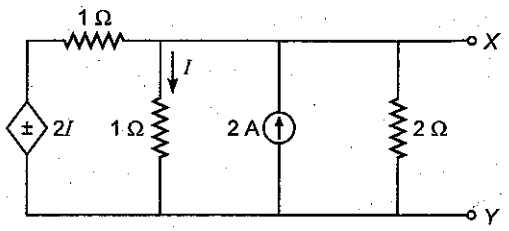
43. The working principle of a Pirani gauge pressure transducer is based on which one of the following?
(a) Humidity of the medium
(b) Thermal conductivity of the medium
(c) Combustibility of the medium
(d) Electrical resistivity of the medium

44. Two series resonant filters are as shown in the figure. Let the 3 dB bandwidth of filter 1 be B_1 and that of filter 2 be B_2. The value of $\frac{B_1}{B_2}$ is



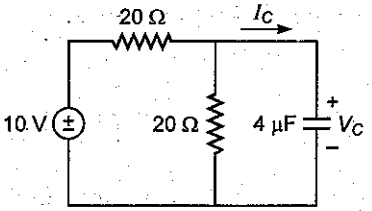
(a) 4 (b) 1 (c) $\frac{1}{2}$ (d) $\frac{1}{4}$

45. For the circuit shown in figure the Thevenin voltage and resistance looking into *X-Y* are



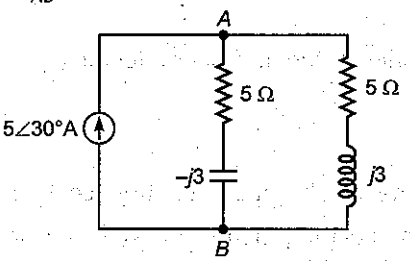
(a) $\frac{4}{3}$ V, 2 Ω (b) 4 V, $\frac{2}{3}$ Ω
(c) $\frac{4}{3}$ V, $\frac{2}{3}$ Ω (d) 4 V, 2 Ω

46. In the circuit shown V_C is 0 at $t=0$. For $t>0$, the capacitor current $I_C(t)$, where t is in second is given by



(a) 0.50 exp (– 25 *t*) mA
(b) 0.25 exp (– 25 *t*) mA
(c) 0.50 exp (– 12.5 *t*) mA
(d) 0.25 exp (– 6.25 *t*) mA

47. In the AC network shown in the figure, the phasor voltage V_{AB} (in volt) is



(a) 0 (b) 5∠30°
(c) 12.5∠30° (d) 17∠30°

48. A *p-n* junction has a built-in potential of 0.8 V. The depletion layer width at a reverse bias of 1.2 V is 2 μm. For a reverse bias of 7.2 V, the depletion layer width will be
(a) 4 μm (b) 4.9 μm
(c) 8 μm (d) 12 μm

49. The DC current gain (β) of a BJT is 50. Assuming that the emitter injection efficiency is 0.995, the base transport factor is
(a) 0.980 (b) 0.985 (c) 0.990 (d) 0.995

50. Consider the sequence

$$x[n] = [-4 - 5j,\ 1 + 2j,\ 4]$$

The conjugate anti-symmetric part of the sequence is

(a) $[-4 - j\,2.5,\ j2,\ 4 - j2.5]$
(b) $[-j\,2.5, 1, j2.5]$
(c) $[-j2.5, j\,20]$
(d) $[-4, 1, 4]$

51. A causal LTI system is described by the difference equation

$$2y[n] = \alpha y[n-2] - 2x[n] + \beta x[n-1]$$

The system is stable only, if

(a) $|\alpha| = 2,\ |\beta| < 2$
(b) $|\alpha| > 2,\ |\beta| > 2$
(c) $|\alpha| < 2$, any value of β
(d) $|\beta| < 2$, any value of α

52. The state variable equations of a system are

$$x_1 = -3x_1 - x_2 = u,\ x_2 = 2x_1 \text{ and } y = x_1 + u.$$

The system is

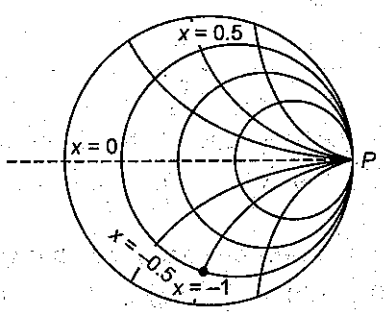
(a) controllable but not observable
(b) observable but not controllable
(c) neither controllable nor observable
(d) controllable and observable

53. Given $A = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$, the state transition matrix e^{At} is given by

(a) $\begin{bmatrix} 0 & e^{-1} \\ e^{-t} & 0 \end{bmatrix}$ (b) $\begin{bmatrix} e^{t} & 0 \\ 0 & e^{t} \end{bmatrix}$

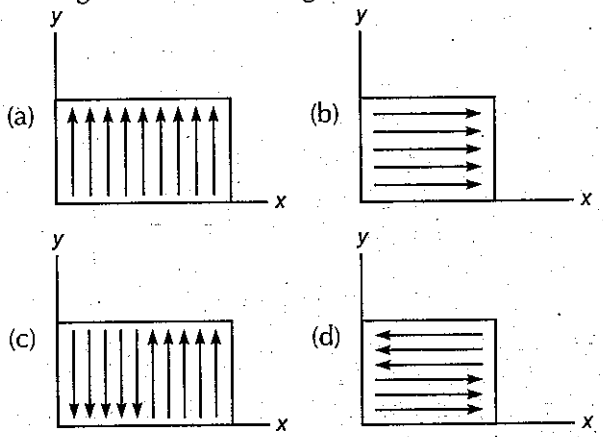
(c) $\begin{bmatrix} e^{-t} & 0 \\ 0 & e^{-t} \end{bmatrix}$ (d) $\begin{bmatrix} 0 & e^{t} \\ e^{t} & 0 \end{bmatrix}$

54. Consider an impedance $Z = R + jX$ marked with point P in an impedance smith chart as shown in figure. The movement from point P along a constant resistance circle in the clockwise direction by an angle 45° is equivalent to

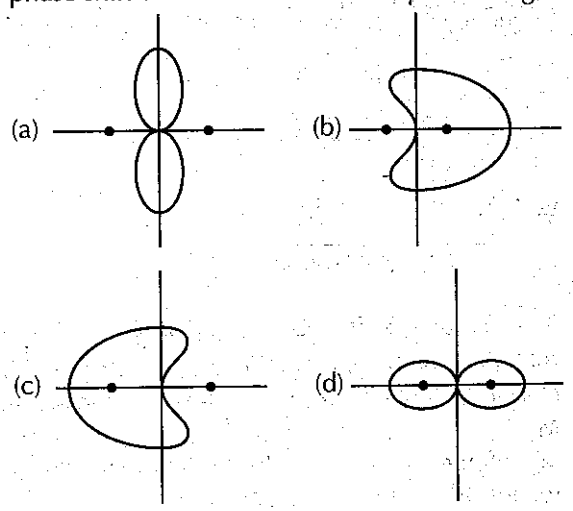


(a) adding an inductance in series with Z
(b) adding a capacitance in series with Z
(c) adding an inductance in shunt across Z
(d) adding a capacitance in shunt across Z

55. Which one of the following does represent the electric field lines for the mode in the cross-section of a hollow rectangular metallic waveguide?



56. Two identical and parallel dipole antennas are kept apart by a distance of $\frac{\lambda}{4}$ in the H-plane. They are fed with equal currents but the right most antenna has a phase shift of + 90°. The radiation pattern is given as



57. Characteristic impedance of a transmission line is 50 Ω. Input impedance of the open-circuited line when the transmission line a short circuited, then value of the input impedance will be

(a) 50 Ω (b) $100 + j\,150\,\Omega$
(c) $7.69 + j\,11.54\,\Omega$ (d) $7.69 - j\,11.54\,\Omega$

58. During transmission over a certain binary communication channel, bit errors occur independently with probability p. The probability of at most one bit in error in a block of n bits is given by

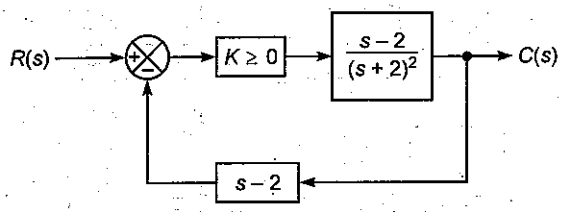
(a) p^n
(b) $1 - p^n$
(c) $np(1-p)^{n-1} + (1-p)^n$
(d) $1 - (1-p)^n$

59. The open-loop DC gain of a unity negative feedback system with closed-loop transfer function $\frac{s+4}{s^2+7s+13}$ is

(a) $\frac{4}{13}$ (b) $\frac{4}{9}$

(c) 4 (d) 13

60. The feedback control system in the figure is stable



(a) for all $K \geq 0$ (b) only, if $K \geq 0$

(c) only, if $0 \leq K < 1$ (d) only, if $0 \leq K \leq 1$

61. Consider two transfer functions $G_1(s) = \frac{1}{s^2 + as + b}$ and $G_2(s) = \frac{s}{s^2 + as + b}$. The 3 dB bandwidth of their frequency responses are, respectively

(a) $\sqrt{a^2 - 4b}, \sqrt{a^2 + 4b}$

(b) $\sqrt{a^2 + 4b}, \sqrt{a^2 - 4b}$

(c) $\sqrt{a^2 - 4b}, \sqrt{a^2 - 4b}$

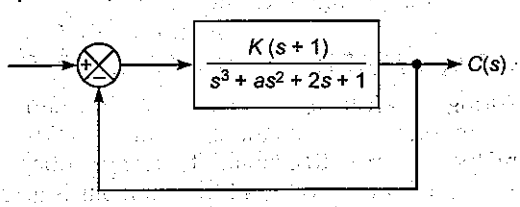
(d) $\sqrt{a^2 + 4b}, \sqrt{a^2 + 4b}$

62. The Nyquist plot of $G(j\omega)$, $H(j\omega)$ for a closed-loop control system, passes through $(-1, j0)$ point in the GH-plane. The gain margin of the system in dB is equal to

(a) infinite (b) greater than zero

(c) less than zero (d) zero

63. The positive values of K and a so that the system shown in figure below oscillates at a frequency of 2 rad/s respectively are

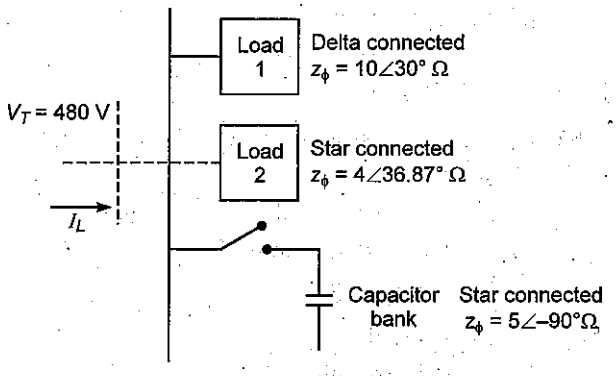


(a) 1, 0.75 (b) 2, 0.75

(c) 1, 1 (d) 2, 2

Statements for Linked Answer Questions 64 and 65

Figure shows a small 480 V distribution system. Assume that the wires in the system have zero impedance.



64. If the switch is opened, the total current supplied to the distribution system by the utility is

(a) 15.2 A (b) 130.4 A

(c) 152 A (d) 13.04 A

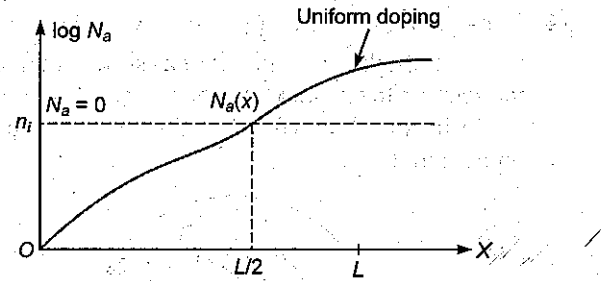
65. If the switch is closed, the total current supplied by utility

(a) increases (b) decreases

(c) remains same (d) Can't be determined

Statements for Linked Answer Questions 66 and 67

A bar of silicon is doped with acceptor as shown below. The doping density varies smoothly and monotonically in the X-direction from $N_A << n_i$ at $x = 0$ to $N_A >> n_i$ at $x = L$. $N_a\left(\frac{L}{2}\right) = n_i$. Around $x = L$, the acceptor profile becomes uniform. The donor density is zero everywhere. This is a thermal equilibrium situation.



66. Where is the hole concentration is greatest?

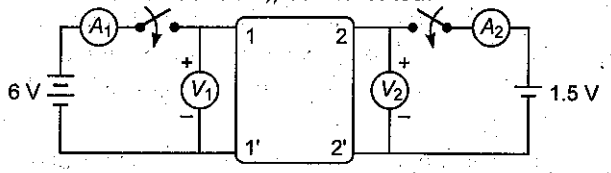
(a) $x = 0$ (b) $x = \frac{L}{2}$ (c) $x = L$ (d) Uniform

67. Where is the electron concentration is greatest?

(a) $x = 0$ (b) $x = \frac{L}{2}$ (c) $x = L$ (d) Uniform

Statements for Linked Answer Questions 68 and 69

A two-port network shown below is excited by external DC source. The voltage and the current are measured with voltmeters V_1, V_2 and ammeters A_1, A_2 (all assumed to be ideal), as indicated.



Under following conditions, the readings obtained are

(i) S_1-open, S_2-closed, $A_1 = 0$, $V_1 = 4.5$ V, $V_2 = 15$ V, $A_2 = 1$A

(ii) S_1-open, S_2-closed, $A_1 = 4$ A, $V_1 = 6$ V, $V_2 = 6$ V, $A_2 = 0$

68. The Z-parameter matrix for this network is

(a) $\begin{bmatrix} 1.5 & 15 \\ 45 & 15 \end{bmatrix}$ (b) $\begin{bmatrix} 15 & 45 \\ 15 & 45 \end{bmatrix}$ (c) $\begin{bmatrix} 15 & 45 \\ 15 & 15 \end{bmatrix}$ (d) $\begin{bmatrix} 45 & 15 \\ 15 & 45 \end{bmatrix}$

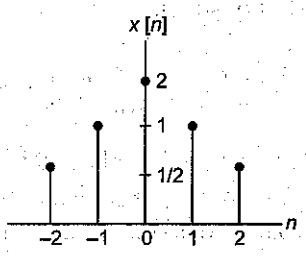
69. The h-parameter matrix for this network is

(a) $\begin{bmatrix} -3 & 3 \\ -1 & 0.67 \end{bmatrix}$ (b) $\begin{bmatrix} -3 & -1 \\ 3 & 0.67 \end{bmatrix}$

(c) $\begin{bmatrix} 3 & 3 \\ 1 & 0.67 \end{bmatrix}$ (d) $\begin{bmatrix} 3 & 1 \\ -3 & -0.67 \end{bmatrix}$

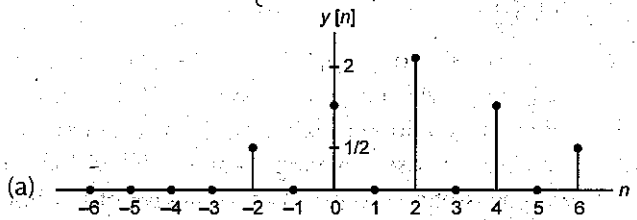
Statements for Linked Answer Questions 70 and 71

A sequence $x(n)$ has non-zero values as shown in the figure.

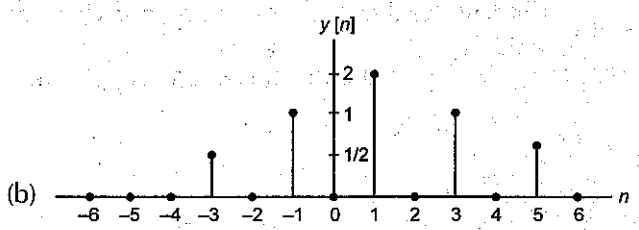


70. The sequence $y(n) = \begin{cases} x\left(\frac{n}{2} - 1\right), & \text{for } n \text{ even} \\ 0, & \text{for } n \text{ odd} \end{cases}$ will be

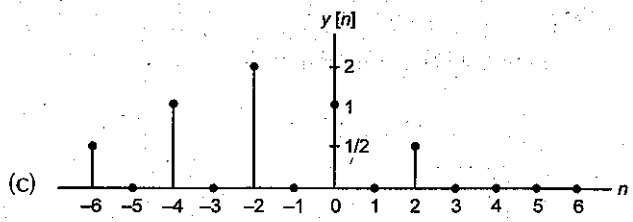
(a)



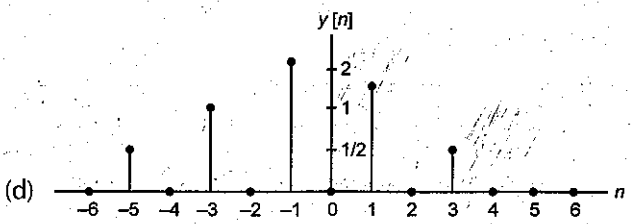
(b)



(c)



(d)



71. The Fourier transform of $y(2n)$ will be

(a) $e^{-j2\omega}[\cos 4\omega + 2\cos 2\omega + 2]$

(b) $\cos 2\omega + 2\cos \omega + 2$

(c) $e^{-j\omega}[\cos 2\omega + 2\cos \omega + 2]$

(d) $e^{-j2\omega}[\cos 2\omega + 2\cos \omega + 2]$

Statements for Linked Answer Questions 72 and 73

Consider a linear system whose state space representation is $\dot{x}(t) = Ax(t)$. If the initial state vector of the system is $x(0) = \begin{bmatrix} 1 \\ -2 \end{bmatrix}$, then the system response is $x(t) = \begin{bmatrix} e^{-2x} \\ -2e^{-2t} \end{bmatrix}$. If the initial state vector of the system changes to $x(0) = \begin{bmatrix} 1 \\ -2 \end{bmatrix}$, then the system response becomes $x(t) = \begin{bmatrix} e^{-t} \\ -e^{-t} \end{bmatrix}$.

72. The eigen value and eigen vector pairs (λ_i, V_i) for the system are

(a) $\left(-1, \begin{bmatrix} 1 \\ -1 \end{bmatrix}\right)$ and $\left(-2, \begin{bmatrix} 1 \\ -2 \end{bmatrix}\right)$

(b) $\left(-1, \begin{bmatrix} 1 \\ -1 \end{bmatrix}\right)$ and $\left(2, \begin{bmatrix} 1 \\ -2 \end{bmatrix}\right)$

(c) $\left(-1, \begin{bmatrix} 1 \\ -1 \end{bmatrix}\right)$ and $\left(-2, \begin{bmatrix} 1 \\ -2 \end{bmatrix}\right)$

(d) $\left(-2, \begin{bmatrix} 1 \\ -1 \end{bmatrix}\right)$ and $\left(1, \begin{bmatrix} 1 \\ -2 \end{bmatrix}\right)$

73. The system matrix is

(a) $\begin{bmatrix} 0 & 1 \\ -1 & 1 \end{bmatrix}$ (b) $\begin{bmatrix} 1 & 1 \\ -1 & -2 \end{bmatrix}$

(c) $\begin{bmatrix} 2 & 1 \\ -1 & -1 \end{bmatrix}$ (d) $\begin{bmatrix} 0 & 1 \\ -2 & -3 \end{bmatrix}$

74. The open-loop function of a unity-gain feedback control system is given by

$$G(s) = \frac{K}{(s+1)(s+2)}$$

The gain margin of the system in dB is given by
(a) 0 (b) 1 (c) 20 (d) ∞

General Aptitude

75. Which of the following options is closest in meaning to the word given below?
Pinnacle
(a) Foothills (b) Pass (c) Peak (d) Excess

76. The question below consists of a pair of related words, followed by four pairs of words. Select the pair which best expresses the relation in the original pair.
Water : Oxygen
(a) Helium : Nitrogen (b) Salt : Sodium
(c) Tree : Plant (d) Food : Hunger

77. Choose the most appropriate word from the options given below to complete the following sentence.
Comparatively few rock musicians are willing to laugh at themselves, although a hint of ..... can boost sales of video clips very nicely.
(a) self-deprecation (b) congeniality
(c) cynicism (d) embarrassment

78. In a direct election between two contestants, 4% of the votes cost are declared to be illegal. One contestant secures 55% of the valid votes and wins with a majority of 240 votes. What is the total number of votes cost?
(a) 2455 (b) 2400
(c) 2500 (d) None of these

79. There are two groups *A* and *B* consisting of 30 and 70 students respectively. If the average weight of group *A* is 30 kg and that of group *B* is 70 kg. Find the average weight of all the students of group *A* and group *B*.
(a) 58 kg (b) 50 kg (c) 40 kg (d) 12 kg

80. A program instituted in a particular state allows parents to prepay their children's future college tuition fee at current rates. The program, then pays the tuition fee annually for the child at any of the state's public colleges in which the child enrolls. Parents should participate in the program as a means of decreasing the cost of their children's college education.
Which of the following, if true, is the most appropriate reason for parents not to participate in the program?
(a) The parents are unsure about which public college in the state, the child will attend
(b) The amount of money accumulated, by putting the prepayment funds in an interest bearing account today will be greater than the total cost of tuition for any of the public colleges when the child enrolls
(c) The annual cost of tuition at the state's public colleges is expected to increase at a faster rate than the annual increase in the cost of living
(d) Some of the state's public colleges are contemplating large increases in tuition next year

81. The value of a silver coin is directly proportional to the $(\text{diameter})^2$ when the thickness is same and is directly proportional to thickness when the diameter is same. If two silver coins have their diameters in the ratio of 4 : 3, then what is the ratio of thicknesses of the two silver coins, if value of the 1st coin is 4 times that of the 2nd coin?
(a) 4 : 9 (b) 9 : 4 (c) 9 : 16 (d) 16 : 9

82. A man can row 15 km upstream and 22 km downstream in 5 h. Also, he can row 20 km upstream and 27.5 km downstream in 6½ h. Find the rate of current and the speed of the man in still water respectively?
(a) 8 km/h, 3 km/h (b) 3 km/h, 8 km/h
(c) 5 km/h, 11 km/h (d) 11 km/h, 5 km/h

83. *A* supplies 20 men who work for 8 h a day for 6 days. *B* supplies 15 men working at 9 h a day for 7 days and C supplies 10 men working 6 h a day for 8 days to do a certain job. If ₹ 636 is paid for all the labours, what is *C*'s share?
(a) ₹ 102 (b) ₹ 128
(c) ₹ 136 (d) ₹ 144

84. In a certain township, 1/5 of the housing units are equipped with cable television, 1/10 of the housing units are equipped with videocassette recorders and 1/3 of those having cable television have videocassette recorder. What fraction of the housing units have neither cable television nor videocassette recorder?
(a) 7/10 (b) 23/30 (c) 11/15 (d) 1/6

Answers with Explanations

1. (c) Since, $z(\cos n\theta) = \dfrac{z(z-\cos\theta)}{z^2 - 2z\cos\theta + 1} = \overline{u}(z)$

$\therefore$ By change of scale property, we have

$$z(a^n \cos n\theta) = \overline{u}\left(\frac{z}{a}\right)$$

$$= \frac{\frac{z}{a}\left(\frac{z}{a} - \cos\theta\right)}{\frac{z^2}{a^2} - 2\frac{z}{a}\cos\theta + 1}$$

$$= \frac{z(z - a\cos\theta)}{z^2 - 2za\cos\theta + a^2}$$

2. (a) The given matrix will become singular, if

$$\begin{vmatrix} 8 & x & 0 \\ 4 & 0 & 2 \\ 12 & 6 & 0 \end{vmatrix} = 0$$

$$\Rightarrow \quad 8(-12) - x(-24) = 0$$

$$\Rightarrow \quad -96 + 24x = 0$$

$$\Rightarrow \quad x = 4$$

3. (a) We know that the graph of even number of vertices always have even degree.

i.e., $\sum_{v \in V_{even}} d(v) = \text{even (integer)}$ ...(i)

Also know that,

Sum of all degrees is twice the number of edges,

$\Rightarrow \quad \Sigma d(v) = 2e$ ($\because e$ number of edges)

$\Rightarrow \quad \sum_{v \in V_{odd}} d(v) + \sum_{v \in V_{even}} d(v) = 2e$

$\Rightarrow \quad \sum_{v \in V_{odd}} d(v) = 2e$ (even integer) $- \sum_{v \in V_{even}} d(v)$ (even integer)

$\Rightarrow \quad \sum_{v \in V_{odd}} d(v) =$ even integer

($\because$ as $x - y$ is even when $x, y \in$ even)

Hence, G has even number of odd degree vertices.

4. (a) Since, 5 objects cannot fit to be put 3 small boxes, these 5 objects can be put in the remaining 6 boxes in ${}^6P_5$ ways.

The remaining 4 objects can be put in ${}^4P_4$ ways.

$\therefore$ Total number of arrangements $= {}^6P_5 \times {}^4P_4$

$= (6 \times 5 \times 4 \times 3 \times 2) \times (4 \times 3 \times 2 \times 1)$

$= 720 \times 24 = 17280$

5. (d) Given, $\overline{A} + AB = 0$ and $AB = AC$

Let $A = 1$ and $B = 0$

Then, $\overline{A} + BA = \overline{1} + 1 \cdot 0$

$= 0 + 0 = 0$

And also, $AB = 1 \cdot 0 = 0$

$AC = 1 \cdot 0 = 0$, then $C = 0$

We have, also given

$AB + A\overline{C} + CD = 1 \cdot 0 + 1 \cdot \overline{0} + 0 \cdot D$

$= 0 + 1 \cdot 1 + 0$

$= 0 + 1 + 0 = 1$

So, $\overline{CD}$ should be 1.

For this, let $D = 0$

$\Rightarrow \quad \overline{CD} = \overline{0 \cdot 0}$

$= \overline{0} = 1$

Hence, $A = 1, B = 0, C = 0, D = 0$

6. (c) We know that,

$\{\wedge, \neg\}$ and $\{\vee, \neg\}$ are functionally complete.

For NAND ($\uparrow$)

$$P \uparrow Q \equiv 7(P \wedge Q)$$

$$7P = P \uparrow P$$

$$P \vee Q = (P \uparrow P) \uparrow (Q \uparrow Q)$$

$$P \vee Q = (P \uparrow Q) \uparrow (P \uparrow Q)$$

So $\{\uparrow\}$ are functionally complete.

7. (a) Given, $C(t) = e^{at}u(t) * e^{bt}u(t)$

Taking Laplace transform,

$$C(s) = \frac{1}{(s-a)} \cdot \frac{1}{(s-b)}$$

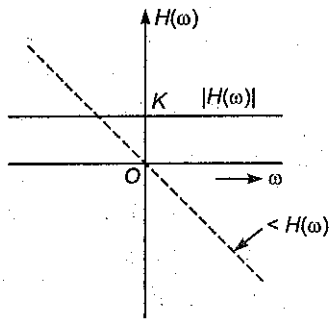
[From time convolution property]

$$C(s) = \frac{1}{a-b}\left[\frac{1}{(s-a)} - \frac{1}{(s-b)}\right]$$

Taking inverse Laplace transform,

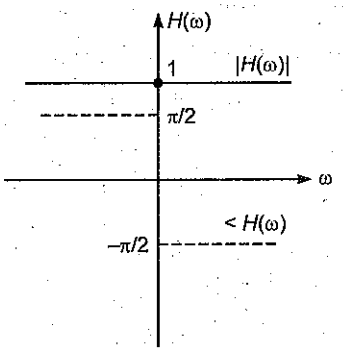
$$C(t) = \frac{1}{a-b}[e^{at} - e^{bt}]u(t)$$

8. (c)



Frequency response of distortionless transmission.

9. (b)



Transfer function of Hilbert transform, *i.e.*, every frequency component is shifted by $-\frac{\pi}{2}$.

10. (d) The VSWR

$$s=\frac{1+\tau}{1-\tau}$$

where τ is reflection coefficient value between 0 and 1.
Hence, s varies from 1 to ∞.

11. (d) If closed-loop transfer function has any zero in the right half of s-plane, then the system is non-minimum phase system.

12. (b) We know that for unity feedback system, closed transfer fucntion

$$T(s)=\frac{G(s)}{1+G(s)}=\frac{s+4}{s^2+7s+13}$$

$$\frac{1+G(s)}{G(s)}=\frac{s^2+7s+13}{s+4}$$

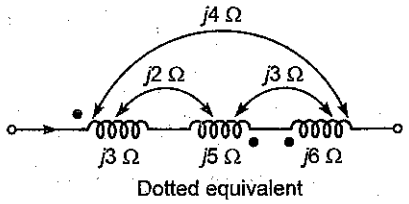
$$\frac{1}{G(s)}=\frac{s^2+7s+13}{s+4}-1=\frac{s^2+6s+9}{s+4}$$

$$G(s)=\frac{s+4}{s^2+6s+9}$$

For DC gain; $s=0$, thus

$$G(0)=\frac{4}{9}$$

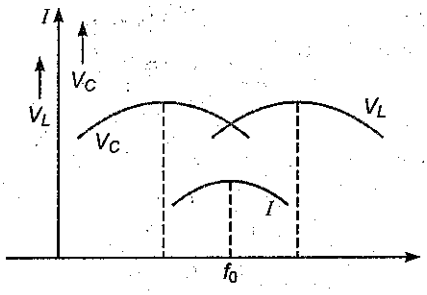
13. (d)



Hence, the equivalent inductive reactance

$$Z=j(3+5+6)-j2-j3+j4$$
$$=j12\Omega$$

14. (c)



15. (c) The Fourier transform is the continuous sum of exponential of the form $e^{j\omega t}$, where frequencies are restricted to the imaginary axis.
The Laplace transform is more generalized, where the frequency variable is $s=\sigma+j\omega$.
If we evaluate the Laplace transform in the imaginary axis of the s-plane ($\sigma=0$).

16. (d) The speed of magnetic field is

$$n_{sync}=\frac{120f_e}{P}=\frac{120\,(50\text{ Hz})}{2}=3000\text{ rev/min}$$

The speed of rotor is

$$n_m=(1-s)\,n_{sync}=(1-0.05)\,(3000\text{ rev/min})$$
$$=2850\text{ rev/min}$$

17. (c) **18.** (b)

19. (d) We have,

$$1+G(s)\,H(s)=0$$

or $$1+\frac{K}{s(s+2)\,(s+3)}=0$$

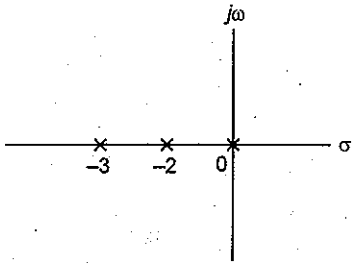
or $$K=-s(s^2+5s^2+6s)$$

$$\frac{dK}{ds}=-(3s^2+10s+6)=0$$

which gives, $$s=\frac{-10\pm\sqrt{100-72}}{6}$$

$$=-0.784,-2.548$$

The location of poles on s-plane is



Since, breakpoint must lie on root locus. So, $s=-0.784$ is possible.

20. (c) If $x(t) * h(t) = y(t)$

Then, $x(t-T_1) * h(t-T_2) = y(t-T_1-T_2)$

Thus, $x(t+5) * \delta(t-7) = x(t+5-7) = x(t-2)$

21. (b) In option (b) the given function is not periodic and does not satisfy Dirichlet condition. So, it can't be expansion in Fourier series.

$x(t) = 2\cos \pi t + 7\cos t$

$$T_1 = \frac{2\pi}{\omega} = 2$$

$$T_2 = \frac{2\pi}{1} = 2\pi$$

$$\frac{T_1}{T_2} = \frac{1}{\pi} = \text{irrational}$$

22. (c)
$$X() = \frac{5-s}{s^2-s-2} = \frac{5-s}{(s+1)(s-2)}$$
$$= \frac{-2}{s+1} + \frac{1}{s-2}$$

Here, three ROC may be possible

$$\text{Re}(s) < -1$$
$$\text{Re}(s) > 2$$
$$-1 < \text{Re}(s) < 2$$

Since, its Fourier transform exists, only $-1 < \text{Re}(s) < 2$ includes imaginary axis. So, this ROC is possible. For this ROC, the inverse Laplace transform is

$$x(t) = [-2e^{-t}u(t) + e^{2t}u(t)]$$

23. (c) Let A, B, C, D be the event when 0(W) and 3(B); 1(W) and 2(B); 2(W) and 1(B); 3(W) and 0(B) are transferred to the second box.

Let E be the event of drawing a black ball from the second box.

$$P(A) = \frac{{}^6C_3}{{}^{10}C_3} = \frac{1}{6}; \quad P(B) = \frac{{}^4C_1 \times {}^6C_2}{{}^{10}C_3} = \frac{1}{2};$$

$$P(C) = \frac{3}{10}; \; P(D) = \frac{{}^4C_3}{{}^{10}C_3} = \frac{1}{30}$$

$$P(A) + P(B) + P(C) + P(D) = \frac{1}{6} + \frac{1}{2} + \frac{3}{10} + \frac{1}{30} = 1$$

$\therefore$ Events A, B, C, D are mutual exclusive and also exhaustive.

$$P\left(\frac{E}{A}\right) = \frac{3}{3} = 1; \quad P\left(\frac{E}{B}\right) = \frac{2}{3}; \quad P\left(\frac{E}{C}\right) = \frac{1}{3}; \quad P\left(\frac{E}{D}\right) = 0$$

Required probability

$$P\left(\frac{C}{E}\right) = \frac{P\left(\frac{E}{C}\right) \times P(C)}{P\left(\frac{E}{A}\right) \times P(A) + P\left(\frac{E}{B}\right) \times P(B) + P\left(\frac{E}{C}\right) \times P(C) + P\left(\frac{E}{D}\right) \times P(D)}$$

$$= \frac{\frac{1}{3} \times \frac{3}{10}}{1 \times \frac{1}{6} + \frac{2}{3} \times \frac{1}{2} + \frac{1}{3} \times \frac{3}{10} + 0 \times \frac{1}{30}}$$

$$= \frac{\frac{1}{10}}{\frac{1}{6} + \frac{1}{3} + \frac{1}{10}}$$

$$= \frac{1}{6}$$

24. (a) It is given that ϕ and ψ satisfy Laplace's equation

$$\Rightarrow \quad \frac{\partial^2\phi}{\partial x^2} + \frac{\partial^2\phi}{\partial y^2} = 0 \qquad \ldots(i)$$

and
$$\frac{\partial^2\psi}{\partial x^2} + \frac{\partial^2\psi}{\partial y^2} = 0 \qquad \ldots(ii)$$

Now, to prove that $(s + it)$ is analytic *i.e.*, we have to show that Cauchy-Reimann equations are satisfied

i.e., $\frac{\partial s}{\partial x} = \frac{\partial t}{\partial y}$ and $\frac{\partial s}{\partial y} = -\frac{\partial t}{\partial x}$

Now, $\frac{\partial s}{\partial x} = \frac{\partial}{\partial x}\left(\frac{\partial\phi}{\partial y} - \frac{\partial\psi}{\partial x}\right)$

$\because$ put given value of s.

$$= \frac{\partial^2\phi}{\partial x \partial y} - \frac{\partial^2\psi}{\partial x^2} = 0$$

$$\Rightarrow \quad \frac{\partial s}{\partial x} = \frac{\partial^2\phi}{\partial x\,\partial y} + \frac{\partial^2\psi}{\partial y^2} \qquad \ldots(iii)$$

$\because$ by using Eq. (ii)

$$\frac{\partial t}{\partial y} = \frac{\partial}{\partial y}\left(\frac{\partial\phi}{\partial x} + \frac{\partial\psi}{\partial y}\right)$$

$\because$ put given value of t.

$$= \frac{\partial^2\phi}{\partial y\,\partial x} + \frac{\partial^2\psi}{\partial y^2} \qquad \ldots(iv)$$

$$\frac{\partial s}{\partial y}=\frac{\partial}{\partial y}\left(\frac{\partial \phi}{\partial y}-\frac{\partial \psi}{\partial x}\right)=\frac{-\partial^2\phi}{\partial x^2}-\frac{\partial^2\psi}{\partial y\,\partial x} \quad \ldots(v)$$

$\because$ by using Eq. (i)

$$\frac{\partial t}{\partial x}=\frac{\partial}{\partial x}\left(\frac{\partial \phi}{\partial x}+\frac{\partial \psi}{\partial y}\right)=\frac{\partial^2\phi}{\partial x^2}+\frac{\partial^2\psi}{\partial x\,\partial y} \quad \ldots(vi)$$

From Eqs. (iii) and (iv)

$$\Rightarrow \quad \frac{\partial s}{\partial x}=\frac{\partial t}{\partial y}$$

From Eqs. (v) and (vi)

$$\Rightarrow \quad \frac{\partial s}{\partial y}=-\frac{\partial t}{\partial x}$$

Cauchy-Riemann equations are satisfied.

Hence, $(s+it)$ is an analytic function.

25. (a) The given differential equation is

$$p=\tan(px-y) \text{ or } y=px-\tan^{-1}p \quad \ldots(i)$$

This equation is in the form of Clairauts equation.

The general solution of this equation is

$$y=cx-\tan^{-1}c \quad \ldots(ii)$$

Differentiating Eq. (ii) with respect to c,

We get, $$0=x-\frac{1}{1+c^2}$$

$$\Rightarrow \quad x=\frac{1}{1+c^2}$$

$$\Rightarrow \quad c=\sqrt{\frac{1-x}{x}} \quad \ldots(iii)$$

Eliminating c between Eqs. (ii) and (iii), the required singular solution is

$$y=\sqrt{\frac{1-x}{x}}\cdot x-\tan^{-1}\sqrt{\frac{1-x}{x}}$$

26. (b) From mean value theorem,

$$\frac{f(1)-f(0)}{1-0}=f'(x), x\in[0,1]$$

$$f(1)=2+\frac{2x}{(5-x^2)^2} \quad \because f(0)=2$$

Lower bound of $f(1)=2+0=2$

Upper bound of $f(1)=2+\frac{2}{(5-1)^2}=2.125$

27. (c) Given, $x=\sqrt[3]{C} \Rightarrow x^3=C$

Let $$f(x)=x^3-C$$

$$f'(x)=3x^2$$

We know that, the Newton-Raphson formula

$$x_{n+1}=x_n-\frac{f(x_n)}{f'(x_n)}$$

$$x_{n+1}=x_n-\frac{(x_n^3-C)}{3x_n^2}$$

$$x_{n+1}=\frac{3x_n^3-x_n^3+C}{3x_n^2}$$

$$x_{n+1}=\frac{2x^3{}_n+C}{3x_n^2}$$

28. (b) Out of 10 ships, one ship is wrecked, *i.e.*, 9 ships out of the 10 ships safe.

$$P(\text{safety})=\frac{9}{10}$$

P(atleast 4 ships out of 5 are safe)

$$=P(4 \text{ or } 5)$$

$$=P(4)+P(5)$$

$$={}^5C_4P^4q^{5-4}+{}^5C_5P^5q^0$$

$$=5\left(\frac{9}{10}\right)^4\cdot\frac{1}{10}+\left(\frac{9}{10}\right)^5$$

$$=\left(\frac{9}{10}\right)^4\left(\frac{5}{10}+\frac{9}{10}\right)$$

$$=\frac{7}{5}\left(\frac{9}{10}\right)^4.$$

29. (a) $f(x)f''(x)-f'(x)f'(x)=0$

$$\therefore \quad \frac{f''(x)}{f'(x)}=\frac{f'(x)}{f(x)}$$

On integrating, we get

$$\log f'(x)=\log f(x)+\log C$$

$$\therefore \quad f'(x)=C\,f(x)$$

From the initial conditions,

$$f'(0)=C\cdot f(0) \text{ or } 2=C\times 1 \quad \therefore C=2$$

$$\Rightarrow \quad \frac{f'(x)}{f(x)}=2$$

On integrating, we get

$$\Rightarrow \quad \log f(x)=2x+D$$

$$\therefore \quad f(x)=e^{2x+D}$$

$$f(0)=1 \quad \therefore \quad D=0$$

$$\Rightarrow \quad f(x)=e^{2x}.$$

30. (c) Given, $\int_0^1 x\log x\,dx$

Let $$t=\log x \Rightarrow x=e^t$$

$$dt=\frac{1}{x}dx \Rightarrow dx=x\,dt$$

$$dx=e^t dt$$

$$\therefore \quad \int_{\infty}^{0} e^{t} \cdot t \cdot e^{t}\, dt = \int_{0}^{\infty} t^{(2-1)}\, e^{2t}\, dt$$

$$= -\frac{\lfloor 2}{(2)^2}$$

$$\because \int_0^{\infty} x^{n-1} e^{ax}\, dx = \frac{\lfloor n}{a^n}$$

$$= \left(-\frac{1}{4}\right)$$

31. (d) We know that,

$$C_j = \left[\frac{e\varepsilon_S N_A N_D}{2(V_{bi} + V_R)(N_A + N_D)}\right]^{1/2}$$

$$C_j \propto \sqrt{\frac{1}{(V_{bi} + V_R)}}$$

Thus, Now, $\frac{C_{j2}}{C_{j1}} = \sqrt{\frac{(V_{bi} + V_R)_1}{(V_{bi} + V_R)_2}} = \sqrt{\frac{1}{4}} = \frac{1}{2}$

or $C_{j2} = \frac{C_{j1}}{2} = \frac{1}{2} = 0.5\,\text{pF}$

32. (b) $\sigma_n = np\mu_n$

$\sigma_p = pq\mu_p$

$\frac{\sigma_p}{\sigma_n} = \frac{\mu_p}{\mu_n} = \frac{1}{3}$

33. (c) $\begin{bmatrix} y_{11} & y_{12} \\ y_{21} & y_{22} \end{bmatrix} = \begin{bmatrix} y_1 + y_3 - y_3 \\ -y_3 \;\; y_2 + y_3 \end{bmatrix}$

$y_{12} = -y_3$

$y_{12} = -\frac{1}{2} = -0.05\,\text{mho}$

34. (c) The energy delivered in 10 min is

$$E = \int_0^t VI\, dt = I \int_0^t V dt = I \times \text{Area}$$

$$= 2 \times \frac{1}{2}(10 + 12) \times 600 = 13.2\,\text{kJ}$$

35. (c) $30 = 5I_a + 3(I_a - 2.5) + 4(I_a - 2.5 + 2)$

$$I_a = \frac{30 + 7.5 + 2}{12} = 3.29\ \text{A}$$

$\Rightarrow \quad 6I_a = 19.75\,\text{A}$

Voltage across 2Ω resistor $= 30 - 19.75$

$= 10.25\,\text{V},$

$$P = \frac{(10.25)^2}{2} = 52.53\,\text{W}$$

36. (b)

Present State		FF		Next State	
Q_A	Q_B	T_A	T_B	Q_A^+	Q_B^+
0	0	0	1	0	1
0	1	1	1	1	0
1	0	1	0	0	0
1	1	1	1	0	0

From table it is clear that it is a MOD-3 counter.

37. (a) 4 bit uses 4 flip-flop.

Total delay $Nt_d = 4 \times 50\,\text{ns} = 200 \times 10^{-9}$

$$f = \frac{1}{200 \times 10^{-9}} = 5\,\text{MHz}$$

38. (c) $R_{th} = \frac{V_{th}}{I_{sc}} = 1.2\Omega$

$$P = \frac{24^2}{(1.2 + 2)^2} \times 2 = 112.5\,\text{W}$$

39. (a) In PMMC,

Current $I = \left(\frac{K}{G}\right)Q$

As the deflection is directly proportional to the current passing through the meter (K and G) being constant, we get a uniform linear scale for the instrument.

40. (b) As its own sweep output is fed to the input both is of same amplitude. Hence, the oscilloscope display is straight line. (In both case, if in phase or out of phase (180°)).

41. (c) As the same amplitude signal one fed into both the horizontal and vertical input, the straight line with 45° slope pattern on CRO.

42. (a) Resolution of 1V range $= \frac{1}{10^4} = 0.0001$ so any reading upto four decimal can be displayed.

Hence, reading of 0.5245 on 1V is 0.5245.

43. (b) The Pirani gauge is a robust thermal conductivity gauge used for the measurement of the pressure in vacuum system. The gauge may be used for pressure between 0.5 torr to 10^{-4} torr. For higher vacuum measurement other instrument such as Penning gauge is used.

44. (d) We know that, bandwidth of series *R-L-C* circuit is *R/L*. Therefore, bandwidth of filter 1 is $B_1 = \frac{R}{L_1}$.

Bandwidth of filter 2 is

$$B_2 = \frac{R}{L_2} = \frac{R}{L_1/4} = \frac{4R}{L_1}$$

Dividing above equation $\frac{B_1}{B_2} = \frac{1}{4}$

45. (d) **46.** (a)

47. (d) Impedance $= (5-3j) \parallel (5+3j)$

$$= \frac{(5-3j)\times(5+3j)}{5-3j+5+3j} = \frac{(5)^2-(3j)^2}{10}$$

$$= \frac{25+9}{10} = 3.4$$

V_{AB} = Current × Impedance

$= 5\angle 30° \times 34 = 17\angle 30°$

48. (a) $w = K\sqrt{V+V_R}$

Now, $2 = K\sqrt{0.8+1.2}$

and $w_2 = K\sqrt{0.8+7.2}$

From above two equations, we get

$$\frac{w_2}{2} = \frac{\sqrt{0.8+7.2}}{\sqrt{0.8+1.2}} = \frac{\sqrt{8}}{\sqrt{2}} = 2$$

or $w_2 = 4\,\mu m$

49. (b) $\alpha = \frac{\beta}{\beta+1} = \frac{50}{50+1} = \frac{50}{51}$

Current gain = Base transport factor × Emitter injection efficiency

$$\alpha = \beta_1 \times \beta_2$$

or $$\beta_1 = \frac{\alpha}{\beta_2} = \frac{50}{51\times 0.995} = 0.985$$

50. (a) We have, $x(n) = [-4-j5, 1+2j, 4]$

$$\uparrow$$

$$x^*(n) = [-4+j5, 1-2j, 4]$$

$$x^*(-n) = [4, 1-2j, -4+j5]$$

$$\uparrow$$

$$x_{cas}(n) = \frac{x(n)-x^*(-n)}{2}$$

$$= \left[-4-j\frac{5}{2},\ 2j,\ 4-j\frac{5}{2}\right]$$

51. (c) We have, $2y = \alpha y(n-2) - 2x(n) + \beta x(n-1)$

Taking *Z* transform, we get

$$2Y(z) = \alpha y(z)z^{-2} - 2\times(z) + \beta\times(z)z^{-1}$$

or $$\frac{Y(z)}{X(z)} = \left(\frac{\beta z^{-1}-2}{2-\alpha z^{-2}}\right)$$

or $$H(z) = \frac{2\left(\frac{\beta}{2}-1\right)}{\left(z^2-\frac{\alpha}{z}\right)}$$

It has poles at $\pm\sqrt{\frac{\alpha}{Z}}$ and zero at 0 and $\frac{\beta}{2}$. For a stable system poles must be inside the unit circle of *z* plane. Thus,

$$\left|\sqrt{\frac{\alpha}{2}}\right| < 1$$

or $$|\alpha| < 2$$

But zero can lie anywhere in plane. Thus, β can be of any value.

52. (d) We have, $\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} -3 & -1 \\ 2 & 0 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 1 \\ 0 \end{bmatrix} u$

and $$y = [1\ \ 0]\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 1 \\ 2 \end{bmatrix} u$$

Here, $A = \begin{bmatrix} -3 & -1 \\ 2 & 0 \end{bmatrix}$, $B = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$ and $C = [1 \quad 0]$

The controllability matrix is

$$Q_c = [B \quad AB]$$

$$= \begin{bmatrix} 1 & -3 \\ 0 & 2 \end{bmatrix}$$

$$\det Q_c \neq 0$$

The observability matrix is

$$Q_0 = [C^T \quad A^T C^T]$$

$$= \begin{bmatrix} 1 & -3 \\ 0 & 1 \end{bmatrix} \neq 0$$

53. (b) $$(sI - A) = \begin{bmatrix} s & 0 \\ 0 & s \end{bmatrix} - \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} s-1 & 0 \\ 0 & s-1 \end{bmatrix}$$

$$(sI - A)^{-1} = \frac{1}{(s-I)^2}\begin{bmatrix} (s-1) & 0 \\ 0 & (s-1) \end{bmatrix}$$

$$= \begin{bmatrix} \frac{1}{s-1} & 0 \\ 0 & \frac{1}{s-1} \end{bmatrix}$$

$$e^{At} = L^{-1}[(sI - A)]^{-1}$$

$$= \begin{bmatrix} e^t & 0 \\ 0 & e^t \end{bmatrix}$$

54. (a) Suppose at point P impedance is

$$Z = r + j(-1)$$

If we move in constant resistance circle from point P in clockwise direction by an angle 45°, the reactance magnitude increases. Let us consider, a point Q at 45° from point P in clockwise direction. Its impedance is

$$Z_1 = r - 0.5j$$

or $$Z_1 = Z + 0.5j$$

Thus, movement on constant r - circle by an $\angle 45°$ in clockwise direction is the addition of inductance in series with Z.

55. (d)

56. (a) The array factor is

$$A = \cos\left[\frac{\beta d \sin\theta + \alpha}{2}\right]$$

Here, $$\beta = \frac{2\pi}{\lambda}$$

$$d = \frac{\lambda}{4}$$

and $$\alpha = 90°$$

Thus, $$A = \cos\left[\frac{\frac{2\pi}{\lambda}\frac{\lambda}{4}\sin\theta + \frac{\pi}{2}}{2}\right]$$

$$= \cos\left[\frac{\pi}{4}\sin\theta + \frac{\pi}{2}\right]$$

57. (d) $$Z_o^2 = Z_{OC} \cdot Z_{SC}$$

$$Z_{SC} = \frac{Z_o^2}{Z_{OC}}$$

$$= \frac{50 \times 50}{100 + j150} = \frac{50}{2 + 3j}$$

$$= \frac{50(2 - 3j)}{13}$$

or $$Z_{SC} = 7.69 - 11.54j$$

58. (c) By Binomial distribution, the probability, if error is

$$p_e = {}^nC_r\, p^r (1-p)^{n-r}$$

Probability of at most one error = Probability of no error + Probability of one error

$$= {}^nC_0\, p^0 (1-p)^{n-0} + {}^nC_1 p^1 (1-p)^{n-1}$$

$$= (1-p)^n + np(1-p)^{n-1}$$

59. (b) For unity negative feedback system, the closed-loop transfer function is

$$\text{CLTF} = \frac{G(s)}{1 + G(s)} = \frac{s+4}{s^2 + 7s + 13},$$

or $$\frac{1 + G(s)}{G(s)} = \frac{s^2 + 7s + 13}{s + 4}$$

or $$\frac{1}{G(s)} = \frac{s^2 + 7s + 13}{s + 4} - 1$$

$$= \frac{s^2 + 6s + 9}{s + 4}$$

or $$G(s) = \frac{s+4}{s^2 + 6s + 9}$$

or DC gain $s = 0$,

thus $$G(0) = \frac{4}{9}$$

60. (c) From the block diagram, transfer function is

$$T(s) = \frac{G(s)}{1 + G(s)H(s)}$$

where $$G(s) = \frac{K(s-2)}{(s+2)^2}$$

and $$H(s) = (s-2)$$

The characteristic equation is

$$1 + G(s)H(s) = 0$$

$$1 + \frac{K(s-2)}{(s+2)^2}(s-2) = 0$$

or $$(s+2)^2 + K(s-2)^2 = 0$$

or $$(1+K)s^2 + 4(1-K)s + 4K + 4 = 0$$

Routh table is shown below for system to be stable

$1 + K > 0$ and $4 + 4K > 0$ and $4 - 4K > 0$

This, gives $-1 < K < 1$

As per question for $0 \le K < 1$

s^2	$1 + K$	$4 + 4K$
s^1	$4 - 4K$	0
s^0	$4 + 4K$	

61. (c)

62. (d) If the Nyquist plot of $G(j\omega)H(j\omega)$ for closed-loop system passes through $(-1 + j0)$ point, the gain margin is 1 and in dB

$$\text{GM} = -20\log 1 = 0$$

63. (b) The characteristic equation is

$$1 + G(s)H(s) = 0$$

$$1 + \frac{K(s+1)}{s^3 + as^2 + 2s + 1} = 0$$

$$s^3 + as^2 + (2+K)s + K + 1 = 0 \quad \ldots(i)$$

The Routh table is shown below for system to be oscillatory stable

$$\frac{a(2+K)-(K+1)}{a}=0$$

or $$a=\frac{K+1}{K+2}$$

Then we have $as^2+K+1=0$

At 2 rad/s, we have

$$s=j\omega \longrightarrow s^2=-\omega^2=-4 \qquad ...(ii)$$

Thus, $$-4a+K+1=0$$

Solving Eqs. (i) and (ii), we get

$$K=2 \quad \text{and} \quad a=0.75$$

s^3	1	$2+K$
s^2	a	$1+K$
s^1	$\frac{(1+K)a-(1+K)}{a}$	
s^0	$1+K$	

64. (c) When the switch is opened, the power supplied to each load is

$$P_1=3\frac{V_\phi^2}{Z}\cos\theta=3\frac{(480\text{V})^2}{10\,\Omega}\cos 30°$$

$$=59.86\text{ kW}$$

$$Q_1=3\frac{V_\phi^2}{Z}\sin\theta=3\frac{(480\text{V})^2}{10\,\Omega}\sin 30°$$

$$=34.56\text{ kW}$$

$$P_2=3\frac{V_\phi^2}{Z}\cos\theta=3\frac{(277\text{V})^2}{4\,\Omega}\cos 36.87°$$

$$=46.04\text{ kW}$$

$$Q_2=3\frac{V_\phi^2}{Z}\sin\theta=3\frac{(277\text{V})^2}{4\,\Omega}\sin 36.87°$$

$$=34.53\text{ kW}$$

$$P_{TOT}=P_1+P_2=105.9\text{ kW}$$

$$Q_{TOT}=Q_1+Q_2=69.09\text{ kW}$$

The apparent power supplied by the utility is

$$S_{TOT}=\sqrt{P_{TOT}^2+Q_{TOT}^2}=126.4\text{ kVA}$$

The power factor supplied by the utility is

$$\text{PF}=\cos\tan^{-1}\left(\frac{Q_{TOT}}{P_{TOT}}\right)=0.838\text{ lagging.}$$

The current supplied by the utility is

$$I_L=\frac{P_{TOT}}{\sqrt{3}\,V_T\,\text{PF}}=\frac{105.9\text{ kW}}{\sqrt{3}\,(480\text{V})\,(0.838)}=152\text{ A}$$

65. (b) The total current supplied by the power system drops when the switch is closed because the capacitor bank is supplying some of the reactive power being consumed by loads 1 and 2.

66. (c) The hole concentration is greatest where N_a is greatest, *i.e.*, $x=L$.

67. (a) The electron concentration is greatest where hole concentration is lowest, which is where N_a is lowest.

68. (c) From the problem statement, we have

$$Z_{11}=\left.\frac{V_1}{i_1}\right|_{i_2=0}=\frac{6}{4}=1.5\,\Omega$$

$$Z_{12}=\left.\frac{V_1}{i_2}\right|_{i_1=0}=\frac{45}{1}=45\,\Omega$$

$$Z_{21}=\left.\frac{V_2}{i_1}\right|_{i_2=0}=\frac{6}{4}=1.5\,\Omega$$

$$Z_{22}=\left.\frac{V_2}{i_2}\right|_{i_2=0}=\frac{15}{1}=15\,\Omega$$

Thus, Z parameter matrix is

$$\begin{bmatrix}Z_{11} & Z_{12}\\ Z_{21} & Z_{22}\end{bmatrix}=\begin{bmatrix}15 & 45\\ 15 & 15\end{bmatrix}$$

69. (a) From the problem statement, we have

$$h_{12}=\left.\frac{V_1}{V_2}\right|_{I_1=0}=\frac{45}{15}=3$$

$$h_{22}=\left.\frac{i_2}{V_2}\right|_{I_1=0}=\frac{1}{15}=0.67$$

From Z matrix, we have

$$V_1=Z_{11}I_1+Z_{12}I_2$$

$$V_2=Z_{21}I_1+Z_{22}I_2$$

If $V_2=0$,

then $$\frac{I_2}{I_1}=\frac{-Z_{21}}{Z_{22}}=\frac{-15}{15}=-1=h_{21}$$

or $$I_1=-I_1$$

Put it in equation for V_1, we get

$$V_1=(Z_{11}-Z_{12})I_1$$

$$\left.\frac{V_1}{I_1}\right|_{V_2=0}=h_{11}=Z_{11}-Z_{12}$$

$$=15-45=-3$$

Hence, h-parameter will be

on $\begin{bmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{bmatrix} = \begin{bmatrix} -3 & 3 \\ -1 & 0.67 \end{bmatrix}$

70. (a) From graph,

$$x(n) = \left[\frac{1}{2}, 1, 2, 1, \frac{1}{2}\right]$$

$$y(n) = x\left(\frac{n}{2} - 1\right), n \text{ even}$$

$$= 0, \text{ for } n \text{ odd}$$

$n = -2, \quad y(-2) = x\left(\frac{-2}{2} - 1\right) = x(-2) = \frac{1}{2}$

$n = -1, \quad y(-1) = 0$

$n = 0, \quad y(0) = x\left(\frac{0}{2} - 1\right) = x(-1) = 1$

$n = 1, \quad y(1) = 0$

$n = 2, \quad y(2) = x\left(\frac{2}{2} - 1\right) = x(0) = 2$

$n = 3, \quad y(3) = 0$

$n = 4, \quad y(4) = x\left(\frac{4}{2} - 1\right) = x(1) = 1$

$n = 5, \quad y(5) = 0$

$n = 6, \quad y(6) = x\left(\frac{6}{2} - 1\right) = x(2) = \frac{1}{2}$

Hence, $y(n)$

$$= \frac{1}{2}\delta(n+2) + \delta(n) + 2\delta(n-2) + \delta(n-4) + \frac{1}{2}\delta(n-6)$$

71. (c) Here, $y(n)$ is scaled and shifted version of $x(n)$ and again $y(2n)$ is scaled version of $y(n)$ giving

$$Z(n) = y(2n) = x(n-1)$$

$$= \frac{1}{2}\delta(n+1) + \delta(n) + 2\delta(n-1) + \delta(n-2) + \frac{1}{2}\delta(n-3)$$

Taking Fourier transform,

$$Z(e^{j\omega}) = \frac{1}{2}e^{j\omega} + 1 + 2e^{-j\omega} + e^{-2j\omega} + \frac{1}{2}e^{-3j\omega}$$

$$= e^{-j\omega}\left(\frac{1}{2}e^{2j\omega} + e^{j\omega} + 2 + e^{-j\omega} + \frac{1}{2}e^{-2j\omega}\right)$$

$$= e^{-j\omega}\left(\frac{e^{2j\omega} + e^{-2j\omega}}{2} + e^{j\omega} + 2 + e^{-j\omega}\right)$$

or $Z(e^{j\omega}) = e^{-j\omega}[\cos 2\omega + 2\cos\omega + 2]$

72. (a)

73. (d)

74. (d) Given system is 2nd order and for 2nd order system GM is infinite.

75. (c) Pinnacle means climax on endeavour or success. So, peak is most closest to pinnacle.

76. (b) As water contains oxygen, similarly salt contains sodium.

77. (a) According to the sentence, very few musicians want to laugh at themselves, but this can boost the sale of video clips. So, option (a) is the best to complete this sentence.

78. (c) Let total number of votes cost be x.

$\therefore$ Total number of valid votes $= 0.96x$

Winner contestant secures 55% of valid votes.

So, loser contestant secures 45% of valid votes.

$\therefore$ Majority $= (55 - 45)\%$ of valid votes

$\Rightarrow \quad 240 = 10\%$ of $0.96x$

$\Rightarrow \quad x = \frac{240}{0.96} \times \frac{100}{10} = 2500$

79. (a) Average weight of group $A = 30$ kg

Total weight of group $A = 30 \times 30 = 900$ kg

Average weight of group $B = 70$ kg

Total weight of group $B = 70 \times 70 = 4900$ kg

Average weight of group A and $B = \frac{900 + 4900}{30 + 70}$

$$= \frac{5000}{100} = 58 \text{ kg}$$

80. (b) The conclusion from the paragraph is that parents can prepay their children future college tuition fee at current rate as it is expected that in future college tuition fee may increase. However, one drawback of this program is that, if interest on principal amount is more than the increase in fee, then parents will not participate in this program.

81. (b) Value $\propto$ (diameter)^2

Value $\propto$ thickness

$\therefore \quad \frac{v_1}{v_2} = \left(\frac{d_1}{d_2}\right)^2 \cdot \left(\frac{t_1}{t_2}\right)$

$\Rightarrow \quad \frac{4}{1} = \left(\frac{4}{3}\right)^2 \cdot \left(\frac{t_1}{t_2}\right)$

$\Rightarrow \quad \frac{t_1}{t_2} = \frac{9}{4}$

82. (b) Let rate of current be x and the speed of man in still water be y.

$\therefore$ Upstream speed $= y - x$

Downstream speed $= y + x$

$$5 = \frac{15}{y-x} + \frac{22}{y+x} \quad \ldots(i)$$

$$6\frac{1}{2} = \frac{20}{y-x} + \frac{27}{y+x} \quad \ldots(ii)$$

On solving Eqs. (i) and (ii), we get

$$y - x = 5 \quad \ldots(iii)$$

$$y + x = 11 \quad \ldots(iv)$$

On solving Eqs. (iii) and (iv), we get

$$y = 8 \text{ km/h}$$

$$x = 3 \text{ km/h}$$

83. (b) Ratio of total man hours supplied by A, B and C

$$= (20 \times 8 \times 6) : (15 \times 9 \times 7) : (10 \times 6 \times 8)$$

$$= (4 \times 8 \times 2) : (3 \times 3 \times 7) : (2 \times 2 \times 8)$$

$$= 64 : 63 : 32$$

$$\therefore C\text{'s share} = \frac{32}{64+63+32} \times 6.36$$

$$= ₹128$$

84. (b) Cable television $= \frac{1}{5}$

Videocassette recorder $= \frac{1}{10}$

Housing both $= \frac{1}{3} \times \frac{1}{5} = \frac{1}{15}$

Housing units having either television or videocassette recorder

$$= \frac{1}{5} + \frac{1}{10} - \frac{1}{15}$$

$$= \frac{6+3-2}{30} = \frac{7}{30}$$

Housing units having neither television nor videocassette recorder

$$= 1 - \frac{7}{30} = \frac{23}{30}$$

GATE Exam

(Graduate Aptitude Test in Engineering)

Crack Paper-2

1 Mark Questions

1. Which of the following propositions is a tautology?

(a) $(P \vee q) \rightarrow P$ (b) $P \vee (q \rightarrow P)$

(c) $P \vee (P \rightarrow q)$ (d) $P \rightarrow (P \rightarrow q)$

2. If A be a finite set of size n, then number of elements in the power set of $A \times A$ is

(a) 2^{2^n} (b) 2^{n^2}

(c) $(2^n)^2$ (d) None of these

3. The number of different spanning trees in complete graph K_n, by Cayley's theorem is

(a) 2^{n-2} (b) n^{n-2}

(c) n^{n-1} (d) None of these

4. If ${}^nP_r = 3024$, then what will be the value of r?

(a) 3 (b) 4 (c) 6 (d) 9

5. Rank of the diagonal matrix

$$\begin{bmatrix} 3 & & & & & \\ & 0 & & & & \\ & & 0 & & & \\ & & & -3 & & \\ & & & & 0 & \\ & & & & & 4 \end{bmatrix}$$ is

(a) 1 (b) 2 (c) 3 (d) 4

6. The correct Laplace transform of the function $f(t) = e^{3t} \sin 2t$ is

(a) $\dfrac{2}{s^2 - 4s + 9}$ (b) $\dfrac{2}{s^2 - 6s + 9}$

(c) $\dfrac{2}{s^2 - 6s + 13}$ (d) $\dfrac{3}{s^2 - 6s + 13}$

7. The value of integral $\int_{-\infty}^{\infty} \delta(t-2) \cos\left(\dfrac{\pi t}{4}\right) dt$ is

(a) 1 (b) π (c) $\dfrac{3\pi}{4}$ (d) 0

8. For a causal $f(K)$, if $(Z-1)F(Z)$ has no pole outside the unit circle, then

(a) $\lim\limits_{N \to \infty} f(N) = \lim\limits_{Z \to 0} ZF(Z)$

(b) $\lim\limits_{N \to \infty} f(N) = \lim\limits_{Z \to 0} (Z-1)F(Z)$

(c) $\lim\limits_{N \to \infty} f(N) = \lim\limits_{Z \to 1} (Z-1)F(Z)$

(d) $\lim\limits_{N \to \infty} f(N) = \lim\limits_{Z \to 1} ZF(Z)$

9. If $R(\tau)$ is the auto correlation function of a real, wide-sense stationary random process, then which of the following is not correct?

(a) $R(\tau) = R(-\tau)$

(b) $|R(\tau)| \leq R(0)$

(c) $R(\tau) = -R(-\tau)$

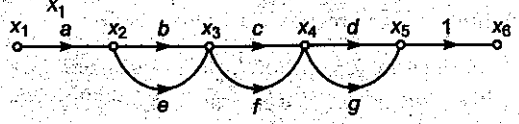
(d) The mean square value of the process is $R(0)$

10. The gain margin for the system with open loop transfer function

$$G(s)H(s) = \frac{2(1+s)}{s^2}, \text{ is}$$

(a) ∞ (b) 0 (c) 1 (d) $-\infty$

11. Consider the signal flow graph shown in figure, the gain $\dfrac{x_5}{x_1}$ is



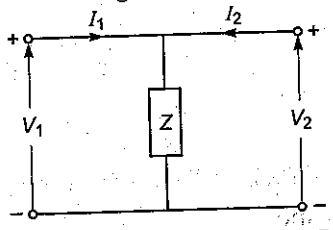
(a) $\dfrac{1-(be+cf+dg)}{abcd}$

(b) $\dfrac{bedg}{1-(be+cf+dg)}$

(c) $\dfrac{abcd}{1-(be+cf+dg)+bedg}$

(d) $\dfrac{1-(be+cf+dg)+bedg}{abcd}$

12. The transmission parameters, *i.e.*, $\begin{bmatrix} A & B \\ C & D \end{bmatrix}$ of the network shown in figure are



(a) $\begin{bmatrix} 1 & 0 \\ 1 & 1/2 \end{bmatrix}$ (b) $\begin{bmatrix} 1 & 0 \\ 1/2 & 1 \end{bmatrix}$

(c) $\begin{bmatrix} Z & 1 \\ 0 & 1 \end{bmatrix}$ (d) $\begin{bmatrix} 1 & 0 \\ Z & 1 \end{bmatrix}$

13. The oscillator step response of a network function $N(s)$ is damped sinusoidal. Pole location of $N(s)$ in the complex s-plane would be
(a) on the imaginary ($j\omega$) axis
(b) on the right-half plane
(c) on the real axis
(d) on the left-half plane excluding the real axis

14. The transfer function of a linear time-invariant system represented by vector-matrix differential equation $\dot{X} = AX + BU$ and $Y = CX + DU$ is given by
(a) $C(sI + A)^{-1}B$ (b) $C(sI - A)^{-1}B + D$
(c) $B(sI - A)^{-1}C$ (d) $B(sI - A)^{-1}C + D$

15. The following sequence currents were recorded in a power system under a fault condition

$$I_{positive} = j1.753 \text{ pu}$$
$$I_{negative} = -j0.6 \text{ pu}$$
$$I_{zero} = -j1.153 \text{ pu}$$

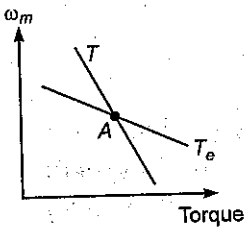
The fault is
(a) line to ground (b) three phase
(c) line to line ground (d) line to line

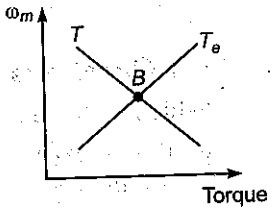
16. For economic operation, the generator with highest positive incremental transmission loss will operate at
(a) the highest positive incremental cost of production
(b) the highest negative incremental cost of production
(c) the lowest positive incremental cost of production
(d) the lowest negative incremental cost of production

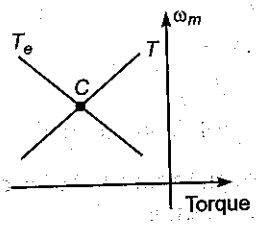
17. During no load test an induction motor draws power
(a) for core, windage and friction loss
(b) for core, loss and copper loss
(c) for copper, windage and friction loss
(d) only for the very small copper loss

18. In multiquadrant operation of drive for the torque and speed positive is called
(a) forward motoring (b) forward braking
(c) reverse braking (d) reverse motoring

19. The stable point of the given steady state speed-torque curve of motor and load is

(a)


(b)


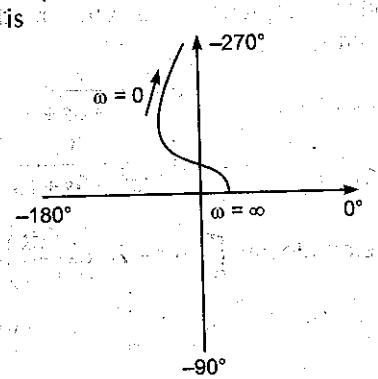
(c)


(d) None of these

20. Which of the following tests must be performed on a transformer to determine its leakage reactance?
(a) Both OC and SC test
(b) OC test only
(c) SC test only
(d) Test by an impedance bridge

21. Which one of the following statements is not true for Zener diode?
(a) Before breakdown having very high resistance
(b) After breakdown has a very small dynamic resistance
(c) Lightly doped
(d) Used to establish a fixed reference

22. The polar plot is shown in figure below. The type of system is



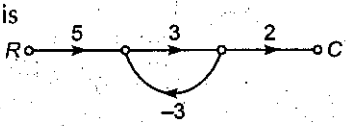
(a) type 0 (b) type 1
(c) type 2 (d) type 3

23. The forward path transfer function of a unit feedback system is

$$G(s)=\frac{K(s+2)}{(s^2+1)(s+4)(s-1)}$$

The range of K, for which there will be only two closed loop right hand poles, is
(a) $-33 < K - 12$ (b) $K < -33$
(c) $K > 2$ (d) $0 < K < 2$

24. In the signal flow graph shown in figure, the transfer function is



(a) 3.75 (b) -3 (c) 3 (d) -3.75

25. The transfer function of a system is given as

$H(z)=\dfrac{4z^{-1}}{\left(1-\frac{1}{4}z^{-1}\right)^2}, |z|>\dfrac{1}{4}$. Then $h[n]$ is

(a) stable (b) causal
(c) stable and causal (d) None of these

2 Marks Questions

26. Given, $P(A \cap \overline{B})=\dfrac{1}{3}$ and $P(A \cup B)=\dfrac{2}{3}$, then $P(B)$ is

(a) 2/3 (b) 1/5 (c) 1/3 (d) 4/5

27. If $u + ir$ is an analytic function, then $u + iv$ will be
(a) always analytic
(b) analytic only, if u = constant
(c) analytic only, if v = constant
(d) analytic only, if both u and v are constants

28. If product of matrix,

$$A=\begin{bmatrix}\cos^2\theta & \cos\theta\cdot\sin\theta\\ \cos\theta\cdot\sin\theta & \sin^2\theta\end{bmatrix} \text{ and}$$

$$B=\begin{bmatrix}\cos^2\phi & \cos\phi\sin\phi\\ \cos\phi\sin\phi & \sin^2\phi\end{bmatrix}$$

is a null matrix, then θ and ϕ differ by an
(a) odd multiple of π (b) even multiple of π
(c) odd multiple of $\pi/2$ (d) even multiple of $\pi/2$

29. The value of ξ in the mean value theorem of
$f(b)-f(a)=(b-a)f'(\xi)$
for $f(x)=Ax^2+Bx+C$ in (a, b) is
(a) $b+a$ (b) $b-a$
(c) $\dfrac{(b+a)}{2}$ (d) $\dfrac{(b-a)}{2}$

30. If $\Delta f(x)=f(x+h)-f(x)$, then a constant $k, \Delta k$ equals
(a) 1 (b) 0
(c) $f(k)-f(0)$ (d) $f(x+k)-f(x)$

31. Complimentary function of

$$x^2\frac{d^2y}{dx^2}+4x\frac{dy}{dx}+2y=x\log x \text{ is}$$

(a) c_1x+c_2 (b) $c_1x^2+c_3$
(c) $c_1x^{-1}+c_2x^{-2}$ (d) $c_1x^2+c_1x$

32. The maximum slope of the curve
$-x^3+6x^2+2x+1$ is
(a) 14 (b) 16 (c) 19 (d) -13

33. The solution of the differential equation
$x\,dy-y\,dx=\sqrt{x^2-y^2}\,dx$ is given by
(a) $\sin^{-1}\left(\dfrac{x}{y}\right)=x-c$ (b) $cx=e^{\sin^{-1}(y/x)}$
(c) $\sin^{-1}(y/x)=x^2c$ (d) $cx=e^{-\sin^{-1}(y/x)}$

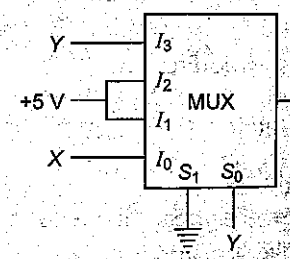
34. Consider the impulse response of two LTI system
$S_1: h_1(t)=e^{-t}u(t)$
$S_2: h_2(t)=e^{-t}\cos 2t\,u(t)$
The stable system is
(a) S_1 (b) S_2
(c) both S_1 and S_2 (d) None of these

35. The contents of accumulator after the execution of following instruction will be

```
MVI  A, B7H
ORA  A
RAL
```

(a) 6EH (b) 6FH (c) EEH (d) EFH

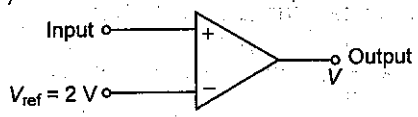
36. The output of the 4×1 multiplexer shown in figure is



(a) $X+Y$ (b) $\overline{X}+\overline{Y}$ (c) $\overline{XY}+X$ (d) XY

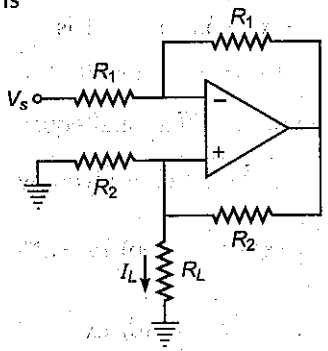
37. To implement $y=ABCD$ using only two-input NAND gates, minimum number of requirement of gate is
(a) 3 (b) 4 (c) 5

38. If the input to the ideal comparator shown in figure is a sinusoidal signal of 8 V (peak to peak) without any DC component, then the output of the comparator has a duty cycle of



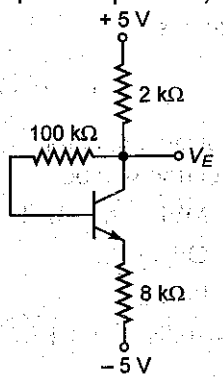
(a) $\frac{1}{2}$ (b) $\frac{1}{3}$ (c) $\frac{1}{6}$ (d) $\frac{1}{12}$

39. In the op-amp circuit given in figure below, the load current I_L is



(a) $-\frac{V_S}{R_2}$ (b) $\frac{V_S}{R_2}$ (c) $-\frac{V_S}{R_L}$ (d) $\frac{V_S}{R_L}$

40. In the circuit shown in figure. Voltage $V_E = 4$ V. The values of α and β are respectively.



(a) 0.943, 17.54 (b) 0.914, 17.54
(c) 0.914, 11.63 (d) 0.914, 11.63

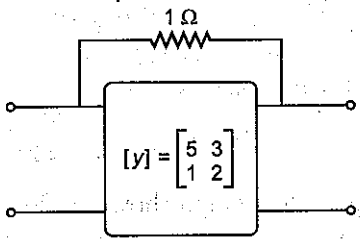
41. Isolation in ICs is required
(a) to make it simple to test circuits
(b) to protect the transistor from possible thermal run away
(c) to protect the components of mechanical damage
(d) to minimize electrical interaction between circuit components

42. In silicon at $T = 300$ K, thermal equilibrium concentration of electron is $n_0 = 5 \times 10^4\ cm^{-3}$. The hole concentration is
(a) $4.5 \times 10^{15}\ cm^{-3}$ (b) $4.5 \times 10^{15}\ m^{-3}$
(c) $0.3 \times 10^{-6}\ cm^{-3}$ (d) $0.3 \times 10^{-6}\ m^{-3}$

43. The Y-parameters of a 2-port network are

$$[Y] = \begin{bmatrix} 5 & 3 \\ 1 & 2 \end{bmatrix} S$$

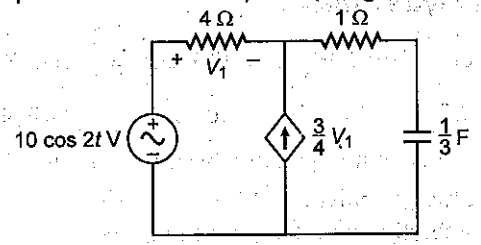
A resistor of 1 Ω is connected across as shown in figure. The new Y-parameters would be



(a) $\begin{bmatrix} 6 & 4 \\ 2 & 3 \end{bmatrix} S$ (b) $\begin{bmatrix} 6 & 2 \\ 0 & 3 \end{bmatrix} S$
(c) $\begin{bmatrix} 5 & 4 \\ 2 & 2 \end{bmatrix} S$ (d) $\begin{bmatrix} 4 & 4 \\ 2 & 1 \end{bmatrix} S$

44. Two mutually coupled coils for which $4 = 2\ \mu H$ $L_2 = 80\ \mu H$ and $K = 1$ have a load of $Z_L = 2 + j10$ connected across L_2. The Z_{in} at the terminals of L_1 is ($\omega = 250$ krad/s)
(a) $-0.19 + j1.46\ \Omega$ (b) $0.05 + j0.01\ \Omega$
(c) $-0.02 + j0.83\ \Omega$ (d) $0.02 + j0.17\ \Omega$

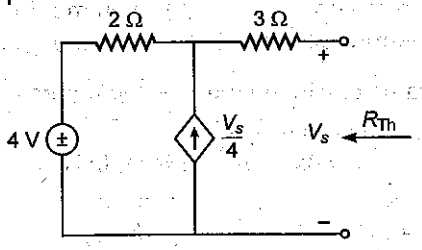
45. The power factor seen by the voltage source is



(a) 0.8 (leading) (b) 0.8 (lagging)
(c) 36.9 (leading) (d) 39.6 (lagging)

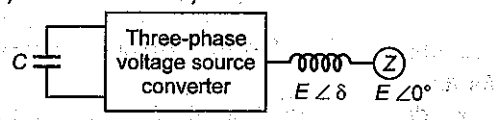
46. A battery has a short-circuit current of 30 A and an open circuit voltage of 24 V. If the battery is connected to an electric bulb of resistance 2 Ω, the power dissipated by the bulb is
(a) 80 W (b) 1800 W
(c) 112.5 W (d) 228 W

47. The equivalent resistance of circuit shown below is



(a) 3 Ω (b) 1.2 Ω (c) 2 Ω (d) 1 Ω

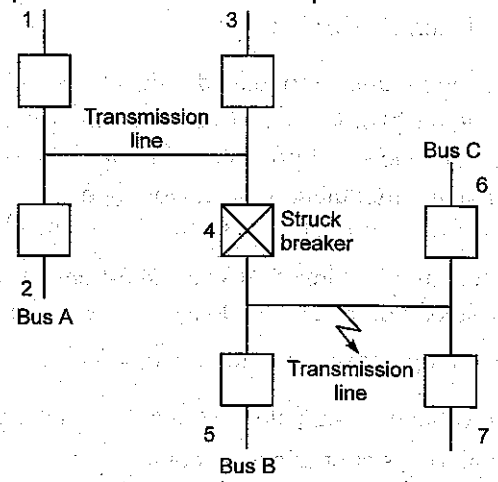
48. The figure below shows a three phase self- commutated voltage source converter connected to a power system. The converter's DC bus capacitor is marked as C in the figure. The circuit is initially operating in steady state with $\delta = 0$ and the capacitor DC voltage is equal to V_{dco}. You may neglect all loses and harmonics. What action should be taken to increase the capacitor DC voltage slowly to a new steady state value?



(a) Make δ positive and maintain it at a positive value
(b) Make δ positive and return it to its original value
(c) Make δ negative and maintain it at a negative value
(d) Make δ negative and return it to its original value

49. The total reactance and total susceptance of a lossless overhead EHV line, operating at 50 Hz, are given by 0.045 pu add 1.2 pu respectively. If the velocity of wave propagation is 3×10^5 km/s, then the approximate length of the line is
(a) 122 km (b) 172 km
(c) 222 km (d) 272 km

50. Consider the protection system shown in the figure below. The circuit breakers numbered from 1 to 7 are of identical type. A single line to ground fault with impedance occurs at the mid point of the line (at point F), but circuit breaker 4 fails to operate (stuck breaker) if the relays are coordinated correctly, a valid sequence of circuit breaker operation is



(a) 1, 2, 6, 7, 3, 5 (b) 1, 2, 5, 5, 7, 3
(c) 5, 6, 7, 3, 1, 2 (d) 5, 1, 2, 3, 6, 7

51. A synchronous generator is feeding a zero power factor (lagging) load at rated current. The armature reaction is
(a) magnetizing (b) demagnetizing
(c) cross-magnetizing (d) ineffective

52. Two transformers are to be operated in parallel such that they share load in proportion to their kVA ratings. The rating of the first transformer is 500 kVA and its per unit leakage impedance is 0.05 per unit. If the rating of second transformer is 250 kVA, its pu leakage impedance is
(a) 0.20 (b) 0.10
(c) 0.05 (d) 0.025

53. The speed of a 4-pole induction motor is controlled by varying the supply frequency while maintaining the ratio of supply voltage to supply frequency (V/F) constant. At rated frequency of 50 Hz and rated voltage of 400 V its speed is 1440 rpm. Find the speed at 30 Hz, if the load torque is constant.
(a) 882 rpm (b) 864 rpm
(c) 840 rpm (d) 828 rpm

54. A single-phase voltage source inverter is controlled in a single pulse width modulated mode with a pulse width of 150° in each half cycle. Total harmonic distortion is defined as

$$\text{THD} = \frac{\sqrt{V_{rms}^2 - V_1^2}}{V_1} \times 100$$

where V_1 is the rms value of the fundamental component of the output voltage. The THD of output AC voltage waveform is
(a) 65.65% (b) 48.42%
(c) 31.83% (d) 30.49%

55. A three-phase, 440 V, 50 Hz AC mains fed thyristor bridge is feeding a 440 V DC 15 kW, 1500 rpm separately excited DC motor with a ripple-free continuous current in the DC link under all operating conditions neglecting the losses, the power factor of the AC mains at half the rated speed is
(a) 0.354 (b) 0.372 (c) 0.90 (d) 0.955

56. A single-phase 230 V, 50 Hz AC mains fed step-down transformer (4 : 1) is supplying power to a half-wave uncontrolled AC to DC converter used for charging a battery (12 V DC) with the series limiting resistor being 19.04 Ω. The charging current is
(a) 3.43 A (b) 1.65 A
(c) 1.22 A (d) 1.0 A

57. A permanent magnet moving coil instrument has a coil dimensions 15 mm × 12 mm. The flux density in the air gap is 1.8×10^{-13} Wb/m^2 and the spring constant is 0.14×10^{-6} N-m/rad. The number of turns required to produce an angular deflection of 90° when a current of 5 mA is flowing through the coil will be
(a) 136 (b) 210 (c) 196 (d) 156

58. Match List I with List II and select the correct answer using the codes given below the lists.

List I	List II
A. Kelvin double bridge	1. Capacitance
B. Wien bridge	2. Self inductance
C. Schering bridge	3. Frequency
D. Maxwell's bridge	4. Low resistance

Codes

	A	B	C	D
(a)	4	2	3	1
(b)	4	3	1	2
(c)	4	2	1	3
(d)	4	1	2	3

59. While measuring the phase difference between the signals $V_1(t) = 10 \sin \omega t$ and $V_2(t) = 10 \sin(\omega t + \phi)$ the Lissajous pattern observed on CRO was a circle. The value of ϕ is

(a) 0 (b) $\frac{\pi}{4}$ (c) $\frac{\pi}{2}$ (d) π

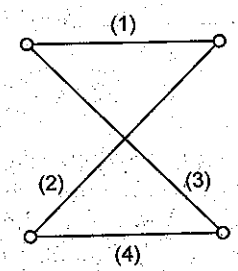
60. Match List I (Relay) with List II (Protection) and select the correct answer using the codes given below the lists.

List I	List II
A. Buchholz relay	1. Feeder
B. Translay relay	2. Transformer
C. Carrier current phase comparison relay	3. Ring main distributor
D. Directional over current relay	4. Long transmission line

Codes

	A	B	C	D
(a)	2	3	4	1
(b)	4	1	2	3
(c)	2	1	4	3
(d)	4	3	2	1

61. In the following graph, the number of trees (P) and the number of cutset (Q) are



(a) $P = 2, Q = 2$ (b) $P = 2, Q = 6$
(c) $P = 4, Q = 6$ (d) $P = 4, Q = 10$

62. The transfer function $H(s) = \frac{V_0(s)}{V_i(s)}$ of an R-L-C circuit is given by

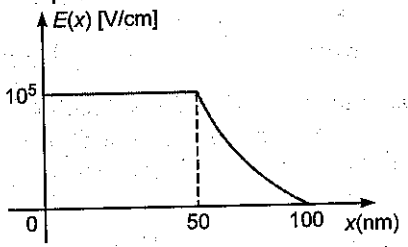
$$H(s) = \frac{10^6}{s^2 + 20s + 10^6}$$

The quality factor (Q-factor) of this circuit is

(a) 25 (b) 50 (c) 100 (d) 5000

Statements for Linked Answer Questions 63, 64 and 65

Consider a p-n junction at zero bias with an electric field distribution as sketch below . The metallurgical junction is placed at $x = 0$.



63. The depletion capacitance at zero bias is

(a) 1.1×10^{-7} F/cm^2 (b) 1.1×10^{-6} F/cm^2
(c) 1.1×10^{-8} F/cm^2 (d) Can't be determined

64. The doping type and level of the region between $50 < x < 100$ nm are

(a) acceptor, 1.3×10^{17} cm^{-2}
(b) donor, 1.3×10^{17} cm^{-2}
(c) acceptor, 1.3×10^{19} cm^{-2}
(d) donor, 1.3×10^{19} cm^{-2}

65. An amplitude modulated signal is given as $x_{AM}(t) = 100\, CP(t) + 0.5\, g(t) \cos \omega_c t$ in the interval $0 \le t \le 1$. One set of possible values of the modulating signal and modulation index would be

(a) t, 0.5 (b) t, 1.0 (c) t, 2.0 (d) t^2, 0.05

Statements for Linked Answer Questions 66 and 67

Consider a unity gain feedback control system whose open-loop transfer function is $G(s) = \frac{as + 1}{s^2}$.

66. The value of a, so that the system has a phase margin equal to $\frac{\pi}{4}$ is approximately equal to

(a) 2.40 (b) 1.40 (c) 0.84 (d) 0.74

67. With the value of a set for a phase margin of $\frac{\pi}{4}$, the value of unit impulse response of the open-loop system at $t = 1$s is equal to

(a) 3.40 (b) 2.40 (c) 1.84 (d) 1.74

Common Data for Questions 68 and 69

The system under consideration is an *RC* low-pass filter (*RC*-LPF) with $R = 1\,k\Omega$ and $C = 1.0\,\mu F$.

68. Let $H(F)$ denotes the frequency response of the *RC*-LPF. Let f_1 be the highest frequency such that $0 \le |f| \le f_1$ $\frac{|H(f_1)|}{H(0)} \ge 0.95$. Then, f_1 (in Hz) is

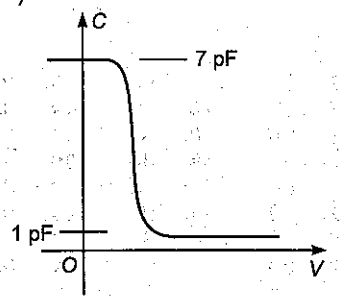
(a) 327.8 (b) 163.9 (c) 52.2 (d) 104.4

69. Let t_g be the group delay function of the given *RC*-LPF and $F_2 = 100$ Hz. Then, t_g (f_2) in ms is

(a) 0.717 (b) 7.17 (c) 71.7 (d) 4.505

Common Data for Questions 70 and 71

The figure shows the high frequency capacitance voltage (*C-V*) characteristics of metal SiO_2/silicon (MOS) capacitor having an area of 1×10^{-4} cm^2. Assume that the permittivities $(\varepsilon_0, \varepsilon_1)$ of silicon and SiO_2 are 1×10^{-12} F/cm and 35×10^{-13} F/cm respectively.



70. The gate oxide thickness in the MOS capacitor is

(a) 50 nm (b) 143 nm (c) 350 nm (d) 1μm

71. The maximum depletion layer width in silicon is

(a) 0.143 μm (b) 0.857 μm
(c) 1 μm (d) 1.143 μm

72. Consider the following statements about the *C-V* characteristics plot :

S_1 : The MOS capacitor has as *n*-type substrate.

S_2 : If positive charges are introduced in the oxide, the *C-V* plot will shift to the left.

Then, which of the following is true?

(a) Both S_1 and S_2 are true
(b) S_1 is true and S_2 is false
(c) S_1 is false and S_2 is true
(d) Both S_1 and S_2 are false

73. The values of voltage (V_D) across a tunnel diode for corresponding to peak and valley currents are V_P, V_D respectively. The range of tunnel-diode voltage for V_D which the slope of its I-V_D characteristics is negative would be

(a) $V_D < 0$ (b) $O \le V_D < V_P$
(c) $V_P \le V_D < V_V$ (d) $V_D \ge V_V$

General Aptitude

74. Which of the following options is closest in meaning to the word given below?

Elegant

(a) Unrefined (b) Tasteless
(c) Inelegant (d) Graceful

75. The question below consists of a pair of related words followed by four pairs of words. Select the pair which best expresses the relation in the original pair.

Die : Dice

(a) Data : Datas (b) Mouse : Mice
(c) Monkey : (d) Dates : Datum

76. Choose the most appropriate word from the options given below to complete the following sentence.

Congress is having great difficulty developing a consensus on energy policy, primarily because the policy objectives of various members of Congress rest on such ..... assumptions.

(a) commonplace (b) trivial
(c) explicit (d) divergent

77. Choose the most appropriate word from the options given below to complete the following sentence.

Because folk art is neither completely rejected nor accepted as an art form by art historians, their final evaluations of it necessarily remain .....

(a) arbitrary (b) equivocal
(c) orthodox (d) unspoken

78. The price per kg of rice increases by 20%. By what percentage should the consumption be decreased such that expenditure remains the same?

(a) 20% (b) 16.67%
(c) 25% (d) 16.33%

79. The conclusion above would be properly drawn, if which of the following were assumed as a premise?

(a) The amount of money citizens spend on products subject to the state tax tends to be equal across income levels
(b) The federal income tax favours citizens with high incomes, whereas the state sales tax favours citizens with low incomes
(c) Citizens with low annual incomes can afford to pay a relatively higher percentage of their incomes in state sales tax, since their federal income tax is relatively low
(d) The lower a state's sales tax, the more it will tend to redistribute income from the more affluent citizens to the rest of the society.

80. A man invests a certain sum of money at 6% pa. simple interest another sum at 7% pa simple interest. His income from interest after 2 yr was ₹ 354. One-fourth of the first sum is equal to one-fifth of the second sum. The total sum invested was
(a) ₹ 2600 (b) ₹ 2700
(c) ₹ 2880 (d) ₹ 2900

81. Mr. Tamatar buys some apples at 5 per rupee from one trader and a similar quantity at 7 per rupee from a different trader. He mixes both the varieties and sells the entire lot at 6 per rupee. What is the profit or loss% that he makes?
(a) $2\frac{7}{9}\%$ (b) $3\frac{7}{9}\%$
(c) $3\frac{2}{9}\%$ (d) $2\frac{6}{9}\%$

82. A man starts from B to K, another from K to B at the same time. After passing each either they complete their journey in $3\frac{1}{3}$ and $4\frac{4}{5}$ h respectively. Find the speed of the second man, if the speed of the first is 12 km/h.
(a) 8 km/h (b) 10 km/h
(c) 15 km/h (d) 24 km/h

83. A and B working separately can do a piece of work in 9 and 12 days respectively. If they work for a day alternatively. A beginning, in how may days the work will be completed?
(a) $7\frac{1}{2}$ days (b) $9\frac{1}{4}$ days
(c) $10\frac{1}{4}$ days (d) $12\frac{1}{4}$ days

Answers

1. (c)	**2.** (b)	**3.** (b)	**4.** (c)	**5.** (d)	**6.** (c)	**7.** (d)	**8.** (c)	**9.** (c)	**10.** (d)
11. (c)	**12.** (b)	**13.** (d)	**14.** (b)	**15.** (c)	**16.** (c)	**17.** (a)	**18.** (a)	**19.** (b)	**20.** (c)
21. (c)	**22.** (d)	**23.** (c)	**24.** (c)	**25.** (c)	**26.** (c)	**27.** (d)	**28.** (c)	**29.** (c)	**30.** (b)
31. (c)	**32.** (a)	**33.** (b)	**34.** (c)	**35.** (a)	**36.** (a)	**37.** (d)	**38.** (b)	**39.** (a)	**40.** (c)
41. (d)	**42.** (a)	**43.** (b)	**44.** (d)	**45.** (a)	**46.** (c)	**47.** (d)	**48.** (d)	**49.** (b)	**50.** (c)
51. (b)	**52.** (c)	**53.** (c)	**54.** (c)	**55.** (a)	**56.** (d)	**57.** (a)	**58.** (b)	**59.** (c)	**60.** (c)
61. (c)	**62.** (b)	**63.** (a)	**64.** (a)	**65.** (a)	**66.** (c)	**67.** (c)	**68.** (c)	**69.** (a)	**70.** (b)
71. (a)	**72.** (b)	**73.** (c)	**74.** (d)	**75.** (b)	**76.** (d)	**77.** (b)	**78.** (b)	**79.** (a)	**80.** (b)
81. (a)	**82.** (b)	**83.** (c)							

GATE Exam

(Graduate Aptitude Test in Engineering)

Crack Paper-3

1 Mark Questions

1. Find the Fourier sine transform of $f(x)$, where

$$f(x)=\begin{cases}1, & 0<x<a\\ 0, & x>a\end{cases}$$

(a) $\sqrt{\frac{2}{\pi}}\left(\frac{\cos st}{s}\right)$ (b) $\sqrt{\frac{2}{\pi}}\left(\frac{1-\cos as}{s}\right)$

(c) $\sqrt{2}(1-\cos as)$ (d) None of these

2. Nullity of the matrix $\begin{bmatrix}1 & 0 & 0\\ 5 & 0 & 0\\ 7 & 3 & 4\end{bmatrix}$ is

(a) 1 (b) 2 (c) 3 (d) 4

3. The proposition of $p\wedge(\sim p\vee q)$ is

(a) a tautology
(b) a contradiction
(c) logically equivalent to $p\wedge q$
(d) None of the above

4. If A and B be the sets and A^c and B^c denote the complements of the sets. A and B, then set $(A-B)\cup(B-A)\cup(A\cap B)$ is equal to

(a) $A\cup B$ (b) $A^c\cup B^c$
(c) $A\cap B$ (d) $A^c\cap B^c$

5. Let $G=G(V,E)$ has five vertices, then the maximum number of m of edges in E, if G is a multigraph?

(a) 5 (b) 2
(c) 10 (d) Finite or infinite

6. How many straight lines can be drawn through 10 points on a circle?

(a) 10 (b) 20 (c) 45 (d) Infinite

7. The Fourier transform of unit step function $u(t)$ is

(a) 1 (b) $\pi\delta(\omega)$
(c) $\pi\delta(\omega)-\frac{1}{j\omega}$ (d) $\pi\delta(\omega)+\frac{1}{j\omega}$

8. The value of the integral $\int_{-\infty}^{\infty} e^{-2(x-t)}\delta(t-2)\,dt$ is

(a) $e^{-2(x-2)}$ (b) $e^{2(x-2)}$
(c) $e^{-2(x+2)}$ (d) $e^{2(x+2)}$

9. The maximum and minimum values of envelope of the AM signal be measured with the help of CRO as E_{max} and E_{min} respectively. The modulation index is given by

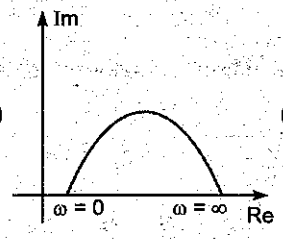
(a) $\frac{E_{max}+E_{min}}{E_{max}-E_{min}}$ (b) $\frac{E_{max}}{E_{min}}$
(c) $\frac{E_{max}-E_{min}}{E_{max}+E_{min}}$ (d) $\frac{E_{min}}{E_{max}}$

10. The unit of $\nabla\times H$ is

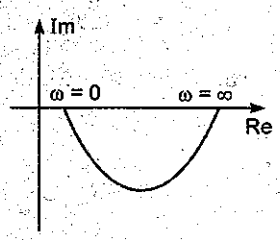
(a) A (b) A/m
(c) A/m^2 (d) A-m

11. Which one of the following corresponds to a lag network?

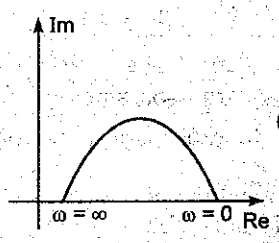
(a)



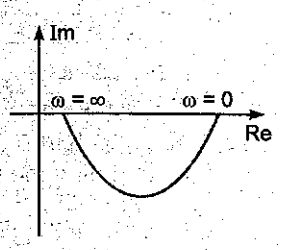
(b)



(c)



(d)

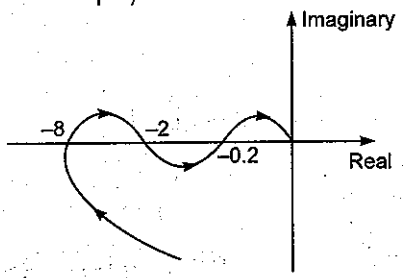


12. Given $A=\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}$, the state transition matrix e^{At} is given by

(a) $\begin{bmatrix}0 & e^{-t}\\ e^{-t} & 0\end{bmatrix}$ (b) $\begin{bmatrix}e^{t} & 0\\ 0 & e^{t}\end{bmatrix}$

(c) $\begin{bmatrix}e^{-t} & 0\\ 0 & e^{-t}\end{bmatrix}$ (d) $\begin{bmatrix}0 & e^{t}\\ e^{t} & 0\end{bmatrix}$

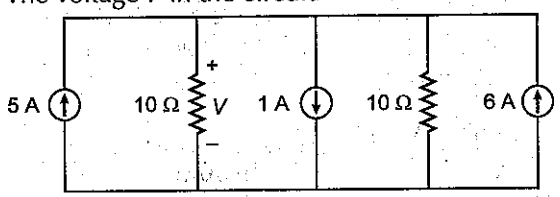
13. The polar diagram of conditionally stable system for open-loop gain $K=1$ is shown in figure The open-loop transfer function of the system is known to be stable. The closed-loop system is stable for



(a) $K<5$ and $\frac{1}{2}<K<\frac{1}{8}$ (b) $K<\frac{1}{8}$ and $\frac{1}{2}<K<5$

(c) $K<\frac{1}{8}$ and $5<K$ (d) $K>\frac{1}{8}$ and $5>K$

14. The voltage V in the circuit



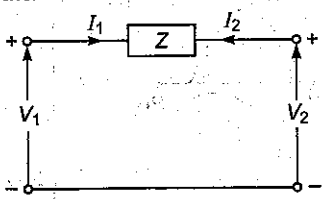
(a) 10 V (b) 50 V (c) 20 V (d) 0 V

15. The two-port network exhibits electrical symmetry, if

(a) $AD-BC=1$ (b) $Z_{12}=Z_{21}$

(c) $h_{12}=-h_{21}$ (d) $A=D$

16. Which of the following parameters doesn't exist for the given circuit?



(a) h-parameter (b) Transmission parameter

(c) z-parameter (d) y-parameter

17. Stringing chart is used in transmission line for

(a) designing the tower

(b) calculating the sag in the conductor

(c) determining the distance between the conductor

(d) the design of insulator string

18. At bridge crossing and near the railway tracks ternary lead alloy cables are used, since

(a) it has low specific gravity

(b) it has high tensile strength

(c) it has low coefficient of thermal expansion

(d) it can withstand shocks and vibration

19. The process of current commutation in a DC machine is opposed by

(a) emf induced in commutating coil

(b) reactance emf

(c) brush resistance

(d) coil resistance

20. In slip test on a synchronous machine, the stator mmf alignment, for maximum current drawn from mains, is

(a) along g-axis (b) along d-axis

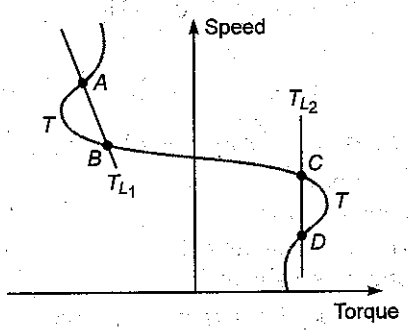
(c) along 45° to g-aixs (d) along 45° to d-axis

21. At low slip, the torque-slip relation is

(a) $T\propto\frac{1}{S^2}$ (b) $T\propto S^2$

(c) $T\propto\frac{1}{S}$ (d) $T\propto S$

22. Figure shows plot of speed *vs* motor and load torques. Then



(a) points A and D unstable, points B and C stable

(b) points A and B unstable, points C and D stable

(c) points B and C unstable, points A and D stable

(d) Can't be determined

23. The armature current and speed of DC separately excited motor will be respectively by halving armature voltage and field current with load torque remaining constant, then

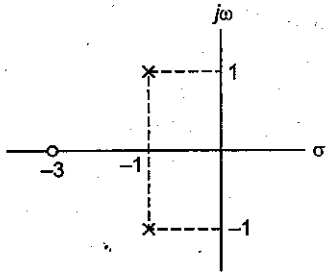
(a) I_a remains same, W_m becomes twice

(b) I_a becomes twice, W_m remains same

(c) I_a, W_m remains uneffected

(d) I_a, W_m both become twice

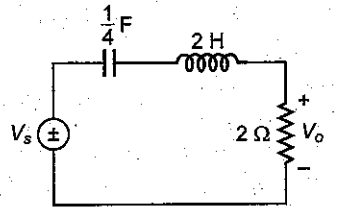
24. The driving point impedance $Z(s)$ of a network has the pole zero location as shown in figure. If $Z(0)=3$, then $Z(s)$ is



(a) $\frac{4(s+3)}{s^2+s+1}$ (b) $\frac{2(s+3)}{s^2+2s+2}$

(c) $\frac{2(s+3)}{s^2+2s+2}$ (d) $\frac{4(s+3)}{s^2+s+2}$

25. In the network of figure, all initial conditions are zero. The damping exhibited by the network is



(a) over damped (b) under damped
(c) critically damped (d) value of voltage requires

2 Marks Questions

26. The value of $y(t)$ as $t \to \infty$ for an initial value of $y(1)=0$ for the differential equation

$$(4t^2+1)\frac{dy}{dt}+8yt-t=0 \text{ is}$$

(a) 1 (b) $\frac{1}{2}$ (c) $\frac{1}{4}$ (d) $\frac{1}{8}$

27. If $(\omega+1)^2=\frac{4}{z}$, the unit circle in the ω-plane corresponds to
(a) ellipse (b) hyperbola
(c) parabola (d) None of these

28. An examination paper has 150 multiple choice questions of one mark each, with each question having four choices. Each incorrect answer fetches 0.25 mark. Suppose 1000 students choose all their answers randomly with uniform probability. The sum total of the expected marks obtained by all these students is
(a) 0 (b) 2550 (c) 7225 (d) 9375

29. For the matrix $P=\begin{bmatrix}3 & -2 & 2\\ 0 & -2 & 1\\ 0 & 0 & 1\end{bmatrix}$, if eigen values is equal to -2, then which of the following is an eigen vector?

(a) $\begin{bmatrix}3\\-2\\1\end{bmatrix}$ (b) $\begin{bmatrix}-3\\2\\-1\end{bmatrix}$ (c) $\begin{bmatrix}1\\-2\\3\end{bmatrix}$ (d) $\begin{bmatrix}2\\5\\0\end{bmatrix}$

30. Use trapezoidal rule to obtain $\int_4^{5.2}\log x\,dx$ using the table given below

x :	4.0	4.2	4.4	4.6	4.8	5.0	5.2
$\log x$:	1.386	1.435	1.482	1.526	1.569	1.609	1.649

(a) 1.8277 (b) 1.9284 (c) 1.6424 (d) 1.9127

31. The function $f(x)=x^5-5x^4+5x^3-1$ has
(a) one minimum and two maxima
(b) two minima and one maximum
(c) two minima and two maxima
(d) one minimum and one maximum

32. Expand this function $\frac{1}{z(e^z-1)}$ for $0<|z|<2\pi$, and obtain coefficients of five terms of the expansion.

(a) $1,-\frac{1}{2},-\frac{1}{12},0,\frac{1}{720}$ (b) $1,-\frac{1}{2},\frac{1}{12},0,\frac{-1}{720}$

(c) $-1,-\frac{1}{2},-\frac{1}{12},0,\frac{1}{720}$ (d) $-1,-\frac{1}{2},\frac{1}{12},0,\frac{-1}{720}$

33. The solution of $\frac{dy}{dx}-y\tan x-y^2\sec x=0$ is given by

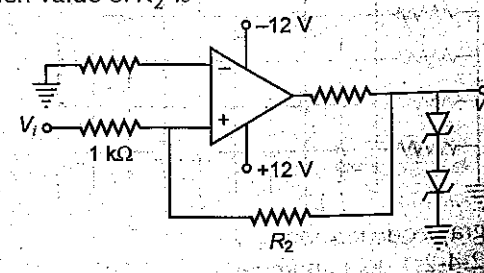
(a) $y^{-1}=\frac{1}{2}\cos x+\frac{1}{2}x\sin x+\frac{1}{2}c$

(b) $y^{-1}=\frac{1}{2}\cos x-\frac{1}{2}(x+c)\sin x$

(c) $y^{-1}=\frac{1}{2}\sin x(x+c)-\frac{1}{2}\cos x$

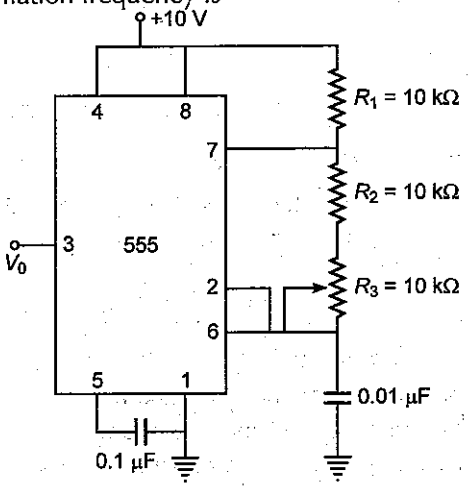
(d) $y^{-1}=\frac{(x+c)}{2}\cos x-\frac{\sin x}{2}$

34. For the comparator circuit shown in figure. The forward diode voltage drop is $V_\gamma=0.7$ V and reverse Zener diode voltage is 5.6 V. If hysteresis width is 1 V, then value of R_2 is



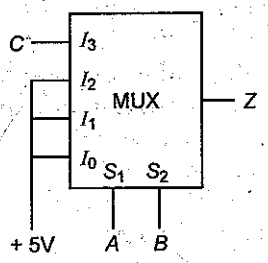
(a) 4 kΩ (b) 12.6 Ω (c) 2 kΩ (d) 6.3 Ω

35. A 555 IC is connected as shown in figure. The range of oscillation frequency is



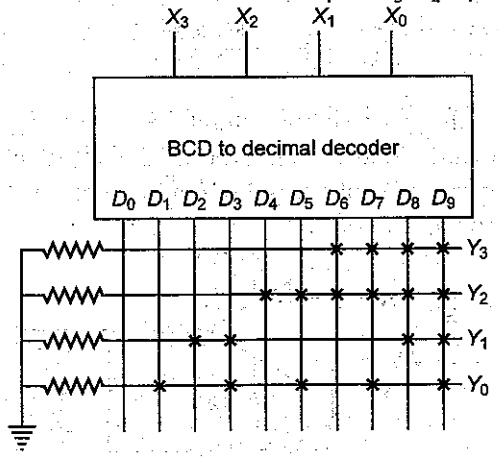
(a) 607 Hz $< f <$ 1.41 kHz
(b) 208 Hz $\le f \le$ 496 Hz
(c) 627 Hz $\le f \le$ 4.81 kHz
(d) 5 kHz $\le f \le$ 9.4 kHz

36. The MUX shown in figure is 4×1 multiplexer, the output Z is



(a) ABC (b) $A \oplus B \oplus C$
(c) $A \odot B \odot C$ (d) $A + B + C$

37. If the inputs X_3, X_2, X_1, X_0 of the ROM in figure are 8-4-2-1 BCD number, then outputs Y_3, Y_2, Y_1 are



(a) gray code number
(b) 2-4-2-1, BCD number
(c) excess 3 code converter
(d) None of the above

38. Consider a unit feedback system with forward path transfer function

$$G(s) = \frac{K(s+3)}{s^4(s+2)}$$

The system is stable for the range of K
(a) $K > 0$ (b) $K < 0$
(c) $K > 1$ (d) always unstable

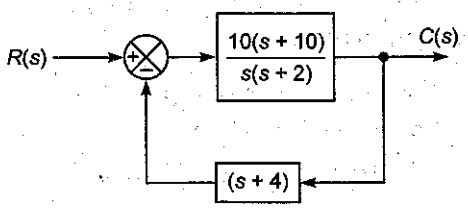
39. A second order system exhibits 100% over-shoot. It's damping coefficient is
(a) 0 (b) 1
(c) less than 1 (d) greater than 1

40. The transfer function $H(s)$ of a stable system is

$$H(s) = \frac{s^2 + 5s - 9}{(s+1)(s^2 - 2s + 10)}$$

The impulse response is
(a) $-e^{-t}\,u(t) + (e^t \sin 3t + 2e^t \cos 3t)\,u(t)$
(b) $-e^{-t}\,u(t) - (e^t \sin 3t + 2e^t \cos 3t)\,u(-t)$
(c) $-e^{-t}\,u(t) - (e^t \sin 3t + 2e^t \cos 3t)\,u(t)$
(d) $-e^{-t}\,u(t) + (e^t \sin 3t + 2e^t \cos 3t)\,u(-t)$

41. For the system shown in figure, the input waveform to yield a constant error is



(a) step (b) ramp
(c) parabolic (d) impulse

42. The state equation of a LTI system is

$$\dot{x} = \begin{bmatrix} 0 & 2 \\ -2 & 0 \end{bmatrix} x + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u$$

The state transition matrix is

(a) $\begin{bmatrix} \cos 2t & \sin 2t \\ -\sin 2t & \cos 2t \end{bmatrix}$ (b) $\begin{bmatrix} \cos 2t & -\sin 2t \\ \sin 2t & \cos 2t \end{bmatrix}$

(c) $\begin{bmatrix} \sin 2t & \cos 2t \\ -\cos 2t & \sin 2t \end{bmatrix}$ (d) $\begin{bmatrix} \sin 2t & -\cos 2t \\ \cos 2t & \sin 2t \end{bmatrix}$

43. A non-uniform linear charge density $\rho_L = 6/(z^2 + 1)$ nC/m lies along the z-axis. The potential at (1, 0, 0) in free space is ($V_\infty = 0$)
(a) 0 (b) 216 V (c) 144 V (d) 108 V

44. An electromagnetic waveform an under water source with perpendicular polarization is incident on a water-air interface at angle 20° with normal to surface.

For water assume $\varepsilon_r = 81$, $\mu_r = 1$. The critical angle θ_c is

(a) 83.62° (b) 6.38°
(c) 42.6° (d) None of these

45. The difference equation representation for a system is

$$Y[n] - \frac{1}{2}Y[n-1] = 2x[n],\ Y[-1] = 3$$

The natural response of system is

(a) $\frac{3}{2}\left(-\frac{1}{2}\right)^n u[n]$ (b) $\frac{2}{3}\left(-\frac{1}{2}\right)^n u[n]$
(c) $\frac{3}{2}\left(\frac{1}{2}\right)^n u[n]$ (d) $\frac{2}{3}\left(\frac{1}{2}\right)^n u[n]$

46. The contents of the accumulator after the execution of the following program will be

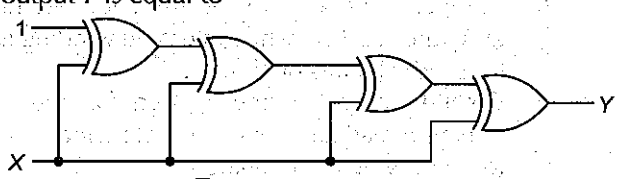
```
MVI A,  C5H
ORA
RAL
```

(a) 45 H (b) C5H
(c) C4H (d) None of these

47. In an 8085 microprocessor, the instruction CMP B has been executed while the contents of accumulator is less than that of register B. As a result carry flag and zero flag will be respectively

(a) set, reset (b) reset, set
(c) reset, reset (d) set, set

48. If the input to the digital circuit shown in figure consisting of a cascade of 20 XOR gate is X, then the output Y is equal to



(a) X (b) $\bar{X}$ (c) 0 (d) 1

49. A computer has the following negative numbers stored in binary form as shown. The wrongly stored number is

(a) – 37 as 1101 1011 (b) – 89 as 1010 0111
(c) – 48 as 1110 1000 (d) – 32 as 1110 0000

50. A lossless transmission line having Surge Impedance Loading (SIL) of 2280 MW is provided with a uniformly distributed series capacitive compensation of 30%. Then, SIL of the compensated transmission line will be

(a) 1835 MW (b) 2280 MW
(c) 2725 MW (d) 3257 MW

51. A lossless power system has to serve a load of 250 MW. There are two generators (G_1 and G_2) in the system with cost curves C_1 and C_2 respectively, defined as follows

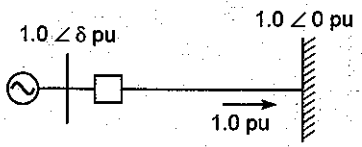
$$C_1(P_{G_1}) = P_{G_1} + 0.055 \times P_{G_1}^2$$

$$C_2(P_{G_2}) = 3P_{G_2} + 0.03 \times P_{G_2}^2$$

where P_{G_1} and P_{G_2} are the MW injections from generators G_1 and G_2 respectively. Thus, the minimum cost dispatch will be

(a) $P_{G_1} = 250$ MW; $P_{G_2} = 0$
(b) $P_{G_1} = 150$ MW; $P_{G_2} = 100$ MW
(c) $P_{G_1} = 100$ MW; $P_{G_2} = 150$ MW
(d) $P_{G_1} = 0$; $P_{G_2} = 250$ MW

52. A lossless single machine infinite bus power system is shown below.



Ths synchronous generator transfers 1.0 per unit of power to the infinite bus. The critical clearing time of circuit breaker is 0.28 s. If another identical synchronous generator is connected in parallel to the existing generator and each generator is scheduled to supply 0.5 per unit of power, then the critical clearing time of the circuit breaker will

(a) reduce to 0.14 s
(b) reduce but will be more than 0.14 s
(c) remain constant at 0.28 s
(d) increase beyond 0.28 s

53. A 230 V, 50 Hz, 4 pole, single-phase induction motor is rotating in the clockwise (forward) direction at a speed of 1425 rpm. If the rotor resistance at standstill is 7.8 Ω, then the effective rotor resistance in the backward branch of the equivalent circuit will be

(a) 2Ω (b) 4Ω (c) 78Ω (d) 156Ω

54. A 400 V, 50 Hz, 30 HP, three-phase induction motor is drawing 50 A current at 0.8 power factor lagging. The stator and rotor copper losses are 1.5 kW and 900 W respectively. The friction and windage losses are 1050 W and the core losses are 1200 W. The air-gap power of the motor will be

(a) 23.06 kW (b) 24.11 kW
(c) 25.01 kW (d) 20.21 kW

55. Which one of the following cannot be the Fourier series expansion of a periodic signal?

(a) $X(t) = 2\cos t + 3\cos 3t$
(b) $X(t) = 2\cos \pi t + 7\cos t$
(c) $X(t) = \cos t + \cos 3t$
(d) $X(t) = 2\cos 1.5\pi t + \sin 3.5\pi t$

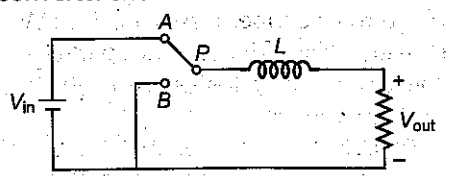
56. Which one of the following is not a property of auto correlation function $[R(\tau)]$?
(a) $R(0) \le R(\tau)$
(b) $R(\tau) = R(-\tau)$
(c) $R(0) = S$ = average power of the waveform
(d) Power spectral density is Fourier transform of auto correlation function for a periodic waveform

57. A system is described by the state equation $\dot{X} = AX + BU$. The output is $Y = CX$, where $A = \begin{bmatrix} -4 & -1 \\ 3 & -1 \end{bmatrix}$, $B = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$, $C = [1, 0]$. The transfer function, $G(s)$ of the system is
(a) $\frac{s}{s^2 + 5s + 7}$ (b) $\frac{1}{s^2 + 5s + 7}$
(c) $\frac{s}{s^2 + 3s + 2}$ (d) $\frac{1}{s^2 + 3s + 2}$

58. The transfer function of a system is $\frac{K}{a_3 s^3 + a_2 s^2 + a_1 s + a_0}$ for the system to be absolutely stable. Then
(a) $a_3, a_2, a_1, a_0 > 0$ and $a_2 a_1 - a_3 a_0 > 0$
(b) $a_3, a_2, a_1, a_0 > 0$ and $a_2 a_1 - a_3 a_0 < 0$
(c) $a_3, a_2, a_1, a_0 > 0$ and $a_2 a_1 - a_3 a_0 = 0$
(d) $a_2, a_0 > 0$ and $a_3, a_1 < 0$

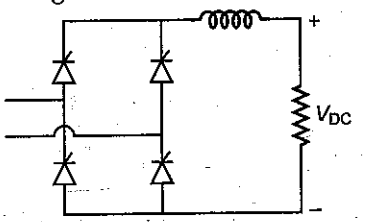
59. The second order system is defined by $\frac{25}{s^2 + 5s + 25}$ is given a step input. The time taken for the output to settle within ± 2% of input is
(a) 1.2 s (b) 1.6 s (c) 2 s (d) 0.4 s

60. The power electronic converter shown in the figure has a single pole double-throw switch. The pole P of the switch is connected alternately to throws A and B. The converter shown is a



(a) step-down chopper (buck converter)
(b) half-wave rectifier
(c) step-up chopper (boost converter)
(d) full-wave rectifier

61. The fully controlled thyristor converter in the figure is fed from a single-phase source. When the firing angle is 0°, the DC output voltage of the converter is 300 V. What will be the output voltage for a firing angle of 60°, assuming continuous conduction?



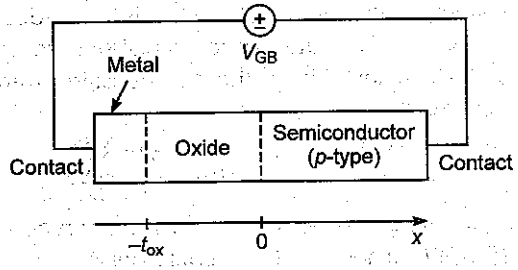
(a) 150 V (b) 210 V (c) 300 V (d) 100 V

62. In transformers, which of the following statements is valid?
(a) In an open circuit test, copper losses are obtained while in short circuit test, core losses are obtained
(b) In an open circuit test, current is drawn at high power factor
(c) In a short circuit test, current is drawn at zero power factor
(d) In an open circuit test, current is drawn at low power factor

63. For a single phase capacitor start induction motor which of the following statements is valid?
(a) The capacitor is used for power factor improvement
(b) The direction of rotation can be changed by reversing the main winding terminals
(c) The direction of rotation cannot be changed
(d) The direction of rotation can be changed by interchanging the supply terminals

64. In a DC machine, which of the following statements is true?
(a) Compensating winding is used for neutralizing armature reaction while interpole winding is used for producing residual flux
(b) Compensating winding is used for neutralizing armature reaction while interpole winding is used for improving commutation
(c) Compensating winding is used for improving commutation while interpole winding is used for neutralizing armature reaction
(d) Compensating winding is used for improving commutation while interpole winding is used for producing residual flux

Statements for Linked Answer Questions 65 and 66
Consider a MOS structure sketched below.

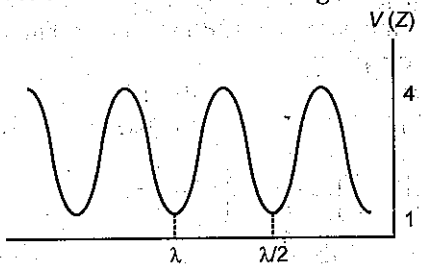


The oxide thickness is $t_{ox} = 50$ nm and the doping level in the substrate is $N_a = 10^{16}\,cm^{-3}$.

65. The hole concentration at $x = 0$ at flat band is
(a) $10^{16}\,cm^{-3}$ (b) $10^{23}\,cm^{-3}$
(c) $3.5\times10^{-15}\,cm^{-3}$ (d) $10^{6}\,cm^{-3}$

66. The hole concentration at $x = 0$ at a condition when the capacitance per unit area of the MOS structure is 50 nF/cm^2, is
(a) $3.5\times10^{-18}\,cm^{-3}$ (b) $3.5\times10^{-15}\,cm^{-3}$
(c) $3.5\times10^{-10}\,cm^{-3}$ (d) $3.5\times10^{-20}\,cm^{-3}$

Statements for Linked Answer Questions 67 and 68
Voltage standing wave pattern in a lossless transmission line with characteristic impedance 50 and a resistive load is shown in figure.



67. The value of the load resistance is
(a) 50 Ω (b) 200 Ω (c) 125 Ω (d) zero

68. The reflection coefficient is given by
(a) – 0.6 (b) – 1 (c) 0.6 (d) zero

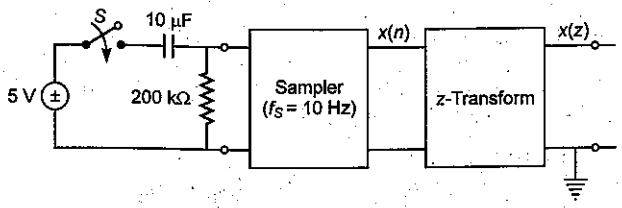
Statements for Linked Answer Questions 69 and 70
The open-loop transfer function of a unity feedback system is given by

$$G(s) = \frac{3e^{-2s}}{s(s+2)}$$

69. The gain and phase cross-over frequencies in rad/s are respectively,
(a) 0.632 and 1.26 (b) 0.632 and 0.485
(c) 0.485 and 0.632 (d) 1.26 and 0.632

70. Based on the above results, the gain and phase margins of the system will be
(a) – 7.09 dB and 87.5°
(b) 7.09 dB and 87.5°
(c) 7.09 dB and – 87.5°
(d) – 7.09 dB and – 87.5°

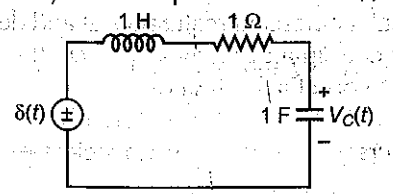
Statements for Linked Answer Questions 71 and 72
In the following network, the switch is closed at $t = 0^-$ and the sampling starts from $t = 0$. The sampling frequency is 10 Hz.



71. The samples $x(n), n = (0, 1, 2, \ldots)$ are given by
(a) $5(1-e^{-0.05n})$ (b) $5e^{-0.05n}$
(c) $5(1-e^{-5n})$ (d) $5e^{-5n}$

72. The expression and the region of convergence of the z-transform of the sampled signal are
(a) $\frac{5z}{z-e^{5}}, |z| < e^{-5}$
(b) $\frac{5z}{z-e^{-0.05}}, |z| < e^{-0.05}$
(c) $\frac{5z}{z-e^{-0.05}}, |z| > e^{-0.05}$
(d) $\frac{5z}{z-e^{-5}}, |z| > e^{-5}$

The following series R-L-C circuit with zero conditions is excited by a unit impulse function $\delta(t)$.



73. For $t > 0$, the output voltage $V_C(t)$ is
(a) $\frac{2}{\sqrt{3}}[e^{-1/2t} - e^{\sqrt{3}/2t}]$ (b) $\frac{2}{\sqrt{3}} e^{\frac{-1}{2}t}$
(c) $\frac{2}{\sqrt{3}} e^{-1/2t} \cos\left(\frac{\sqrt{3}}{2}t\right)$ (d) $\frac{2}{\sqrt{3}} e^{-1/2t} \sin\left(\frac{\sqrt{3}}{2}t\right)$

74. For $t > 0$, the voltage across the resistor is
(a) $\frac{1}{\sqrt{3}}[e^{\sqrt{3}/2t} - e^{-1/2t}]$
(b) $e^{-1/2t}\left[\cos\left(\frac{\sqrt{3}}{2}t\right) - \frac{1}{\sqrt{3}}\sin\left(\frac{\sqrt{3}t}{2}\right)\right]$
(c) $\frac{2}{\sqrt{3}} e^{-1/2t} \sin\left(\frac{\sqrt{3}t}{2}\right)$
(d) $\frac{2}{\sqrt{3}} e^{-1/2t} \cos\left(\frac{\sqrt{3}}{2}t\right)$

75. The system with the open-loop transfer function $G(s)\cdot H(s) = \frac{1}{s(s^2+s+1)}$ has a gain margin of
(a) – 6 dB (b) zero (c) 35 dB (d) 6 dB

General Aptitude

76. Which of the following options is farthest in meaning to the word given below?

Gloomy

(a) Discouraging (b) Disguising
(c) Bright (d) Tragic

77. The question below consists of a pair of related words followed by four pairs of words. Select the pair which best expresses the relation in the original pair.

Paper : Blackboard

(a) Filter : Air (b) Pencil : Pen
(c) Eraser : Chalk (d) Polish : Shoe

78. Choose the most appropriate word from the options given below to complete the following sentence.

It has been argued that politics as ....., whatever its transcendental claims, has always been the systematic organization of common hatreds.

(a) a theory (b) an ideal
(c) a practice (d) a contest

79. Choose the most appropriate word from the options given below to complete the following sentence.

Although adolescent maturational and developmental states occur in an orderly sequence, their timing with regard to onset and duration.

(a) lasts (b) carries
(c) falters (d) accelerates

80. The annual salaries of Mr. John, Mr. Adam and Mr. Joe are in the ratio 2 : 3 : 5. If the salary of Mr. Joe is ₹ 90000 more than that of Mr. John, then the monthly salary of Mr. Adam is

(a) ₹ 7500 (b) ₹ 75000
(c) ₹ 90000 (d) None of these

81. The ice on the front windshield of the car had formed when moisture condensed during the night. The ice melted quickly after the car was warmed up the next morning because the defrosting vent, which blows only on the front windshield, was turned on full force.

Which of the following, if true, most seriously jeopardizes the validity of the explanation for the speed with which the ice melted?

(a) The side windows have no ice condensation on them
(b) Even though no attempt was made to defrost the back window, the ice there melted at the same rate as did the ice on the front windshield
(c) The speed at which ice on a window melts increases as the temperature of the air blown on the window increases
(d) The warm air from the defrosting vent for the front windshield cools rapidly as it dissipates throughout the rest of the car

82. The cost of turfing a uniform circular road round a circular garden at ₹ 20 per sq. meter is ₹ 21560 and the area of the garden is 1386 sq. meter. Find the breadth of the circular road.

(a) 7 m (b) 14 m
(c) 10 m (d) None of these

83. $\left[1-\frac{1}{3}\right]\left[1-\frac{1}{4}\right]\left[1-\frac{1}{5}\right]\ldots\left[1-\frac{1}{n}\right]=?$

(a) $\frac{2}{n}$ (b) $\frac{1}{n}$ (c) $\frac{3}{n}$ (d) $n-\frac{1}{n}$

84. Two casks of liquid contain liquid of mixture of liquids A and B. In the first cask $A : B :: 7 : 3$ and in the second cask $A : B :: 3:1$. In what ratio must the mixtures from the two casks be taken to give a mixture in which $A : B :: 11: 4$?

(a) 1 : 2 (b) 1 : 3
(c) 2 : 3 (d) 2 : 1

85. A and B can complete a piece of work in 80 and 120 days respectively. They together start the work but A left after 20 days. After another 12 days C joined B and now they complete the work in 28 more days. In how many days C can complete the work, working alone?

(a) 92 days (b) 102 days
(c) 112 days (d) 124 days

Answers

1. (b)	**2.** (a)	**3.** (c)	**4.** (a)	**5.** (d)	**6.** (c)	**7.** (c)	**8.** (a)	**9.** (c)	**10.** (c)
11. (d)	**12.** (b)	**13.** (b)	**14.** (b)	**15.** (d)	**16.** (c)	**17.** (b)	**18.** (a)	**19.** (b)	**20.** (a)
21. (d)	**22.** (a)	**23.** (b)	**24.** (b)	**25.** (b)	**26.** (d)	**27.** (c)	**28.** (d)	**29.** (a)	**30.** (a)
31. (d)	**32.** (b)	**33.** (b)	**34.** (b)	**35.** (c)	**36.** (d)	**37.** (b)	**38.** (d)	**39.** (a)	**40.** (d)
41. (a)	**42.** (a)	**43.** (d)	**44.** (b)	**45.** (c)	**46.** (a)	**47.** (a)	**48.** (d)	**49.** (c)	**50.** (b)
51. (c)	**52.** (b)	**53.** (a)	**54.** (c)	**55.** (b)	**56.** (a)	**57.** (a)	**58.** (a)	**59.** (b)	**60.** (a)
61. (a)	**62.** (d)	**63.** (b)	**64.** (b)	**65.** (a)	**66.** (b)	**67.** (c)	**68.** (a)	**69.** (d)	**70.** (d)
71. (b)	**72.** (c)	**73.** (d)	**74.** (b)	**75.** (b)	**76.** (c)	**77.** (c)	**78.** (c)	**79.** (b)	**80.** (c)
81. (b)	**82.** (a)	**83.** (a)	**84.** (a)	**85.** (c)					